QUESTION 1

Oh no! Your very good friend, Probass the fish, is being chased by many fishermen! In Probass's magic fishmobile, he must cover a certain distance, A meters, in a certain amount of time, B seconds. Given x is greater than 10, for each part, find the rate the fishmobile must go in m/s when:

$$W: \quad \text{A} = x^3 - 7x - 6 \text{ , B} = x^2 - x - 6$$
$$X: \quad \text{A} = x^3 - 8x^2 + 11x + 20 \text{ , B} = x^2 - 9x + 20$$
$$Y: \quad \text{A} = x^3 - 37x - 84 \text{ , B} = x^2 + 7x + 12$$
$$Z: \quad \text{A} = x^3 - 2x^2 - 43x - 40 \text{ , B} = x^2 + 6x + 5$$

Find the sum of the roots of $\dfrac{W \cdot Y \cdot Z}{X}$.

QUESTION 2

A school named Decisive Academy has three different courses (math, geography, and history) and students are required to take at least one course. You survey a certain number of students to figure out the courses they are taking. You learn that 30 students take math, 27 students take geography, and 19 take history. You also learn that 12 students take math and geography, 7 students take geography and history, and 5 students take history and math. Finally, you learn that 3 students take all three courses. Let:

$$
\begin{aligned}
A &= \text{the total number of students that take exactly two courses} \\
B &= \text{the total number of students that only take math} \\
C &= \text{the total number of students surveyed at Decisive Academy} \\
D &= \text{the total number of students who only take one course}
\end{aligned}
$$

Find A + B + C + D.

QUESTION 3

Starting with 2, add 1 for every true statement, and subtract 2 for every false statement:

$I.$ $a = a$ is an example of the Symmetric Property of Equality

$II.$ $g = e$ and $e = f$, then $g = f$ is an example of the Transitive Property of Equality

$III.$ $(a + b) + c = a + (b + c)$ is an example of the Associative Property of Equality

$IV.$ $a(b + c + d) = ab + ac + ad$ is an example of the Distributive Property of Equality

What is the final value?

QUESTION 4

Prabhas tried to create a story for this question, but he got sleepy and took a nap. He dreamt of being teleported outside of a secret math temple with the following engraved on the wall:

Given the following functions:

$$f(x) = 3x^3 + 1$$
$$g(x) = 3x^2 + 9x + 23$$

Let:

$$A = g(3)$$
$$B = g(f(1))$$
$$C = f^{-1}(32)\text{, rounded to the nearest integer}$$
$$D = \text{the lowest value that } g(x) \text{ can get to, while using integer x-values}$$

Find A + B + C + D.

QUESTION 5

Let:

$$\begin{aligned} A &= 4 \cdot i^{200} \\ B &= \text{the simplification of } \frac{\sqrt{5}+2\sqrt{3}}{\sqrt{2}} \text{ with a rationalized denominator} \\ C &= \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{\ldots}}}} \\ D &= \sqrt{12-\sqrt{12-\sqrt{12-\sqrt{\ldots}}}} \end{aligned}$$

Find $\sqrt{A+\frac{B}{\sqrt{2}}+C^2+D^3}$.

QUESTION 6

Dylan is trying to juggle, practice his penmanship, and write his own test. The time it takes for him to write his test is inversely proportional to the cube root of the time he spends juggling, is and directly proportional to double the time he spends practicing his penmanship. If he spends 1 hour juggling and 4 hours practicing his penmanship, he would spend 8 hours writing his test. Let:

A = the amount of time he spends on his penmanship, if he spends 7 hours on writing his test, and 8 hours on juggling (round to nearest whole number)

B = the amount of time he spends on writing his test, if he spends 27 hours on juggling and 10 hours on his penmanship (round to nearest whole number)

C = the amount of time he spends on juggling, if he spends 2 hours on writing his test, and 6 hours on his penmanship (round to nearest whole number)

D = the difference between the square root of the first positive integer that is both a perfect square and a perfect cube and the opposite of the 3rd smallest prime number

Find A - B + C + D.

QUESTION 7

Let:

$$\begin{aligned} A &= \text{the number of prime numbers between 1-105, inclusive} \\ B &= \text{the sum of all the numbers between 1-100, inclusive} \\ C &= \text{the sum of all the even numbers between 1-50, inclusive} \\ D &= \text{the number of odd numbers between 1-153, inclusive} \end{aligned}$$

Find A + B - C + D.

