For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. How many regular polygons have more sides than diagonals?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

2. There is a triangle with side lengths of 6, 8, and 10. The Euler line of this triangle intersects the triangle at two points. What is the distance between these two points?

   (A) 3 (B) 4 (C) 4.5 (D) 5 (E) NOTA

3. A circle goes through the points $(5, 7)$ and $(10, -3)$. Let C be the center of the circle. What is the equation that describes the locus of all possible placements of C?

   (A) $y = 0.5x - 1.75$ (B) $x^2 + y^2 = 74$ (C) $x^2 + y^2 = 109$ (D) $y = -0.5x + 5.75$ (E) NOTA

4. Deekshita is packing her suitcase to attend a Jonas Brothers Concert. The suitcase is in the shape of a quadrilateral. To help Deekshita pass airline regulations, find the sum of the interior and exterior angles in any quadrilateral, in radians.

   (A) $360°$ (B) 3π (C) 4π (D) 5π (E) NOTA

5. Farzan's favorite snack is a bag of chips. He receives a peculiar chip container that can be modeled as two concentric circles. Farzan measures a chord of the larger circle, that is also tangent to the smaller circle, and sees that it has a length of 12. What is the maximum positive difference in area of the two circles?

   (A) 36 (B) 36π (C) 72 (D) 72π (E) NOTA

6. Solve for a:

$$\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots = \frac{a}{3^0} + \frac{a^2}{3^1} + \frac{a^3}{3^2} + \cdots$$

   (A) 0.6 (B) 0.8 (C) 1 (D) 1.2 (E) NOTA

7. Rohan is flexing his trigonometry knowledge by asking you to simplify the expression below. Surprise Rohan and give the correct answer.

$$2\left(\sin(30°) + \cos(30°)\right)\left(\sin(60°) + \cos(60°)\right)\left(\sin(45°) + \cos(45°) + \tan(45°)\right)$$

   (A) $2 + 2\sqrt{2} + \sqrt{3} + \sqrt{6}$ (B) $1 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$ (C) $1 + \sqrt{2} + \sqrt{3} + \sqrt{6}$ (D) 0 (E) NOTA

8. What is the sum of all distinct numbers x that satisfy the following conditions: $0 < x < 1$, x only contains one 2, and x contains at least one zero in its decimal representation (such as 0.2 and 0.0002)?

   (A) 0.21 (B) 0.22 (C) $0.\overline{2}$ (D) 0.23 (E) NOTA

9. What is the product of the inradius and circumradius in a triangle with side lengths 13, 14, and 15?

   (A) 30 (B) 32.5 (C) 33.6 (D) 40 (E) NOTA

10. Tanmay is making Tanvi a birthday card. In order to cut out the perfect shape for the card, he plots four points on a grid. The points are $(1, 3)$, $(2, 5)$, $(5, -1)$, and $(12, 0)$, and the card is in the shape of a convex quadrilateral. What is the area of Tanmay's card?

    (A) 10 (B) 16 (C) 28.5 (D) 57 (E) NOTA

11. James models Aaron's pencil, and realizes that it is a triangle with sides 2, x, and 6. After measuring the area, James is shocked to find that he gets x square units. Find x^2.

(A) 30 (B) 32 (C) 34 (D) 36 (E) NOTA

12. What is the sum of the lengths of the altitudes in a triangle with side lengths 20, 21, and 29?

(A) $14\frac{14}{29}$ (B) $34\frac{14}{29}$ (C) $35\frac{14}{29}$ (D) $55\frac{14}{29}$ (E) NOTA

13. Let n be called a *special number* if it is a positive integer, and the sum of the digits of 2^n is n. It can be shown that 70 is a special number. What is the only other special number?

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

14. Dylan, the compulsive liar, gives you four statements to evaluate:

1.The midpoints of the sides of a quadrilateral form a parallelogram.
2.The surface area of a cone with radius 3 and height 4 is 15π.
3.The volume of a sphere is $\dfrac{S\sqrt{S}}{6\sqrt{\pi}}$, where S is the surface area.
4.Euler is the father of geometry.

What is the sum of the number labels of the true statements? (For example, if you thought only statements 1 and 4 are true, put 5 as your answer.)

(A) 4 (B) 5 (C) 8 (D) Cannot be determined (E) NOTA

15. A right triangle and three non-congruent, regular pentagons are drawn such that the three pentagons are outside the triangle, and each of the pentagons share a side with the right triangle. If the areas of the two smaller pentagons are 60 and 80, what is the area of the largest pentagon?

(A) 100 (B) 140 (C) 180 (D) Cannot be determined (E) NOTA

16. A sphere is circumscribed about a cone with radius 15 and height 25. What is the surface area of the sphere?

(A) 1024π (B) 1080π (C) 1120π (D) 1156π (E) NOTA

17. Shreyas is bored during a lecture and decides to draw random shapes on his paper, and see how many times they intersect. He is drawing a square, a line, and a circle. Which number could he not get when he counts the number of intersection points on his paper?

