QUESTION 1

Starting with the value of 0, add the value of each true statement and subtract the value of each false statement below to compute the final answer.

(729) In a cyclic quadrilateral, the sine of the adjacent angles always sum to 0.

(10) Given that v and w are four-dimensional vectors, v · w = $|v||w|\sin(\theta)$.

(23) A non-singular matrix has a nonzero determinant.

(2) The polar graph of $r = 10 - 10\sin(\theta)$ can be classified as a cardioid.

(9) The distance between line $3x + 4y + 12z = 19$ and point (1, 2, 5) is 4.

(1000) The rectangular coordinate (6, -6$\sqrt{3}$) is equivalent to the polar coordinate (-12, $\frac{2\pi}{3}$).

(573) A probability vector's components add up to 1.

QUESTION 2

Given $f(x) = -3\cos(x\pi) + 1$, let:

$$\begin{aligned} A &= \text{the frequency of } f(x) \\ B &= \text{the sum of the minimum and maximum values of } f(x) \\ C &= \text{the amplitude of } f(x) \\ D &= \text{the vertical shift of } f(x) \end{aligned}$$

Find $ABCD$.

QUESTION 3

Given that all angles are between 0 and 2π, let:

$$\begin{aligned} X &= \text{an angle in the third quadrant such that } \tan(X) = \frac{7}{24} \\ Y &= \text{an angle in the second quadrant such that } \cos(Y) = -\frac{9}{15} \\ Z &= \sin(243^\circ) + \sin(117^\circ) \end{aligned}$$

Find $\cos(X+Y) + \sqrt{2}\sin(\frac{X}{2})$ - Z.

QUESTION 4

Let:

$$\begin{aligned}
R &= \text{the sum of the components of the resulting vector of } <2,6,1> + <3,9,4> \\
I &= \text{the magnitude of } <7,24,25> \\
C &= \text{the dot product of vectors } <10,0,8> \text{ and } <1,9,5> \\
K &= \text{the volume of a tetrahedron with vertices } (0,0,0),\ (3,2,7),\ (1,4,5), \text{ and } (5,1,8)
\end{aligned}$$

Find $\dfrac{RI^2}{25}$ + CK.

QUESTION 5

Given ellipse M has equation $5x^2 + 4y^2 - 25x + 4y + \frac{49}{4} = 0$ and parabola N has equation $3x^2 - 4x + \frac{4}{3} + 6y = 0$, let:

$$\begin{aligned} A &= \text{the eccentricity of } M \text{ squared} \\ B &= \text{the eccentricity of } N \\ C &= \text{the length of the focal radius of } N \\ D &= \text{the sum of the coordinates of the center of } M \end{aligned}$$

Order these letters from greatest to least.

QUESTION 6

The following question is a relay-styled question. As you solve each part, use that answer to solve the next part. Assume all angles are between 0 and 2π.

Sanjita, the functions enthusiast, writes down $f(x) = \dfrac{x^4 - 3x^3 + 3x^2 - x}{48x^3 - 52x^2 + 18x - 2}$. Rohan, the annoying Sanjita enthusiast, asks her to solve for the number of asymptotes of $f(x)$. Given she gets the question correct, let A equal the answer Sanjita tells Rohan.

Nihar is angry that Tanvi and Tanusri keep talking, so he tells them to evaluate $\dfrac{(-1 - i\sqrt{3})^A}{(1+i)^2}$ in order to silence them. They find that the correct answer is $(n)(\text{cis } B)$.

Deekshita, bored out of her mind, draws a triangle and labels the vertices X, Y, and Z. Segment XY has a length of 6, segment XZ has a length of 8, and the sum of $\angle$XYZ and $\angle$XZY is equal to B in degrees.

Find the area of $\triangle XYZ$.

QUESTION 7

Josh has a special coin that has a $\frac{3}{4}$ probability of landing on heads. If the coin is flipped 10 times, let the probability that the total number of times the coin lands on heads is less than 3 be $\frac{A}{2^B}$. Assume that the fraction is in simplest form.

Akash and Vishnav, best buds, have a total of 15 distinguishable pieces of candy. They're feeling quite generous and decide to give away all of their candy to their 5 other friends. Let C equal the number of ways there are to distribute the pieces of candy among the friends, given that everyone of them must receive at least one piece of candy.

Compute AB + C.

QUESTION 8

Let:

$$\begin{aligned}
A &= \text{the number of petals on the graph } r = 3\cos(10\theta) \\
B &= \text{cosine of the angle between plane } 8x + 6y + 10z = 2 \text{ and plane } 3x + 4y + 5z = 7 \\
C &= \text{the area of } 4x^2 - 3xy + 2y^2 = 1 \\
D &= \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}, \text{ where } a, b, c, \text{ and } d \text{ are the roots of } y = 2x^4 + 14x^2 - 10x + 1
\end{aligned}$$

Find 50(A + B + D) + C.

QUESTION 9

Let:

$$\begin{aligned}
A &= \lim_{x\to\infty}(1+\frac{3}{x})^x \\
B &= \lim_{x\to 3}\frac{x^3-10x^2+27x-18}{x^2-7x+12} \\
C &= \lim_{x\to 81}\frac{3-\sqrt{x}}{2\sqrt{3}-2x^{1/4}} \\
D &= \sum_{x=0}^{\infty}\frac{7^x}{x!}
\end{aligned}$$

Find $A+B+C+D$.

QUESTION 10

Given $x = 3\cos(\theta)$ and $y = 2\sin(\theta)$, let:

$$
\begin{aligned}
A &= \text{the area of the figure} \\
B &= \text{the length of the major axis} \\
C &= \text{the sum of the coordinates of the focus with the greater abscissa} \\
D &= \text{the eccentricity of the figure}
\end{aligned}
$$

Find $AB + CD$.

QUESTION 11

Let:

$$
\begin{aligned}
A &= \text{the distance between polar coordinates X and Y, given that X} = (2,\, 30^\circ) \text{ and Y} = (8,\, 150^\circ) \\
B &= \text{the sum of } m \text{ and } n\text{, given that } i - 2i^2 + 3i^3 - \ldots + 2019i^{2019} = m + ni \\
C &= \text{the sum of the coordinates when the polar coordinate } (6,\, 60^\circ) \text{ is written as a rectangular coordinate} \\
D &= |z|\text{, given that } z = \frac{5-3i}{2+4i}
\end{aligned}
$$

Compute $A^2 + BC + D^2$.

QUESTION 12

Given:

$$M = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 0 & 7 \\ 8 & 6 & 5 \end{bmatrix}$$

Let:

$$\begin{aligned} A &= \text{tangent of twice the angle of the counterclockwise rotation to remove the } xy \text{ term from } 3x^2 - 10xy + 2y^2 + 9y - 1 = 0 \\ B &= \text{tangent of the angle between lines } 2y + 6x = 4 \text{ and } 5x + y = 7 \\ C &= \text{the product of the eigenvalues of matrix M} \\ D &= \text{the determinant of the inverse of matrix M} \end{aligned}$$

Calculate A - $\frac{1}{B}$ + CD.

QUESTION 13

Given that $\log(2) = 0.3$ and $\log(3) = 0.48$, and no further approximations are made, let:

$$\begin{aligned} A &= \log(150) \\ B &= \log_4(54) \\ C &= \log_9(288) \\ D &= \log(360) \end{aligned}$$

Compute $5000(A + B + C + D)$.

QUESTION 14

Evaluate:

$$\begin{vmatrix} 5 & 4 & 7 & 9 \\ 2 & 4 & 1 & 2 \\ 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & 3 \end{vmatrix}$$