QUESTION 8

Let:

A = the coordinates of the intersection between the perpendicular bisector of a line segment with endpoints (2, 4) and (10, 12), and the line with equation $y = 2x + 7$

B = the coordinates of a point P that lies on the line segment S with endpoints (4, 6) and (24, 58), such that the distance between P and the point (4,6) is 25% of the length of S

C = the coordinates of the intersection point with the smaller y-value of the equations $y = x^2 + 6x + 12$ and $y = 2x^2 + 10x + 12$

Find the area of the triangle with endpoints A, B, and C.

QUESTION 9

Farzan is trying to guess the passwords (multiple) to someones computer, which are sequences of positive, integral digits. Let:

A = the largest numerical password such that it is 4 digits long, even, is a multiple of 3, has a tens digit that is 3 less than the units digit, has a hundreds and thousands digit that are multiples of the tens digit, and has a thousands digit that is greater than the hundreds digit

B = the second password, if it is 4 digits long, is a power of 6, and the first three digits are the same number

C = the third password, if it is six digits long, if each digit is the same number, and the number is a root of the equation $x^3 - 73x - 72$

D = the fourth password, if it is 5 digits long, if it is known that the number is a multiple of 4, and if the three-digit number formed by the ten thousands, thousands, and hundreds digits is the square of the two-digit number formed by the tens and units digits. The units digit also equals the sum of its factors

Find the 4 digit number that contains the first digits of A, B, C and D in that order.

QUESTION 10

Given that:

$$\text{if } x > y,\ x\&y = x^y - y^x + xy - yx + 6yx^2 - 5x^2y + 1$$
$$\text{if } x < y,\ x\&y = y^x - x^y + yx - xy + 6xy^2 - 5y^2x + 1$$
$$\text{if } x = y,\ x\&y = 2^x + 2^y + 5^x + 5^y$$

Let:

$$\begin{aligned} A &= 1\&2 \\ B &= 4\&3 \\ C &= 3\&(2\&1) \\ D &= 1\&(2\&3) \text{ - } (1\&2) \end{aligned}$$

Find A - B + C + D.

QUESTION 11

Let:

$$\begin{aligned} A &= \text{the number of real solutions to } \sqrt{x+9}+3=x \\ B &= \text{the number of the real solutions to } |x+3|+3=4x+2 \\ C &= \text{the largest real solution to } ||x+5|+|2x+3||=16 \end{aligned}$$

Find A + B + C.

QUESTION 12

Starting with $x = 2019$, for every true statement, subtract 12 from x, and for every false statement, add 10 to x.

a. e is a transcendental number

b. 2023 is a prime number

c. there are 25 whole numbers between -3 and 24, inclusive

d. π is a rational number

e. $2^{30} > 3^{20}$

f. the last two digits of 125^2 is 25

g. the quadratic formula is $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$

h. i^{203} is a rational number

What is the final value of x?

QUESTION 13

Given the test scores:

$$\{52, 39, 65, 99, 101, 22, 63, 65, 39, 96, 48, 46, 67, 95, 65\}$$

Let:

A = the sum of the mode and range of these scores

B = the sum of the median and the maximum of these scores

C = the mean, rounded to the nearest integer, if every other score (the first score, the third score...) was increased by 5

D = the mean, rounded to the nearest integer, if the answers to parts A and B were added to the set of scores

Find the median of the set {A, B, C, D, 55}.

QUESTION 14

A group of people are painting a building, and their names are Molly, Polly, Dolly, and Bob. Molly can paint a building in 9 hours, Polly can paint a building in 8 hours, Dolly can paint a building in 7 hours, and Bob can scrape away the paint on a building in 6 hours. Let:

A = the amount of time (in hours) it takes Molly, Polly, and Dolly to paint a building

B = the amount of time (in hours) it takes Molly and Dolly to paint a building while Bob is scraping

C = the amount of time (in hours) after 12 AM that it takes for Molly, Dolly, and Polly to paint the whole building, if Dolly starts at 1:00 AM, Molly joins at 3:30 AM, and Bob begins scraping at 4:00 AM

Find 191A + 33B + 341C.