QUESTION 1

Two points on the curve $y = x^4 - 2x^2 - x$ have a common tangent line. Let $A =$ the sum of the ordinates of these two points.

A ladder will be carried down a hallway that is 8 feet wide. At the end of the hallway, the ladder will need to make a $90°$ turn into a hallway that is 64 feet wide. Let $B =$ the maximum length of the ladder that will be carried horizontally through the corner described.

Find $A \cdot B$. Express your answer in simplest radical form, if needed.

QUESTION 2

Let:

$$
\begin{aligned}
A &= \text{the maximum speed of a runner with position function } p(t) = -\frac{1}{3}t^3 + 4t^2 - 15t + 4 \text{ from time } t = 1 \text{ to time } t = 5. \\
B &= \text{the minimum velocity of a runner with velocity function } v(t) = \frac{1}{4}t^4 - t^3 + \frac{1}{2}t^2 - 3t + 4. \\
C &= \text{the total distance traveled by a runner with velocity function } v(t) = t^2 - 4t + 3 \text{ from time } t = 0 \text{ to time } t = 5. \\
D &= \text{the maximum velocity of a runner with a position function } p(t) = -\frac{1}{20}t^5 + \frac{1}{3}t^4 - t^3 + 2t^2 + 5t + 9.
\end{aligned}
$$

Find $A + 4B + C + D$.

QUESTION 3

Let:

$$
\begin{aligned}
A &= \int_2^3 \frac{x^2+1}{x^3+3x}dx \\
B &= \int_2^3 \frac{3x+6}{x^2+4x-3}dx \\
C &= \int_0^3 ((2x^2)(3^{\frac{1}{3}x^3+1}))dx \\
D &= \int_0^{\sqrt{\pi}} x\sin(x^2)dx
\end{aligned}
$$

Find $3A + \frac{2}{3}B + \frac{118092}{C} + \ln(D)$.

QUESTION 4

Let:

$$
\begin{aligned}
A &= \lim_{x\to\ 0^+} \left(2^x + 3x\right)^{\frac{4}{x}} \\
B &= \lim_{x\to\frac{\pi}{4}} \frac{\ln(\tan(x))}{\sin(x) - \cos(x)} \\
C &= \lim_{x\to\infty} \frac{\ln(t+5)}{\log_3 t} \\
D &= \left(\lim_{x\to 4} \frac{x^{2019} - 4x^{2018} + x\log_2(x) - 4\log_2(x)}{x-4}\right) - 2
\end{aligned}
$$

Find $\ln(A) + B^2 + e^C + \log_2(D)$.

QUESTION 5

Let:

A = the probability that a point within the region bounded by $f(x) = -x^2 + 5x$, $x = 0$, $x = 5$, and $y = 0$ has an abscissa ≥ 4.5, given that the abscissa is ≥ 2.

B = the value at the 25th percentile of the probability density function defined by

$$f(x) = \begin{cases} \frac{1}{2}\sin(x) & \text{if } x \text{ is between 0 and } \pi\text{, inclusive} \\ 0 & \text{otherwise} \end{cases}$$

C = the mean value of the function $f(x) = 4x^3 + 6x + 3$ over the interval $[0, 3]$.

D = the value at the 75th percentile of the probability density function defined by

$$f(x) = \begin{cases} \frac{1}{4}|\sin(x)| & \text{if } x \text{ is between 0 and } 2\pi\text{, inclusive} \\ 0 & \text{otherwise} \end{cases}$$

Find $324A + 6B + C + 2D$.

QUESTION 6

Let:

A = the volume of the solid generated by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.

B = the volume of the solid generated by rotating the region bounded by $y = 4x^2 - x^3$ and $y = 0$ about the y-axis.

A circle, X, with radius 2 is centered at the origin, with an arbitrary diameter, y. A solid is formed by creating vertical, parallel, semicircular cross-sections at every point on y. Each cross-section has a base that passes through and is perpendicular to y and has endpoints on the circumference of X. Let $C =$ the volume of this solid.

