For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. If $f(x) = 3x^5 - 4x^3 + 2x^2 - 1 + \arctan(3x) + x^x$, find $f'(1)$.

(A) $\frac{73}{10}$ (B) $\frac{83}{10}$ (C) $-\frac{17}{10}$ (D) $\frac{81}{10}$ (E) NOTA

2. Find the sum of the slope and the y-intercept of the tangent line to the following relation at the point $(2, -1)$:

$$3x^2y - 2x^3 + 3x^2y^2 + 2y - 6xy + 12 = 3xy^2$$

(A) $\frac{11}{10}$ (B) $-\frac{29}{11}$ (C) $\frac{3}{4}$ (D) $-\frac{21}{10}$ (E) NOTA

3. Evaluate:

$$\lim_{x \to 0} \sqrt[x^2]{(1+3x)^{3x}}$$

(A) e^3 (B) 0 (C) e^6 (D) e^9 (E) NOTA

4. Rohan likes to push himself to his limit, but Tanusri always yells at him for doing so. Instead, Tanusri suggests Rohan solve the following limit. Evaluate:

$$\lim_{x \to 0} \left(x^4 \cos\left(\frac{2}{x}\right)\right)$$

(A) Does Not Exist (B) 2 (C) 4 (D) 8 (E) NOTA

5. Princeton is a very prestigious university. While he was there, Nihar discovered that the current year was 2019. Evaluate:

$$\lim_{x \to \infty} \left(\left(\frac{2016^x + 2017^x + 2018^x}{2019} \right)^{\frac{1}{x}} \right)$$

(A) 1 (B) 2017 (C) 2018 (D) 2019 (E) NOTA

6. Evaluate:

$$\int_{-\frac{1}{3}}^{0} \frac{3(\sin(5x))^2 \ln(3x+2) + 15x \sin(5x)\cos(5x)(\ln(3x+2))^2 + 10\sin(5x)\cos(5x)(\ln(3x+2))^2}{3x+2} dx$$

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) NOTA

7. Tanvi and Sanjita are an integral part of each other's lives. Evaluate:

$$\int_{-1}^{7} \frac{\sqrt{7+6x-x^2}}{4} dx$$

(A) 2π (B) 4π (C) $\frac{7}{4}$ (D) 4 (E) NOTA

8. Anirudh wants to build a rectangular pen for his pet dodo bird. He has 600 feet of fencing, but plans to build the pen such that one side is blocked off by a perfectly straight and infinitely long river. What is the largest area, in square feet, Anirudh can construct for his dodo bird?

(A) 45000 (B) 40000 (C) 31250 (D) 30000 (E) NOTA

9. Find the area enclosed between the following two functions:

$$\begin{aligned} f(x) &= 5x^2 - 12x - 16 \\ g(x) &= 4x^2 - 4x - 28 \end{aligned}$$

(A) $\frac{112}{3}$ (B) $\frac{11}{3}$ (C) $\frac{38}{3}$ (D) $\frac{32}{3}$ (E) NOTA

10. Kira has an odd water bottle in the shape of an inverted cone with radius 3 units and height 12 units. While filling her water bottle up from the water fountain, Jeffrey points out that the total volume of water, in cubic units, ejected from the water fountain is described by the function $\frac{\pi}{3}t^2 + \frac{14\pi}{3}t$, beginning at $t = 0$ seconds. Find the rate, in units per second, at which the height of the water in her water bottle is rising after 2 seconds.