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

18. A sphere is inscribed in a regular hexahedron, and a regular octahedron is inscribed in the sphere. What is the ratio of the volume of the octahedron to the hexahedron?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{7}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{5}$ (E) NOTA

19. What is the distance from $(0,0)$ and $(x^2-y^2, 2xy)$ when $x \geq y \geq 0$?

(A) $(x+y)^2$ (B) $\sqrt{x^4+y^4}$ (C) $(x-y)^2$ (D) $xy(x+y)$ (E) NOTA

20. Tanusri was relearning geometry for fun, and she came across the concept of lattice points. Define a positive integer to be a *Tanusri sphere number* if the equation $x^2+y^2+z^2=D$, when graphed, does not pass through any lattice points. What is the least Tanusri sphere number?

(A) 7 (B) 9 (C) 11 (D) 13 (E) NOTA

21. Eric's favorite theorem is the Pythagorean Theorem, which only works for right triangles. In similar spirit, he defines a *right quadrilateral* as a quadrilateral with sides a, b, c, and d that satisfies $a^2 + b^2 + c^2 = d^2$. A certain right quadrilateral has distinct, integer side lengths, and perimeter 18. What is the product of its side lengths?

    (A) 210 (B) 240 (C) 250 (D) 252 (E) NOTA

22. Rayyan's favorite toy is a plush model of a cube. However, he accidentally chopped it into two pieces with a knife! Which of the following cannot be a cross-section of a cube that he created?

    (A) triangle (B) quadrilateral (C) pentagon (D) hexagon (E) NOTA

23. Akhil wanted to create a unique hula hoop. The hula hoop can be modeled by the circumcircle of a triangle with vertices in the coordinate plane at $(0, 6)$, $(-4, 0)$, and $(3, 0)$. What is the radius of the model of his hula hoop?

    (A) $\frac{\sqrt{65}}{2}$ (B) $\frac{\sqrt{70}}{2}$ (C) $3\sqrt{2}$ (D) $\frac{5\sqrt{3}}{2}$ (E) NOTA

24. Shrung and Nitish are thinking of starting a company. They want the logo to be a dodecagon inscribed inside a unit circle. In order to help them calculate the cost, find the area of this dodecagon.

    (A) $\sqrt{6}$ (B) $2\sqrt{2}$ (C) 3 (D) $2\sqrt{3}$ (E) NOTA

25. Regular hexagon $A_1A_2A_3A_4A_5A_6$ is drawn. There is a point P inside the hexagon such that $[\triangle A_1A_2P] = 6$, $[\triangle A_3A_4P] = 10$, and $[\triangle A_5A_6P] = 11$. Find $[\triangle A_2A_3P][\triangle A_4A_5P][\triangle A_6A_1P]$.

    (A) 624 (B) 648 (C) 660 (D) 672 (E) NOTA

26. Akash and Vishnav love geometry, and present you with the following question:

    In $\triangle ABC$, denote its incircle as ω. Let the midpoint of $\overline{BC}$ be M. Segment AM intersects ω at two distinct points, P and Q, such that $AP < AQ$. It is known that $AP : PQ : QM = 1 : 3 : 1$. The sides of the triangle are positive integers a, b, and c, and $\gcd(a, b, c) = 1$. Find $a + b + c$.

    (A) 84 (B) 90 (C) 96 (D) 128 (E) NOTA

27. Shubham enjoys making right, square pyramids. When he made a particular square pyramid, with base $ABCD$, having side length 2, and apex P, he noticed that if you let M be the midpoint of $\overline{BC}$, then $\angle APM = 90^\circ$. What is the ratio of the surface area to the volume of this pyramid?

    (A) $\frac{3}{2}\left(\sqrt{6} + \sqrt{2}\right)$ (B) $3 + 3\sqrt{2}$ (C) $4 + 4\sqrt{2}$ (D) $6 + 6\sqrt{2}$ (E) NOTA

28. In $\triangle ABC$, $AB = 13$, $BC = 14$, and $AC = 15$. Let M_1 and M_2 be the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. The line passing through M_1 and M_2 intersects the circumcircle of $\triangle ABC$ at two places, P_1 and P_2, such that $M_1P_1 < M_1P_2$. Find $M_1P_1 + M_2P_2$.

    (A) $6 - \sqrt{10}$ (B) $7 - 2\sqrt{5}$ (C) 7 (D) $5\sqrt{10} - 7$ (E) NOTA

29. In equilateral $\triangle ABC$, point P lies inside the triangle such that $AP = 3$, $BP = 5$, and $\angle APB = 120^\circ$. Find PC.

    (A) $\sqrt{19}$ (B) $\sqrt{21}$ (C) $2\sqrt{6}$ (D) 7 (E) NOTA

30. Karthik, while making this test, was eating a weirdly-shaped waffle that was a regular heptagon, which had a side length of 1. Seven of the diagonals have length x, and the other seven have length y, where $y > x$. If x is approximately 1.802, approximate y to two decimal places.

    (A) 2.11 (B) 2.16 (C) 2.19 (D) 2.25 (E) NOTA