

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Evaluate

$$\int_0^{\pi} \frac{x \sin(x)}{1 + \cos^2(x)} dx$$

- (A) $\frac{\pi^2}{2}$ (B) $\frac{\pi^2}{4}$ (C) $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{16}$ (E) NOTA

2. Evaluate

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \sin^2\left(\frac{x}{2}\right) dx$$

- (A) $\frac{4+\sqrt{2}-4\pi}{8}$ (B) $\frac{4-2\sqrt{2}-\pi}{8}$ (C) $\frac{-2-\pi}{8}$ (D) $\frac{-2\sqrt{2}-\pi}{8}$ (E) NOTA

3. Vamsi and Akash are obsessed with watching TV shows. They recently began watching a new show called “Vector Vampires” in which each character is a blood sucking vector. Vamsi’s favorite character is \vec{a} and Akash’s favorite character is \vec{b} . Given $\vec{a} = 5i - 3j + 4k$ and $\vec{b} = i + 7j - 4k$, find the vector projection of Akash’s favorite character vector onto Vamsi’s.

- (A) $\left\langle -\frac{16}{5}, \frac{48}{25}, -\frac{64}{25} \right\rangle$ (B) $\left\langle \frac{16}{5}, -\frac{48}{25}, \frac{64}{25} \right\rangle$ (C) $\left\langle -\frac{16}{33}, -\frac{112}{33}, \frac{64}{33} \right\rangle$ (D) $\left\langle \frac{16}{33}, \frac{112}{33}, -\frac{64}{33} \right\rangle$ (E) NOTA

4. One day Vamsi and Akash got into an intense argument over whose favorite character truly is the best “Vector Vampire.” They decide the only just way to decide is to compare their magnitudes. Using the vectors detailed in the previous question, find whose favorite character vector has a higher magnitude.

- (A) Vamsi’s (B) Akash’s (C) Both have the same magnitude
(D) Cannot be determined (E) NOTA

5. Mihir and Farzan are learning the art of trigonometry, but Farzan boasts that he is already the Trigonometry Boss. In the middle of one of Farzan’s spiels on his talents, Mihir asks him to prove himself by evaluating the expression:

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

Given Farzan truly is the Trigonometry Boss and correctly evaluates all expressions involving trigonometric functions, what is Farzan’s response to Mihir’s test question?

- (A) $\frac{2}{\sin^2 2x}$ (B) $\frac{4}{\sin^2 2x}$ (C) $\frac{2}{\sin^2 x}$ (D) $\frac{4}{\sin^2 x}$ (E) NOTA

6. Find the product of the amplitude and the phase shift of the equation $y = \frac{1}{2} \sin(5x - 5) + 2$.

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) NOTA

7. Identify which polar curve is represented by the following equation.

$$r = 2 - 4 \cos x$$

- (A) Cardioid (B) Rose Curve (C) Lemniscate (D) Trisectrix (E) NOTA

8. What is the area of the polygon that is formed by the solutions to the equation $z^3 = 8$ when graphed on the complex plane.

- (A) $2\sqrt{2}$ (B) 2π (C) 3π (D) $3\sqrt{3}$ (E) NOTA

9. Anurag and Vamsi are both huge Keanu Reeves fans, but interestingly, they hate *The Matrix*. Their hatred motivates them to correctly inverse any and every matrix they come across. Upon finding out about Anurag and Vamsi's unusually tendency, Prem decided to use their hatred to annoy them. In order to irritate them, Prem confronted them with the following matrix and asked for the number in the 1st row and 3rd column of its inverse.

$$\begin{bmatrix} 5 & -6 & 2 \\ 4 & 3 & -3 \\ 6 & 6 & 5 \end{bmatrix}$$

Help the two Keanu Reeves fans by correctly answering Prem's question for them.