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) NOTA

11. Evaluate:

$$\int_{-6}^{0} \frac{\sin(14x+42)\cos(16x+48)\ln|x+3|}{2016} dx$$

(A) Divergent (B) -2019 (C) 0 (D) 6 (E) NOTA

12. Evaluate:

$$\int_{0}^{\sqrt[4]{7}} \frac{x^3}{\sqrt[3]{1+x^4}} dx$$

(A) $\frac{9}{8}$ (B) $\frac{\ln(7)}{4}$ (C) $\frac{\ln(7)}{8}$ (D) $\frac{3}{2}$ (E) NOTA

13. Farzan wants to prove that Prabhas is ReRe™ and asks him to solve many derivatives. However, Prabhas asks Dylan for help, so he becomes very skilled at solving seemingly time-consuming derivatives. Evaluate the derivative of:

$$y = \left(\sqrt{5x^2-9}\right)\left(\sqrt[3]{16x^3-12x^2-316}\right)\left(\sqrt[5]{12x-4}\right)$$

with respect to x at $x = 3$.

(A) $\frac{134}{5}$ (B) $\frac{1859}{5}$ (C) $\frac{1858}{5}$ (D) $\frac{144}{5}$ (E) NOTA

14. Let R be the area bounded by:

$$\begin{aligned} y &= \ln(x) \\ y &= 0 \\ x &= e^2 \end{aligned}$$

Find the volume of the solid with base R and semi-circular cross sections perpendicular to the x-axis.

(A) $\frac{\pi e^2 - \pi}{4}$ (B) $\frac{\pi e^2 - \pi}{2}$ (C) $\frac{\pi e^2}{4}$ (D) $\frac{\pi e^2 + \pi}{4}$ (E) NOTA

15. Brandon is doing some heavy lifting. In particular, he wants to lift a massive point load of 50 grams, attached to a rope with mass 5 grams per centimeter, completely out of a hole 40 centimeters deep. How much energy, in joules, does Brandon require to perform this task? Assume gravitational acceleration to be 10 meters per second per second.

(A) 0.4 (B) 0.6 (C) 0.24 (D) 60000 (E) NOTA

16. Evaluate the following:

$$\frac{d}{dx}\left[\int_{2x}^{5x^2}\left(3t^2\sin(7t)+\cos(2t)\right)dt\right]\Bigg|_{x=0}$$

(A) 0 (B) -1 (C) -2 (D) -3 (E) NOTA

17. Lindsay oddly enjoys the shape of plum pudding. Find the volume of the shape formed when the region in Quadrant 1 bound by $y=-x^3+3x^2+x-3$ and the x-axis is revolved around the y-axis.

(A) 8π (B) 16π (C) $\frac{124}{15}\pi$ (D) $\frac{248}{15}\pi$ (E) NOTA

18. Jason and Venkat also enjoy remarking on such strange shapes. Find the length of the curve $x=\frac{2}{3}(y-1)^{\frac{3}{2}}$ on the interval $1\leq y\leq 4$.

(A) 2 (B) $\frac{9}{2}$ (C) $2\sqrt{3}$ (D) $\frac{14}{3}$ (E) NOTA

19. Approximate a root of x^2-3x-7 by using two steps of Newton's Method with an initial value of $x_0=0$.

(A) $-\frac{113}{69}$ (B) $-\frac{3}{2}$ (C) $\frac{456}{2779}$ (D) $-\frac{114}{5}$ (E) NOTA

20. Joshua often thinks of cookies. While solving the previous problem, he thought of the law of cooling that also bears Newton's name. Assuming this law holds true for Joshua's cookies, find the temperature of the cookies after 20 minutes if they began at 100^o C, cooled to 60^o C after 10 minutes, and were kept in a room with constant ambient temperature of 20^o C.

(A) 60^o C (B) 50^o C (C) 40^o C (D) 30^o C (E) NOTA

21. Find the surface area of the solid formed when the curve $y=\sqrt{4-x^2}$ from $-1\leq x\leq 1$ is rotated around the x-axis.

