For all questions, answer choice (E) NOTA means that none of the given answers is correct. For all problems, $i = \sqrt{-1}$. Good Luck!

1. If $f(n)$ is equal to the remainder when $x^4-3x^3-ax^2+bx-12$ is divided by $x-n$ and $f(1) = f(2) = f(3)$, what is $f(4)$?

   (A) 8 (B) 24 (C) 48 (D) -48 (E) NOTA

2. Mihir and Farzan are playing a game with a 100-sided die, each side labeled with a distinct number from 1 to 100 inclusive. Mihir wins if he rolls a prime number, while Farzan wins if he rolls a multiple of 6 or a multiple of 20. If they take turns rolling the die until someone wins with Farzan rolling first, what is the probability that Mihir wins?

   (A) $\frac{1}{4}$ (B) $\frac{1}{5}$ (C) $\frac{3}{20}$ (D) $\frac{1}{2}$ (E) NOTA

3. Which of the following statements about the function $f(x)$ are true?

$$f(x) = 3x^4 + 4x^2 - 7x^3 + x - 12$$

   I $f(x)$ is a quadratic polynomial.
   II By Descartes' Rule of Signs, $f(x)$ must have exactly 1 negative root.
   III By the Rational Root Theorem, $f(x)$ has 22 possible rational roots.
   IV By the Fundamental Theorem of Algebra, $f(x)$ must have 4 real roots.
   V By Vieta's Theorems, the sum of the roots when taken two at a time is $-\frac{7}{3}$.
   VI The graph of the polynomial intersects the y-axis at $(0, -12)$.

   (A) II, VI (B) II, III, VI (C) II, IV, V, VI (D) II, III, V, VI (E) NOTA

4. A polygon exists that is bounded by each of the asymptotes of $\frac{3x^2-x-4}{x^2-x-2}$ and $\frac{2x^3+7x^2-19x-60}{x^2-4x-32}$. Find the area of this polygon.

   (A) $\frac{289}{4}$ (B) $\frac{841}{4}$ (C) 132 (D) 156 (E) NOTA

5. Shreyas has two pizzas: one a circle with a latus rectum of 18 inches and one an ellipse with minor axis of 12 inches and 16 inches between its two foci. The circular pizza is cut into 9 slices of equal area while the elliptical pizza is cut into 5 slices of equal area. How many slices of the circular pizza are equivalent in area to 3 slices of the elliptical pizza?

   (A) 1 (B) $\frac{8}{3}$ (C) $\frac{16}{5}$ (D) 4 (E) NOTA

6. Given $27(3^{x-1}+1) = 9^{\log_3 7} - 9^x$, if x is expressed in the form $\log_a b$, where a and b are both integers less than 100, what is the maximum value of $a+b$?

   (A) 5 (B) 13 (C) 35 (D) 97 (E) NOTA

7. The elements of 2 x 2 matrix A form an arithmetic sequence when read from left to right, and then top to bottom. If the determinant of the matrix is -18 and the sum of its elements is 6, what is the sum of all possible values of $a_{1,2}$?

   (A) -3 (B) 0 (C) 3 (D) 6 (E) NOTA

8. What is the product of the smallest value in the domain of $f(x)$, the largest value in the domain of $f(x)$, and the sum of all integers in the domain of $f(x)$?

$$f(x) = \frac{\sqrt{9 - g(x)}}{x^2 + 4x - 21}$$
$$g(x) = \sqrt{x^2 - 9}$$

(A) -630 (B) -360 (C) 0 (D) $-\infty$ (E) NOTA

9. The Mihir-value (m-value for short) of a set is the sum of the means of all possible 2-item subsets of the set. For instance, $m(6, 2, 4) = 12$. What is $\frac{m(1, 2, 3, ...99, 100)}{101}$?

(A) 2475 (B) 2500 (C) 2525 (D) 5050 (E) NOTA

10. What is the shortest distance between the midpoint of the two holes in the graph of $f(x) = \frac{x^4 + 2x^3 - 7x^2 - 8x + 12}{x^3 + 2x^2 - x - 2}$ and the oblique asymptote of $f(x)$?