- (A) $-\frac{2}{135}$ (B) $\frac{2}{135}$ (C) $\frac{4}{135}$ (D) $-\frac{4}{135}$ (E) NOTA
10. What is the unit vector in the direction of the vector $\langle 9, 40 \rangle$?
- (A) $\langle 9, 41 \rangle$ (B) $\langle 1, 1 \rangle$ (C) $\langle \frac{1}{9}, \frac{1}{40} \rangle$ (D) $\langle \frac{9}{41}, \frac{40}{41} \rangle$ (E) NOTA
11. Tanusri wants to eat a cookie, but no one in class has cookies for her to take. To relieve herself from despair, Tanusri throws the table in front of her. Given that she threw the table with a constant force of 523 N for a duration of 1.5 s, what was the impulse from the throw? Answers are expressed in $N \cdot s$. (Hint: Impulse = the integral of a force over the time interval for which it acts)
- (A) 523 (B) 784.5 (C) 0 (D) -523 (E) NOTA
12. In his quest to become the UFC Bantamweight Champion, Shubham drinks masala chaas from an inverted cone that was originally filled to the brim. Given that the cone is 40 m tall, has an opening radius of 16 m, and Shubham drinks the masala chaas at a rate of $20 \frac{m^3}{s}$, what is the rate at which the masala chaas level is decreasing when it is at half of its original height? Answers are expressed in $\frac{m^3}{s}$.
- (A) $\frac{5\pi}{8}$ (B) $\frac{15\pi}{12}$ (C) $\frac{5\pi}{12}$ (D) $\frac{5\pi}{16}$ (E) NOTA
13. Tanvi is obsessed with the function $\tan(v)$. Evaluate

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{12}} \left(\tan(v) + \frac{1}{\tan(v)} \right) dv$$

- (A) $\ln(2)$ (B) $\ln(2 + \sqrt{3})$ (C) $\ln\left(\frac{1}{2}\right)$ (D) $\ln\left(\frac{\sqrt{3}}{2}\right)$ (E) NOTA
14. Sanjita realizes she actually doesn't like $\tan(v)$ very much and falls in love with the function $\text{sanj}(y)$. Given that $\text{sanj}(y) = \frac{\cosh(y)}{\sinh(y)}$, evaluate
- $$\int_0^{\frac{\pi}{4}} (\text{sanj}(y) - \tan(v)) dv$$
- (A) $\frac{\ln(2)}{2} + \ln\left(\sinh\left(\frac{\pi}{4}\right) \cosh\left(\frac{\pi}{3}\right)\right)$
 (B) $\frac{\ln(2)}{2} + \ln\left(\sinh\left(\frac{\pi}{3}\right) \cosh\left(\frac{\pi}{4}\right)\right)$
 (C) $\frac{\ln(2)}{2} + \ln\left(\sinh\left(\frac{\pi}{4}\right)\right)$
 (D) $\frac{\ln(2)}{2} + \ln\left(\cosh\left(\frac{\pi}{4}\right)\right)$
 (E) NOTA

15. Tanvi is shocked at Sanjita's outrageous behavior and decides to find the surface area of Sanjita's abundant hair to make her feel bad. Given that Sanjita's hair can be modeled by the solid obtained by rotating $y = \sqrt{12 - x^2}$ from $-3 \leq x \leq 3$ about the x -axis, what value should Tanvi obtain?
- (A) $6\pi\sqrt{3}$ (B) 24π (C) 48π (D) $24\pi\sqrt{3}$ (E) NOTA

16. Sanjita then decides to whine wholeheartedly as if her limit has been reached. To learn more about herself through introspection, Sanjita decides to evaluate her limit:

$$\lim_{x \rightarrow 1} x^{\left(\frac{1205x}{\sin(x-1)}\right)}$$

What value should she obtain?

- (A) Does Not Exist (B) 1205e (C) e^{1205} (D) 1205 (E) NOTA
17. Rayyan, who has a kind and gentle soul, decides he wants to make Tanvi cry for her rude behavior with Sanjita. He succeeds. Evaluate the limit of the loudness of Tanvi's wails, which is represented by

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin(x) - \cos(x)}{\cos(2x)}$$

- (A) 0 (B) $\frac{\sqrt{2}}{4}$ (C) $\frac{-\sqrt{2}}{2}$ (D) ∞ (E) NOTA
18. Tanvi was then put into jail by a French-speaking Cameroonian. Tanvi can only escape from jail by using the Chain Rule to find the derivative of $t(h) = \tan(523h + 17)$ at $h = -\frac{17}{523}$. What value should Tanvi obtain to escape?
- (A) Undefined (B) 0 (C) 523 (D) 8891 (E) NOTA

19. Rayyan knows that he has a bigger heart than Tanvi and wants to find the difference between the area of his heart and the area of Tanvi's heart to prove that fact. Given that Rayyan's heart is represented by the function $r = 4 + 4 \sin(\theta)$ and Tanvi's heart is represented by the function $r = 1 + 1 \cos(\theta)$, what value should Rayyan obtain?
- (A) $\frac{15\pi}{2}$ (B) 15 π (C) $\frac{45\pi}{2}$ (D) 30 π (E) NOTA

