

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Tanvi is mowing her lawn, when Tanmay walks out of his house and realizes that she had mowed the lawn in the shape of a polar curve. Tanvi realizes that the curve had an equation of $r = 7 - 5\sin(\theta)$. Identify which type of curve Tanvi accidentally mowed into her lawn.

(A) Lemniscate (B) Cardioid (C) Dimpled Limacon (D) Convex Limacon (E) NOTA

2. Evaluate $(2i - \sqrt{2})^5$.

(A) $-88i - 4\sqrt{2}$ (B) $-88i + 4\sqrt{2}$ (C) $88i - 4\sqrt{2}$ (D) $88i + 4\sqrt{2}$ (E) NOTA

3. Akash and Vishnav enjoy the outdoors, where they often take long walks in straight lines across Tallahassee, which can be modeled as a Cartesian plane. Akash walks along a path on line $2x + y = 1$ and Vishnav walks along a path on line $9x + 3y = 27$. Find the cosine of the angle between their two paths, assuming both of their paths extend infinitely across the plane.

(A) $\frac{7\sqrt{50}}{50}$ (B) $\frac{1}{7}$ (C) $\frac{1}{15}$ (D) $\frac{\sqrt{50}}{50}$ (E) NOTA

4. Sina is answering a question which results in the complex number $16\sqrt{3} - 16i$. As a test, Josh quizzes Sina and asks him what all of the 5th roots of this complex number are. However, Sina only names 4 of them correctly. Which of the following could be the 5th root that Sina incorrectly said?

(A) $2\text{cis}\left(\frac{11\pi}{30}\right)$ (B) $2e^{\frac{47i\pi}{30}}$ (C) $2\text{cis}\left(\frac{53\pi}{30}\right)$ (D) $2\text{cis}\left(\frac{59\pi}{30}\right)$ (E) NOTA

5. When simplified, $\frac{3x - 8}{x^2 - 3x + 2}$ can be expressed as $\frac{A}{B} - \frac{C}{D}$ where A and C are positive integers and B and D are linear expressions with integer coefficients. Compute $A - B + C + D$.

(A) 1 (B) 2 (C) 4 (D) 6 (E) NOTA

6. Sanjita hates riding on roller coasters, but was forced onto the Euclinator by her friends. To get her mind off of the ride, she decides to think about the roller coaster as a graph, as it resembled the function $y = 28\cos(\theta) + 45\sin(\theta) + 131$. Given the function found by Sanjita is in feet, find the maximum height of the roller coaster.

(A) 146 (B) 184 (C) 327 (D) 328 (E) NOTA

7. Shreyas is really hungry and asks Tanusri for a Bagel Bite. Tanusri tells Shreyas that he can have a Bagel Bite if he can find the sum of

$$3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120} \dots$$

Given that Shreyas is a genius and answers every math question correctly when hungry, which of the following is his answer to Tanusri?

(A) $3e$ (B) $\frac{111}{2}$ (C) $\frac{113}{2}$ (D) e^3 (E) NOTA

8. Find the sum of the eigenvalues of the following 4x4 matrix:

$$\begin{bmatrix} 11 & -72 & 28 & 18 \\ 1 & 12 & -129 & 0 \\ -21 & -2 & 8 & 15 \\ 24 & 32 & -3 & 0 \end{bmatrix}$$

(A) 31 (B) 59 (C) 92 (D) 118 (E) NOTA

9. Every Wednesday, Nihar and Deekshita go to Lucky Goat at a random time between 1 PM and 6 PM. Given Nihar and Deekshita spend exactly 45 minutes and 105 minutes there, respectively, following their independent random arrivals at the coffee house, what is the probability they will be there at the same time?

(A) $\frac{17}{40}$ (B) $\frac{171}{400}$ (C) $\frac{23}{40}$ (D) $\frac{229}{400}$ (E) NOTA

10. How many petals does the rose curve $r = 8 \cos(15\theta)$ have?

(A) 8 (B) 15 (C) 16 (D) 30 (E) NOTA

11. Rohan is a true artist who often fills the boards with drawings of his favorite conic, the ellipse. Which of the following could be a possible equation for his drawings?

