QUESTION 1

Given an equilateral triangle XYZ with side length 4, let:

$$\begin{aligned} A &= \text{the area of } \triangle \text{XYZ} \\ B &= \text{the perimeter of } \triangle \text{XYZ} \\ C &= \text{the area of the circumcircle of } \triangle \text{XYZ} \\ D &= \text{the area of the incircle of } \triangle \text{XYZ} \end{aligned}$$

Find the value of $\dfrac{A}{\sqrt{3}} + B + \dfrac{C}{D}$.

QUESTION 2

Let:

A = the volume of a rectangular prism with faces of area 30, 35, and 42

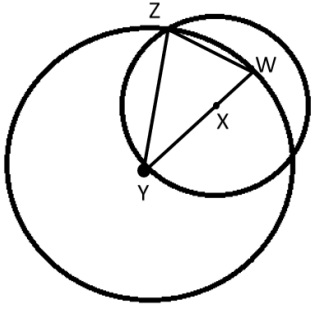
B = the sum of the coordinates of the centroid of the triangle formed by the points (2, 4), (5, 7), (11, 1)

C = the area of the annulus of two concentric circles, given that the chord of the larger circle tangent to the smaller circle has length $\frac{8}{\sqrt{\pi}}$

D = the area of a sector of a circle of radius 5 with an angle of 2 radians

Find the value of $AB + CD$.

QUESTION 3



In the above diagram, Circle X, which has a radius of 4, intersects the center of Circle Y. ZX is perpendicular to YW. Let:

$$\begin{aligned} A &= \text{the area of circle X} \\ B &= \text{the area of circle Y} \\ C &= \text{the area inside circle Y but outside circle X} \\ D &= \text{the area inside circle X but outside circle Y} \end{aligned}$$

Find the value of $A + B + C + D$.

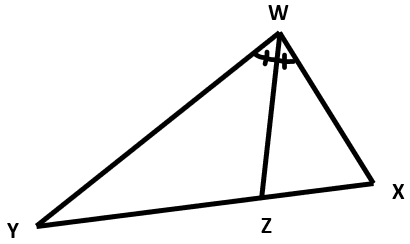
QUESTION 4

Let:

A = the smaller angle in degrees between the minute and hour hands of a clock when the time is 1:15 P.M.
B = the larger angle in degrees between the minute and hour hands of a clock when the time is 2:05 A.M.
C = the smaller angle in degrees between the minute and hour hands of a clock when the time is 9:26 P.M.
D = the larger angle in degrees between the minute and hour hands of a clock when the time is 6:01 A.M.

Find the value of $A + B + C + D$.

QUESTION 5



In the above diagram, WX = 3, XY = 10, and the length of YZ is four times the length of XZ. Let:

$$\begin{aligned} A &= \text{the length of XZ} \\ B &= \text{the length of YZ} \\ C &= \text{the length of YW} \\ D &= \text{the length of WZ} \end{aligned}$$

Find $(ABC) + D$.

QUESTION 6

Each of the following statements are associated with a point value. Beginning from 0, add up the values for each false statement:

(15) The Euler Line doesn't pass through the incenter of a scalene triangle.

(7) If a statement is true, then the converse of its contrapositive is always true.

(-20) The sum of the two pairs of opposite sides in a tangential quadrilateral are equal.

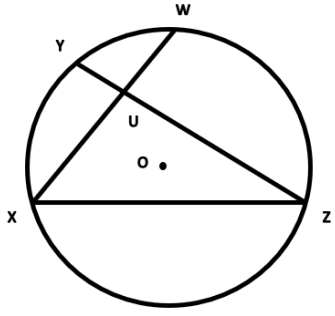
(2) A polyhedron with 5 faces and 6 vertices has 8 edges.

(-11) A triangle with sides of length 9, 40, 41 is a right triangle.

(37) The maximum number of sections that can be created in a circle with 5 cuts is 16.

What is the resulting sum?

QUESTION 7



In the figure shown above, chords XW and YZ in Circle O intersect at point U. $\angle$YZX measures 20°. Let:

A = the measure of $\angle$WXZ in degrees, given that the measure of $\angle$YUW is numerically equal to the square of the area of a polygon with 6 lattice points in its interior and 10 on its boundary

B = the measure of $\angle$WOZ in degrees

C = the measure of segment UZ, if UX has a length of 5, UW has a length of 3, and UY has a length equal to the length of the circumradius of a right triangle with legs of length 3 and 4 (Ignore information provided in parts A and B)

Find $\frac{A * B}{C}$.