20. Tanvi believes that she has a bigger brain than Rayyan and wants to find the difference between the volume of her brain and the volume of Rayyan's brain to validate her belief. Given that Tanvi's brain is a regular tetrahedron with a side length of 10 and Rayyan's brain is a regular hexahedron with a side length of 8, what value should Tanvi obtain?
- (A) $\frac{250\sqrt{2}-1536}{3}$ (B) $\frac{250\sqrt{3}-1536}{3}$ (C) $\frac{100\sqrt{3}-512}{3}$ (D) $\frac{100\sqrt{3}-3072}{6}$ (E) NOTA

21. Clueless Anirudh tilts his head whenever he is confused. Evaluate the maximum y -value on the graph of his head, which is modeled by the function $x^2 - xy - y^2 = 9$.
- (A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt{3}$ (D) $4\sqrt{3}$ (E) NOTA

22. Anjali wants to perfectly paint her nails Orange and Blue. She places her right thumb on graph paper and determines that it forms the solid obtained by rotating the region bounded by $x = y^2 - 9$ and $x = 3 - y$ about the line $x = 12$. What is the volume of her right thumb?
- (A) $\frac{21658\pi}{15}$ (B) $\frac{22981\pi}{15}$ (C) $\frac{1764\pi}{5}$ (D) $\frac{15876\pi}{5}$ (E) NOTA

23. In the race of the century, Shreyas the turtle and Vishal the walrus compete to reach 300 m up the opposite shore of a river, which is 100 m wide, from their common starting point. Shreyas and Vishal can run along the bank of the river at $4 \frac{\text{m}}{\text{s}}$ and $5 \frac{\text{m}}{\text{s}}$ respectively, while Shreyas and Vishal can swim across the river at $2 \frac{\text{m}}{\text{s}}$ and $3 \frac{\text{m}}{\text{s}}$ respectively. Given that both of them can swim or run any distance, what is the absolute value of the difference between the fastest times each one can finish the race in?
- (A) $\frac{25\sqrt{3}-15}{3}$ s (B) $20\sqrt{2}$ s (C) 25 s (D) $\frac{75\sqrt{3}-35}{3}$ s (E) NOTA

24. Shunmukha is vigorously training for the annual Rickards Speed Walking Competition. He does so on the Rickards track that encloses a patch of land. Given the following equations represent the walking track of negligible width, what is the area of the land that the track encloses. All answers are in units².

$$x(t) = 5 \cos(t)$$

$$y(t) = 4 \sin(t)$$

- (A) 10π (B) 16π (C) 20π (D) 25π (E) NOTA
25. Which of the following statements regarding the eccentricity (e) of a curve formed by the intersection of a cone with a plane parallel to its slant edge could be true?
- (A) $e = 0$ (B) $0 < e < 1$ (C) $e = 1$ (D) $e > 1$ (E) NOTA
26. Find the smallest positive value of x that satisfies the following equation:

$$\cos 3x + 3 \cos x = 4$$

- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) π (D) 2π (E) NOTA
27. Tanvi's favorite hobby is graphing functions and her brother Tanmay's favorite hobby is making it difficult for Tanvi to graph functions. One day, when Tanvi was getting ready to sketch the function $f(x) = x^2 - 30x + 221$, Tanmay decided to mess with her by dividing the right hand side of her function by $x^2 - 32x + 247$. Given Tanvi accurately depicts the function following Tanmay's alteration, find the sum of the vertical asymptotes of Tanvi's drawing?
- (A) -32 (B) -30 (C) 30 (D) 32 (E) NOTA
28. Find the number of terminal zeroes in the base-12 representation of 2019!.
- (A) 1005 (B) 1006 (C) 1007 (D) 1008 (E) NOTA
29. Dylan is a complex guy and he has a great imagination. Whenever he is free he fantasizes about questions involving imaginary elements. One day he confronts Prabhas with a product of one of his imagination sessions. Help Prabhas prove he too has a great imagination by solving the following question that Dylan presented him with.

Which of the following is equal to i^{2i} ?

- (A) $i \sin i$ (B) $\cos i$ (C) $\operatorname{cis} i$ (D) $i \operatorname{cis} i$ (E) NOTA
30. Congratulations! You have reached the end of the test! Your final task is to simplify:

$$\frac{\sin x}{2 + 2 \cos x} + \frac{\sec x + 1}{2 \tan x}$$

- (A) $\csc x$ (B) $2 \csc x$ (C) $\sec x$ (D) $2 \sec x$ (E) NOTA