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $\frac{3\sqrt{2}}{4}$ (E) NOTA

11. How many integer values of x satisfy the equation $|x + 3| + |x - 4| < | - x + 12|$?

(A) 13 (B) 14 (C) 15 (D) 16 (E) NOTA

12. Given that $z = \frac{5 - 12i}{3 + 4i}$, if $\overline{z}$ is expressed as $a + bi$, what is $a + b + |z|$?

(A) $-\frac{24}{25}$ (B) 0 (C) $\frac{42}{25}$ (D) $\frac{88}{25}$ (E) NOTA

13. If $\frac{\log 5 + \log 25}{1 - \log 2} = \frac{\log_a 7}{\log_b 49}$, what is $\log_b a^2$?

(A) $\frac{1}{36}$ (B) $\frac{1}{6}$ (C) 12 (D) 36 (E) NOTA

14. What is $f(-2021) + f(2019)$?

$$f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 - 2x + 1$$

(A) -14 (B) -2 (C) 14 (D) 14143 (E) NOTA

15. On a coordinate plane, the point $(3, 9)$ is labelled A and the point $(3, -1)$ is labelled B. A locus is constructed of every point P in the plane such that $|PA - PB| = 6$. What is the sum of the x-intercepts of the two asymptotes of this locus?

(A) $-\frac{7}{3}$ (B) $\frac{7}{4}$ (C) 6 (D) 8 (E) NOTA

16. Evaluate $\sum_{n=-\infty}^{3} \frac{1}{n^2 - 11n + 28}$.

(A) $\frac{11}{18}$ (B) $\frac{11}{6}$ (C) $\frac{22}{3}$ (D) $\frac{7}{4}$ (E) NOTA

17. Given $f(x) = x^4 + 2x^3 - 3x^2 + 8x + 4$, let r be the cube of the sum of the reciprocals of the roots of $f(x)$, and s be the reciprocal of the sum of the cubes of the roots of $f(x)$. A quadratic polynomial $g(x)$ has roots r and s, and a constant term of -16. What is the leading coefficient of $g(x)$?

(A) 50 (B) 100 (C) -100 (D) 400 (E) NOTA

18. The graphs of $\log(x) + \log(y) = 3$ and a circle centered at the origin with radius $10\sqrt{61}$ are drawn in the coordinate plane. What is the sum of the x- and y-coordinates of a point at which the two graphs intersect?

(A) 45 (B) 61 (C) 81 (D) 90 (E) NOTA

19. A ball is dropped from 60 feet above the ground. Each time it hits the ground, it bounces to $\frac{2}{3}$ the height of its last bounce. When the ball is at the peak of its second bounce, a second ball is dropped from that height and bounces in the same pattern as the first ball. At the peak of each consecutive bounce, another ball is released from the height of that bounce. In feet, what is the total distance covered by all the balls?

(A) 180 (B) 360 (C) 540 (D) 720 (E) NOTA

20. What is the remainder when $f(x) = x^{2019} + 3x^{2018} + 2x^{2017} - 12$ is divided by $x^2 + x - 2$?

(A) 2 (B) $x - 4$ (C) $x - 12$ (D) $2x - 8$ (E) NOTA

21. Point A is the maximum point (with respect to the y-coordinate) in the graph of $0.5x^2+2y+5x = 3.5$. Points B and C are the maximum and minimum points (again with respect to the y-coordinate), respectively, of $x^2+y^2 = 8x+7y-8$. What is the tangent of angle CAB?

