QUESTION 1

Consider the functions:

$$\begin{aligned} A(x) &= 2x^2 + 1. \\ B(x) &= 2010x + 1337. \\ C(x) &= \sin x. \\ D(x) &= (x+1)^2. \end{aligned}$$

Compute the value of $A'(1) + B'(3) + C'(0) + D'(2)$.

QUESTION 2

Let

$$\begin{aligned} A &= \lim_{x\to 0} \frac{3x}{2x+1}. \\ B &= \lim_{x\to\infty} \frac{2x^2+3x^3}{4x^4}. \\ C &= \lim_{x\to 6} \frac{x^2-36}{x-6}. \\ D &= \lim_{x\to\infty} \frac{8x}{\sqrt{4x^2+2010}}. \end{aligned}$$

Compute the value of $A+B+C+D$.

QUESTION 3

Consider the functions $f(x)$ and $g(x)$ which are both continuous and differentiable over the reals. The table below gives the the values of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ for $x = \{1, 2, 3, 4\}$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	10	11	8	1
2	0	5	6	9
3	2	0	4	2
4	3	1	4	12

Let

$$\begin{aligned} A &= \frac{d}{dx}(f(g(4))). \\ B &= f'(g(4)). \\ C &= \frac{d}{dx}\frac{f(x)}{g(x)} \text{ at } x = 1. \\ D &= \frac{d}{dx}g(f(f(x))) \text{ at } x = 4. \end{aligned}$$

Compute the value of $A + B + C + D$.

QUESTION 4

Let

$$\begin{aligned} A &= \frac{d}{dx}\left(\sum_{i=0}^{\infty}\frac{x+i}{3^i}\right) \text{ at } x = 2010. \\ B &= \lim_{y\to 0}\frac{(x-y)^3 - x^3}{y}, \text{at } x = 2. \\ C &= \lim_{x\to 2}\frac{f(x)-f(2)}{\sqrt{x}-\sqrt{2}}, \text{ where } f(x) = x^3. \end{aligned}$$

Compute $\dfrac{AC}{B}$.

QUESTION 5

The line normal to the curve $y = e^{-2x}$ at $x = 2\ln 2$ has a $y-$intercept of $A + B\ln 2$, where A is rational. Compute the value of AB.

QUESTION 6

Let

$$\begin{aligned} A &= \text{the area between the curves } y = x^3 - 4x^2 + 4x \text{ and } y = x. \\ B &= \int_{-\pi/2}^{3\pi/2} \cos(x)dx \\ C &= \int_{\ln(e^2)/2}^{e^{\sin(0)}} \frac{\sin(\ln(x))dx}{x} \\ D &= \text{the area between the curves } y = 2x \text{ and } y = 2x^3 - 8x^2 + 8x. \end{aligned}$$

Compute $A + B + C + D$.

QUESTION 7

Let

$$\begin{aligned} A &= \text{the largest possible area of a rectangle with integer side lengths and a perimeter of 18.} \\ B &= \text{the largest possible area of a rectangle with a perimeter of 18.} \\ C &= \text{the largest possible value of } (x+y)^2 \text{ such that } x^2+y^2=25 \\ D &= \text{the smallest possible value of } (x+y)^2 \text{ such that } x^2+y^2=25 \end{aligned}$$

Compute the value of $B - A + C + D$.

QUESTION 8

Consider the function $f(x) = x^3 - 2x^2 + 3x - 4$ and its inverse $f^{-1}(x) = g(x)$. Let

$$\begin{aligned} A &= f'(3). \\ B &= g(-2). \\ C &= g'(-2). \end{aligned}$$

Compute the value of $A + B + C$.

QUESTION 9

Let

$$
\begin{aligned}
A &= \text{the number of inflection points of } y(x) \text{ where } y''(x) = (x-1)^2(x+3)(x^2+4). \\
B &= \text{the sum of the critical values of } y(x) \text{ where } y'(x) = x^2 + 7x - 10. \\
C &= \text{the number of critical values of } y(x) \text{ where } y(x) = (x-2)^2(x+1)^3. \\
D &= \text{the maximum value of } y(x) \text{ where } y(x) = e^{-x^2}.
\end{aligned}
$$

Compute the value of $A + B + C + D$.

QUESTION 10

At time $t = 0$, Felipe Massa, whose Ferrari is traveling at 300 kilometers per hours, is attempting to pass Michael Schumacher, whose Mercedes is traveling at 330 kilometers per hour. In order to catch and pass Schumacher, Massa must travel a total of 72 kilometers more than Schumacher starting at time $t = 0$. Luckily, Massa has a constant acceleration of 6 kilometers per hour-squared, while Schumacher travels at a constant speed. How many hours will it take for Massa to catch up to Schumacher?

QUESTION 11

The following table gives the values of a continuous function $f(x)$ on the interval $[0, 8]$:

x	0	1	2	3	4	5	6	7	8
$f(x)$	C	A	L	C	U	L	U	S	2010

where

$$A = \int_1^e \frac{da}{a} \qquad C = \int_1^5 c\,dc \qquad L = \int_0^\pi \sin(l)dl \qquad S = \int_4^{25} \frac{ds}{2\sqrt{s}} \qquad U = \int_0^8 (u^3 - 2u^2 + 1)du.$$

Using a midpoint Riemann sum with 4 rectangles, estimate the value of $\int_0^8 f(x)dx$.

QUESTION 12

Let $f(x) = x$ if $x \neq \pm\infty$ and $f(x) = 0$ if $x = \pm\infty$.
If

$$\begin{aligned} A &= \text{the minimum value of } g_1(a) = a^2 + 2a + 3 \\ B &= \text{the minimum value of } g_2(b) = b^3 + b \\ C &= \text{the minimum value of } g_3(c) = c - \sqrt{c} \\ D &= \text{the minimum value of } g_4(d) = d + \frac{1}{d} \end{aligned}$$

then compute the value of $f(A) + f(B) + f(C) + f(D)$.

QUESTION 13

Let $F(x) = \int_{\pi}^{x^2} \sin\theta\cos\theta d\theta$. Compute $F'(\sqrt{\pi})$.

QUESTION 14

Consider a function of several variables defined as $f(x, y, z) = f_1(x)f_2(y)f_3(z)$. Then we define the *partial derivative* $\frac{\partial f}{\partial x}$ as $\frac{\partial f}{\partial x} = \left(\frac{d}{dx}f_1(x)\right) f_2(y)f_3(z)$. Similarly, $\frac{\partial f}{\partial y} = \left(\frac{d}{dy}f_2(y)\right) f_1(x)f_3(z)$ and $\frac{\partial f}{\partial z} = \left(\frac{d}{dz}f_3(z)\right) f_1(x)f_2(y)$.

Let $f(x, y, z) = xy^3z^2$. Compute $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$.