(A) $5x^2 - 8xy + 3y^2 + 17x - 82y = 14$ (B) $2x^2 + 10xy + 8y^2 + 9x - 48y - 21 = -4$
 (C) $2x^2 - 18xy - 3y^2 - 2x - 12y - 15 = 12$ (D) $7x^2 + 6xy + 2y^2 - 13x + 18y = 16$ (E) NOTA

12. Evaluate

$$\lim_{x \rightarrow -2} \frac{2x^4 + x^3 - 8x^2 - x + 6}{x + 2}$$

(A) -21 (B) -18 (C) 0 (D) 18 (E) NOTA

13. One day Vishal was playing with his Lincoln Logs when his brother Viswa ridiculed him for playing with kids toys. To display the power of his Lincoln Logs, he asks Viswa a question involving logs: "Given $x = \frac{\pi}{8}$, what is $\log_2(-\cos 5x) + \log_2(\sin 3x)$?" Help Viswa prove the lameness of Lincoln Logs by finding the answer to Vishal's question.

(A) -2 (B) $-\frac{3}{2}$ (C) -1 (D) 1 (E) NOTA

14. Anurag and Vamsi walk into Ms. Pickett's class and see the following math problem on the white board:

$$\text{Given } f(3) = 18, f(4) = 23, f(5) = 37, \text{ and } f(6) = 49, \text{ find } f(7).$$

As they are extremely competitive individuals, they decide to have a race to see who can do the question the fastest. Unfortunately, they are not very good at math, so they both end up finding the wrong answer. It turns out Anurag found the degree of the function $f(x)$ and Vamsi miscalculated and ended up with a number 2 greater than the correct answer. Find the product of Anurag's answer, Vamsi's answer, and the correct answer.

(A) 150 (B) 2400 (C) 4800 (D) 7200 (E) NOTA

15. Find the area of the polar curve: $r^2 = \frac{64}{4 \cos^2 \theta + 9 \sin^2 \theta}$.

(A) 10π (B) 12π (C) 15π (D) 16π (E) NOTA

16. Rayyan, Rahul, and Anirudh are all obsessed with vectors, each with their own favorite 3 dimensional vector, $\langle -3, 9, 1 \rangle$, $\langle 10, 5, -12 \rangle$, and $\langle 8, -7, 0 \rangle$ respectively. As all three of them are huge nerds they decided to entertain themselves with a problem using their three favorite vectors. They looked to find the dot product of the vector orthogonal to both Rayyan's and Anirudh's favorite vectors, and Rahul's favorite vector. Which of the following is the answer to their question?

(A) -720 (B) -718 (C) 720 (D) 722 (E) NOTA

17. Dylan is quite an odd guy; he refuses to do any math question that does not involve an odd function. Accordingly, Mr. Juhasz decided to give Dylan a math question that he would do. He asked, "At what y coordinate does $x = 0.5$ intersects the odd function $f(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$, given that A, B, C, D, E , and F are all integers and $f(x)$ goes through points $(-1, -18)$, $(2, -48)$, and $(3, 138)$?" Given Dylan always gets any question involving an odd function correct, which of the following is his answer?

- (A) $\frac{91}{18}$ (B) $\frac{95}{18}$ (C) $\frac{83}{16}$ (D) $\frac{87}{16}$ (E) NOTA

18. Farzan, Mihir and Prabhas are all addicted to Doritos. As a gift, Ms. Pickett buys 7 Nacho Cheese Doritos bags, 9 Cool Ranch Doritos bags, and 4 Spicy Sweet Chili Doritos bags for them. She then randomly splits the bags of chips between the three Doritos enthusiasts. In how many different ways could the chips bags have been split up, given each one of them did not necessarily receive a bag and Mihir got at least one of his two favorite flavors (Nacho Cheese and Spicy Sweet Chili) and no Cool Ranch?

- (A) 2100 (B) 2500 (C) 2800 (D) 3600 (E) NOTA

19. Karthik, Shubham, and Shrung are all on the same plane located at points $(2, -9, 5)$, $(-4, 6, 10)$, and $(-1, 17, 7)$, respectively. Which of the following is the equation of this plane?