(A) 16π (B) 14π (C) 8π (D) 4π (E) NOTA

22. Find the volume of the shape formed when the graph of $r=2\sin(5\theta)$ is revolved around the line $x=3$.

(A) $12\pi^2$ (B) $6\pi^2$ (C) $2\pi^2$ (D) π (E) NOTA

23. Anish enjoys his parameters. Given the following set of parametric equations, find the second derivative of y with respect to x when $t=0$:

$$\begin{aligned} x &= e^{-7t}+2 \\ y &= 6e^{2t}+e^{-3t}-4t \end{aligned}$$

(A) $\frac{68}{49}$ (B) $-\frac{68}{7}$ (C) $-\frac{5}{7}$ (D) $\frac{33}{49}$ (E) NOTA

24. The Rickards Invitational represents the beautiful convergence of students of mathematics from across the state. Beginning at 7, proceeding down the list in order, multiply by 3 for each series that converges and subtract 4 for each series that diverges.

I $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{-1}$

II $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^3-7n-6}\right)$

III $\displaystyle\sum_{n=3}^{\infty}\left(\frac{e^{4n}}{(n-2)!}\right)$

IV $\displaystyle\sum_{n=0}^{\infty}\left(\frac{n^{1-3n}}{4^{2n}}\right)$

(A) -9 (B) 81 (C) 15 (D) 23 (E) NOTA

25. Find the interval of convergence for the following power series:

$$\sum_{n=1}^{\infty}\left(\frac{6^n(4x-1)^{n-1}}{n}\right)$$

(A) $\left(-\frac{1}{24},\frac{1}{24}\right)$ (B) $\left(0,\frac{1}{2}\right)$ (C) $\left(\frac{5}{24},\frac{7}{24}\right)$ (D) $\left(\frac{1}{12},\frac{1}{4}\right]$ (E) NOTA

26. Deekshita and Vishnu are asked to solve the following differential equation with initial condition $y(3) = 0$ and to calculate $y(5)$:

$$\frac{dy}{dx} = e^{-y}(2x-4)$$

Deekshita (correctly) finds the exact value of $y(5)$. Vishnu is lazy and decides to (correctly) estimate $y(5)$ using Euler's method with two equivalent steps. Find the absolute value of the difference between Deekshita's answer and Vishnu's answer.

(A) $\frac{2e^2\ln(3)-2e^2-4}{e^2}$ (B) $4-2\ln(3)$ (C) $\frac{2e^2+4-e^2\ln\left(\frac{5}{3}\right)}{e^2}$ (D) $\frac{2e^2+4-e^2\ln(7)}{e^2}$ (E) NOTA

27. Evaluate:

$$\int_0^1 e^{\cos^{-1}(x)}dx$$

(A) 0 (B) $\frac{1+e^{\pi}}{2}$ (C) $\frac{1-e^{\pi}}{2}$ (D) $\frac{1+e^{\frac{\pi}{2}}}{2}$ (E) NOTA

28. Evaluate:

$$\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+\dots$$

(A) $\frac{\pi}{4}$ (B) $\frac{1-\pi}{4}$ (C) $\frac{4-\pi}{4}$ (D) $\frac{\pi-4}{4}$ (E) NOTA

29. There exists a function $y(x)$ such that:

$$\frac{dy}{dx}\cos(x)+y\sin(x) = 2\cos^3(x)\sin(x)-1$$

Given that $y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$ and $0 \le x < \frac{\pi}{2}$, find $y(0)$.

(A) $-\frac{1}{2}$ (B) $\frac{9}{2}$ (C) $\frac{11}{2}$ (D) $\frac{13}{2}$ (E) NOTA

30. Akash is standing on the shore of a circular lake with radius 3 miles. He wants to get to Vishnav, who is standing on the shore of the lake diametrically opposing Akash. Akash can run at a rate of 10 miles per hour or row at a rate of 5 miles per hour. Find the minimum amount of time, in hours, needed for Akash to reach Vishnav by utilizing some combination of running and rowing.

    (A) $\dfrac{6\sqrt{3}+2\pi}{10}$ (B) $\dfrac{6\sqrt{3}+\pi}{10}$ (C) $\dfrac{6}{5}$ (D) $\dfrac{3\pi}{10}$ (E) NOTA