QUESTION 8

Denote Circle R as the circle with the equation, $(x-2^{1010})^2+(y-2^{1010})^2=2^{2020}$. There exists two distinct circles in the first quadrant that are externally tangent to Circle R and both of the coordinate axes. Let **A** be the area of the larger circle, and let **B** be the area of the smaller circle. Let:

C = the length of the common chord of two intersecting circles, given that their equations are $x^2+y^2=16$ and $(x-4\sqrt{3})^2+y^2=16$

D = the smallest distance between the graphs of $x^2+y^2-14x-4y+49=0$ and $x^2+y^2+6x-6y-7=0$

Find $\dfrac{A}{B}+C+D$.

QUESTION 9

Let:

A = the number of permutations of the word "PACHAKUTIK"

B = the number of different ways that 6 distinct beads can be placed on a ring

C = the number of different ways that you can arrange a laptop, a water bottle, a clock, and two identical pens in a straight line, given that the laptop and the clock must be next to each other

D = the probability that Fitz and Simmons are at the lab at the same time, given that they both arrive at the lab at a random time between 2 P.M. and 3 P.M., and once arrived at the lab, Fitz stays at the lab for 12 minutes before leaving and Simmons stays at the lab for 6 minutes before leaving

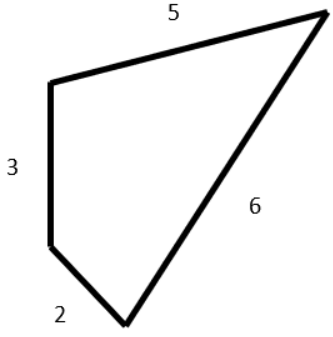
Find $\frac{A}{B} + 10C * D$.

QUESTION 10

Let:

A = the number of sides in a polygon with all interior angles measuring 156°

B = the length of one of the largest diagonal in an octagon with side length 12



The above quadrilateral is cyclic.

C = the area of the quadrilateral

D = the product of the two diagonals of the quadrilateral

Find $A + B^2 + C^2 + 3D$.

QUESTION 11

Buchanan lives on the coordinate plane at the point (1, 6) and his school is located at the point (4, 2).

Buchanan leaves his home to go to school, and after covering 2/5ths of the total distance, he realizes that he dropped his keys somewhere along the way. After back-tracking 1/6th of the distance he has travelled so far, he finds his keys, and proceeds to go to his school. Let **A** be the total distance he travels.

Buchanan's friend Sam lives nearby. The distance from Sam's house to Buchanan's house is $\sqrt{26}$, and the distance from Sam's house to the school is $\sqrt{13}$. Let **B** be the sum of the coordinates of Sam's house, given that the x-coordinate of his house is greater than 4.

For this part, assume Sam's house is located at the point (6, 6). A circular path is built that goes through Buchanan's house, Sam's house, and the school. Let **C** be the radius of this path.

Find the value of ABC.

QUESTION 12

Let:

A = the area of a rhombus with diagonals of length 5 and 4

B = the area of an octagon with a side length of 5

C = the length of the segment parallel to the two bases of a trapezoid, with base lengths of 6 and 10, which passes through the point at which the diagonals intersects

D = the surface area of a right conical frustum with base radii of 1 and 5, and a slant height of 5

Find $3A + B + 2C + D$.

QUESTION 13

Let:

A = the value of $\sin \frac{\pi}{4}$

B = the value of $\cos \frac{7\pi}{12}$

C = the negative value of $\cos\theta$ if $\sin^2\theta$ is equal to 0.36

D = the length of the side opposite to angle D in triangle DEF if angle E measures 60 degrees, angle D measures 45 degrees, and the side opposite to E has a length of 6

Find $\frac{A}{2} - B + C\sqrt{6} + D$.

QUESTION 14

Let:

$$\begin{aligned} A &= \text{the sum of the factors of 5148} \\ B &= \text{the third triangular number} \\ C &= \text{the fifth star number} \end{aligned}$$

Find $A - (BC)$.