- (A) $70x + 12y + 131z = 668$ (B) $100x + 3y + 111z = 728$ (C) $-3x + 14y + 22z = 388$
 (D) $30x - 9y - 20z = 60$ (E) NOTA

20. How many times does $y = \frac{x-1}{14}$ intersect the graph of $y = \sin(x)$?

- (A) 5 (B) 6 (C) 7 (D) 11 (E) NOTA

21. Find the area of the dodecagon enclosed by the 12th roots of $2048i\sqrt{3} - 2048$ on the Argand plane.

- (A) 12 (B) $12\sqrt{3}$ (C) 15 (D) 18 (E) NOTA

22. Find the sine of the angle between lines A and B defined by the following parametric equations:

$$A = \begin{cases} x = -3t - 7 \\ y = 23 - 2t \\ z = t + 11 \end{cases}$$

$$B = \begin{cases} x = 13 - 4t \\ y = -t - 8 \\ z = 5t \end{cases}$$

- (A) $\sqrt{42}$ (B) $\frac{\sqrt{681}}{42}$ (C) $3\sqrt{227}$ (D) $\frac{\sqrt{42}}{42}$ (E) NOTA

23. Given $\sin \theta = \frac{\sqrt{3}}{5}$, find the square root of $4\sin^8 \theta + 4\sin^4 \theta \cos^2 \theta + \cos^4 \theta$.

- (A) $\frac{\sqrt{3}}{25}$ (B) $\frac{\sqrt{25}}{25}$ (C) $\frac{2\sqrt{17}}{25}$ (D) $\frac{568}{625}$ (E) NOTA

24. There are three tangential circles of distinct sizes centered at A, B , and C . Given circle A has a radius of 7, circle B has a radius of 4, and $\triangle ABC$ has an area of $2\sqrt{182}$, find the radius of circle C .

- (A) 1 (B) $\sqrt{2}$ (C) 2 (D) 3 (E) NOTA

25. Hitesh decides to invest all his betting earnings at a continuously compounded interest rate of 18.75%. Five years later, Tanusri, annoyed with Hitesh constantly talking about how much money he has made, decides to give him a question that will keep him occupied. She asks him to calculate the natural log of the quotient obtained by dividing the total amount he has now by his initial investment. Given Hitesh's initial investment was \$4,500 and he does not have any money other than his betting earnings, what is the answer to Tanusri's question to the nearest hundredths?
- (A) 0.94 (B) 0.95 (C) 0.96 (D) 0.97 (E) NOTA
26. What is the volume of a parallelepiped with vertices $(1, 3, 5)$, $(2, 0, 4)$, $(4, 1, 3)$, and $(3, 5, 2)$?
- (A) 15 (B) 36 (C) 42 (D) 54 (E) NOTA
27. The 6th roots of $2048i\sqrt{2} - 2048\sqrt{2}$ are graphed on the Argand plane. These points are labelled A, B, C, D, E , and F going clockwise such that C and D fall in Quadrant I. What is the distance between B and E ?
- (A) 4 (B) $4\sqrt{2}$ (C) 8 (D) $8\sqrt{2}$ (E) NOTA
28. Two ladders of distinct length lay on the side of a wall, both touching the wall on opposite sides at the same elevation. The shorter ladder has a length of 24 feet and is elevated off the ground at an angle of 48.59 degrees, while the longer ladder is elevated off the ground at an angle of 17.46°. Given A equals the distance between the point at which the shorter ladder touches the ground and the wall, and B equals the distance between the point at which the longer ladder touches the ground and the wall, find $\frac{A}{B}$. ($\sin 17.46^\circ = 0.3$, $\sin 48.59^\circ = 0.75$)
- (A) $\frac{\sqrt{13}}{13}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{2}{5}$ (D) $\frac{\sqrt{10}}{5}$ (E) NOTA
29. Carson's favorite drinks are apple juice and orange juice. One day he decides to have a drink and pours himself a glass full of apple juice (glass A) and two thirds of a glass of orange juice (glass B). After drinking three fourths of his apple juice in glass A , he decides to experiment some variations. He first pours half of his juice from glass B into glass A . Following a taste of one fifth of this new mixture in glass A , he decides its not up to his juice standards. He then pours half of this fruit juice mixture from glass A to glass B . After drinking a third of the juice in glass B , Carson rejoices as he has created the perfect drink, and puts the remains of glass B into the fridge. Let M equal the proportion of apple juice that he has drank so far, and let N equal the proportion of apple juice in the perfect drink. Find $M - N$.
- (A) $\frac{1}{2}$ (B) $\frac{124}{325}$ (C) $\frac{198}{325}$ (D) $\frac{2}{3}$ (E) NOTA
30. Given that all angles are in degrees, evaluate:
- $$\sin(30) \cot(15) \sin(165) \tan(135) \cos(3240)$$
- (A) $-\sqrt{2 + \sqrt{3}}$ (B) $-\sqrt{2}$ (C) $-\frac{\sqrt{2 + \sqrt{3}}}{4}$ (D) $-\frac{\sqrt{2}}{4}$ (E) NOTA