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 1 (E) NOTA

22. If $\left(\frac{12}{\sqrt[3]{49} - \sqrt[3]{35} + \sqrt[3]{25}}\right)^3$ is expressed as $\sqrt[3]{x} + \sqrt[3]{y} + \sqrt[3]{z}$, find $x + y + z$. (Hint: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$)

(A) 12 (B) 420 (C) 432 (D) 2148 (E) NOTA

23. Evaluate:

$$\sum_{z=2}^{5}\sum_{x=1}^{15}\sum_{y=0}^{\infty}\frac{x}{z^y}$$

(A) 420 (B) 730 (C) 855 (D) 960 (E) NOTA

24. The focus of a parabola is the point $(5, 2)$ and its directrix is the line $x = 1$. The focus of the parabola is also one of the foci of an ellipse and the directrix of the parabola coincides with the minor axis of the ellipse. If the endpoints of the ellipse's minor axis have the same y-coordinate as those of the parabola's latus rectum, what is the distance between the rightmost point of the ellipse and the leftmost point of the parabola?

(A) $4\sqrt{2}$ (B) $2\sqrt{2} + 4$ (C) $4\sqrt{2} - 2$ (D) $2\sqrt{2} + 2$ (E) NOTA

25. Justin gets a super-combo at China First, which allows him to purchase exactly 12 items of food. Each item of food can either be rice, noodle, chicken, or veggie. If Justin chooses the items randomly, what is the probability that he gets at least 2 chicken and at least 2 veggie items?

(A) $\frac{11}{12}$ (B) $\frac{73}{91}$ (C) $\frac{356}{455}$ (D) $\frac{1013}{1024}$ (E) NOTA

26.

$$A = \begin{bmatrix} 2 & x & x+3 \\ 6 & -5 & 5 \\ 2x & 2 & x^2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} x & 2 \\ 10 & 4 \end{bmatrix}$$

Given the above values of A, B, and C, what is the product of the values of x that satisfy the equation $|A| = |B^{-1}C + C|$?

(A) 6 (B) 16 (C) $\frac{64}{3}$ (D) $\frac{128}{3}$ (E) NOTA

27. What is the coefficient of the $\frac{yz\sqrt{x}}{x^3}$ term in the expanded form of $(4x+3y+2z)^{-\frac{1}{2}}$?

(A) 1 (B) 2 (C) $\frac{3}{8}$ (D) $\frac{9}{64}$ (E) NOTA

28.

$$f(x) = \begin{cases} 1 + \sqrt{4-x^2}, & 0 \le x \le \sqrt{2}, \\ -\mid x-6 \mid +7, & \sqrt{2} < x \le 10, \\ 3 \text{ - } \sqrt{-x^2+24x-140}, & 10 < x \le 12 \end{cases}$$

Find the area beneath the above piecewise function and bound by the y-axis, the x-axis and $x = 12$.

(A) $50 - \frac{\pi}{2}$ (B) $24 + 2\pi$ (C) $100 + \pi$ (D) 40 (E) NOTA

29. Evaluate:

$$\frac{\prod_{n=1}^{4}(1+i)^n}{\sum_{n=1}^{2019} ni^n}.$$

(A) $i+1$ (B) $2i + \frac{1}{505}$ (C) $\frac{i+4}{1010}$ (D) $\frac{8+8i}{505}$ (E) NOTA

30. An assassin, duke, captain, contessa and ambassador are playing a game involving an infinitely large pot of money. The players begin with 0 coins and each turn, a player draws one coin from the pot, unless they can use an ability, in which case, they will do that instead. Additionally, if a player has 7 coins or more at the beginning of their turn, they will eliminate the player with the most money besides them and lose 7 coins. Each player has certain abilities:

- Assassin: When beginning a turn at 3 coins, will eliminate the player with the most coins beside them and be set back to 0 coins. However, if the player the Assassin tries to eliminate is the Contessa, the person with the next-most coins (not including the Assassin) will be eliminated instead.
- Duke: Takes 3 coins each turn.
- Captain: Each turn, steals 2 coins from the player with the most coins besides them.
- Contessa: Cannot be eliminated by the Assassin's ability.
- Ambassador: Copies the role of a player after they are eliminated until the next player is eliminated.

The players play in the same order they are listed in above. Who is the last one left standing?

(A) Assassin (B) Duke (C) Captain (D) Contessa (E) NOTA