A square, W, has a side length of 2. A diagonal of W is labelled z. A solid is formed by creating vertical, parallel cross-sections perpendicular to every point on z. Each cross-section is an equilateral triangle and has a base that is perpendicular to z and has endpoints on the perimeter of W. Let $D =$ the volume of this solid.

Find $15(A + B + C) + 3D$.

QUESTION 7

Let:

$$\begin{aligned} A &= \int_{-1}^{1} \left(\sqrt{1-x^2}\right) dx \\ B &= \int_{0}^{1} \frac{\arctan(x)}{x^2+1} dx \\ C &= \int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x}}{\sqrt{x}} dx \end{aligned}$$

Find $32(A+B+C)$.

QUESTION 8

Let:

$$
\begin{aligned}
A &= \text{the arc length of the function } f(x) = \frac{x^4}{4} + \frac{1}{8x^2} \text{ on the interval } [1, 2]. \\
B &= \text{the area of the surface obtained by revolving the function } f(x) = \sqrt{x} \text{ about the } x\text{-axis, on the interval } [0, 2]. \\
C &= \text{the maximum volume of a cylinder inscribed in a sphere with a radius of 2.} \\
D &= \text{the maximum volume of a cylinder inscribed in a cone with a radius of 2 and a height of 3.}
\end{aligned}
$$

Find $64A + 9(B + C + D)$.

QUESTION 9

The following statements have point values indicated by the numbers in the parentheses to the left of each statement. Starting with 0, add the point value of each true statement, and subtract the point value of each false statement.

(12) The sum of the reciprocals of all prime numbers diverges.

(-15) A point with abscissa c is a critical point of a function $f(x)$ if and only if $f(c)$ exists and $f'(c) = 0$.

(-7) Every bounded monotonic sequence converges.

(5) An unbounded sequence does *not* necessarily diverge.

(4) The harmonic series converges.

(23) Any partial sum of the harmonic series diverges.

(1) The first fundamental theorem of calculus states that if a function f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

What is the final number of points?

QUESTION 10

Let:

$$A = \lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{n}{n^2+r^2}\right)$$

$$B = 1 \text{ if the following series is convergent or } -1 \text{ if it is divergent: } \sum_{n=1}^{\infty}\frac{1}{n\ln(n)}$$

$$C = \lim_{x\to 0}\frac{\sin(x)-x+\frac{x^3}{6}}{\frac{1}{5}x^5}$$

$$D = \text{the radius of convergence of } \sum_{n=1}^{\infty}\frac{3^n}{n+1}(3x-7)^n$$

Find $12A + B + 72(C + D)$.

QUESTION 11

Let:

$$\begin{aligned} A &= \frac{d}{dx}(3x^3 + 4x^2 + 6) \text{ at } x = 1. \\ B &= \frac{d}{dx}(\arctan(x)) \text{ at } x = \frac{1}{2} \\ C &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec(x)dx \\ D &= \int_1^2 \ln(x)dx \end{aligned}$$

Find $A + 5B + e^C + e^{(D+1)}$.

QUESTION 12

Consider the following differential equation, with the initial condition $y(2) = 0$: $\dfrac{dy}{dx} = e^{-y}(3x^2 - 6)$. Let $A = y(6)$.

Consider the following differential equation, with the initial condition $y(1) = 1$: $(xy + y^2)\dfrac{dy}{dx} = y^2$. Let $B = x$ when $y = 3$.

Find $A - B$.

QUESTION 13

Evaluate:

$$\frac{d}{dx}\left(\arctan(x^3+8e^{-x}+3)+\arctan\left(\frac{1}{x^3+8e^{-x}+3}\right)\right)$$

at $x=\dfrac{1}{3}$.

QUESTION 14

Evaluate:

$$\lim_{x\to\infty}\left(\frac{x!}{x^x}\right)^{\frac{1}{x}}$$