

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- Over the past few years, the number of students enrolled at James S. Rickards High has been increasing. The number of students has been increasing by 20 percent every year. Currently, there are 1500 students enrolled. How many students will be enrolled in 3 years?  
(A) 3600 (B) 2400 (C) 2592 (D) 1092 (E) NOTA
- Let  $x = -4$ ,  $y = 4$ , and  $a = 3i$ . Find the area of an ellipse in the complex plane, given that it has foci  $x$ ,  $y$  and a point on its perimeter  $a$ .  
(A)  $15\pi$  (B)  $20\pi$  (C)  $25\pi$  (D)  $28\pi$  (E) NOTA
- Find the remainder when  $9x^4 + 4x^3 + 22x^2 + x$  is divided by  $3x + 2$ .  
(A)  $-\frac{262}{27}$  (B)  $-\frac{127}{27}$  (C)  $\frac{262}{27}$  (D)  $\frac{127}{27}$  (E) NOTA
- Karthik is visiting Sandy's house. Its unique shape piqued his interest, so he decided to find the volume of it. It comprises of a hemisphere with a radius of 12 feet and a cylinder cut in half with a radius of 5 feet and height of 6 feet (which do not intersect). What is the volume of Sandy's house?  
(A)  $1227\pi$  (B)  $2304\pi$  (C)  $150\pi$  (D)  $2454\pi$  (E) NOTA
- Just like Sandy's house, Kevin's conical house is quite unique. It has a vertical cross section perpendicular to the circular base that has an area of  $48\sqrt{3}$  square inches. Given that the cross section is an equilateral triangle and that the plane of the cross section is 4 inches away from the axis of the cone, find the volume of Kevin's house.  
(A)  $450\pi$  (B)  $288\sqrt{3}\pi$  (C)  $504\pi$  (D)  $512\pi$  (E) NOTA
- Find the sum of all the (real) solutions to  $|x - 3| + |x - 4| = 7$ .  
(A) 7 (B) 0 (C) 14 (D) 4 (E) NOTA
- Three circles of radius 6 are in the same plane such that each circle passes through the center of the other two. Find the common area among the three circles.  
(A)  $72\pi - 72\sqrt{3}$  (B)  $9\pi$  (C)  $36\pi - 18\sqrt{3}$  (D)  $18\sqrt{3}$  (E) NOTA
- Phineas's head is in the shape of triangle  $PHI$  with  $PH = 2$ ,  $HI = 4$ , and  $PI = 3$ . Given that  $N$  is the midpoint of  $PH$ , what is the length of  $NI$ ?  
(A)  $\frac{7}{2}$  (B)  $3\sqrt{2}$  (C)  $\frac{\sqrt{46}}{2}$  (D) Does Not Exist (E) NOTA
- Eric's favorite soccer ball is a regular icosahedron with 20 faces and 3 equilateral triangles meeting at each vertex. What is the product of the number of faces, vertices, and edges of a regular icosahedron?  
(A) 4800 (B) 5400 (C) 6400 (D) 7200 (E) NOTA
- Lines  $AB$  and  $CD$  intersect at  $E$ , Given that  $CE = 5$ ,  $DE = 7$ ,  $AE = 8$ , and points  $A, B, C, D$  are concyclic, what is the length of  $BE$ ?  
(A)  $\frac{35}{8}$  (B)  $\frac{56}{5}$  (C)  $\frac{4}{4}$  (D)  $\frac{17}{4}$  (E) NOTA
- Find the sum of the digits of the answer to  $1004^3 - 996^3$ .  
(A) 18 (B) 19 (C) 17 (D) 9 (E) NOTA

12. Ananya is interested in the Binomial Theorem and learns that it is valid for non-integer exponents. Using this new knowledge, she finds the sum of the coefficients of all the terms of the binomial expansion:  $(x - y)^{10}$ . What is her answer?
- (A) 0 (B) 1 (C) 10 (D)  $10^{10}$  (E) NOTA
13. Let  $a$  and  $b$  be real numbers such that  $a^2 + b^2 = 1$ . Find the maximum value of  $3(1 - a) + 4(1 - b)$ .
- (A) 2 (B) 5 (C) 7 (D) 12 (E) NOTA
14. Eric wants to read the logs in Among Us. However, in this version of Among Us, he must compute the value of  $\frac{\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \dots + \log \frac{99}{100}}{\log_4(1024)}$  to read the logs. Given that Eric is successful in reading the logs, what was his answer?
- (A)  $-1$  (B)  $-0.5$  (C)  $0.5$  (D) 1 (E) NOTA
15. The asymptotes of a hyperbola have equations of  $x = y + 8$  and  $x = -y - 6$ . Given that the hyperbola passes through  $(10, -4)$  and that its equation is  $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$  with  $a, b > 0$ , find  $\frac{ab}{|h+k|}$ .
- (A) 8 (B) 9 (C) 10 (D) 12 (E) NOTA
16. Akhil and Nitish are on their way to Rickards High School in their respective vehicles. Line  $X$  represents Akhil's path and Line  $Y$  represents Nitish's path. Line  $X$  has equation  $y = 2x + 5$  and Line  $Y$  has x-intercept 3 and y-intercept 7. At the exact moment that Line  $X$  and Line  $Y$  intersect, evaluate the sum of the coordinates of the point at which they meet?
- (A)  $\frac{77}{13}$  (B)  $\frac{73}{13}$  (C)  $\frac{83}{13}$  (D)  $\frac{84}{13}$  (E) NOTA
17. Tanmay is known for using alliteration in his confusing statements. He approaches Karthik and asks him to find the inverse of the contrapositive of the converse of the following statement, "If Tanmay tells twice, then he twice tells." Assuming that he tells the correct answer to Tanmay, what does Karthik say?
- (A) "If Tanmay does not tell twice, then he does not twice tells."  
(B) "If Tanmay tells twice, then he twice tells."  
(C) "If Tanmay twice tells, then he tells twice."  
(D) "If he twice tells, then Tanmay tells twice."  
(E) NOTA
18. Evaluate  $|3 + 2i|$
- (A)  $\sqrt{13}$  (B)  $\sqrt{15}$  (C)  $\sqrt{17}$  (D)  $\sqrt{19}$  (E) NOTA
19. Danny has a peculiar obsession with logarithms. In the middle of Mr. Juhasz's class, he runs up to the board and writes this question "What is the characteristic of  $\log_3 226$ ?". Help out Danny's class and select the correct answer to that question.
- (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA
20. Let  $ERIC$  be a rectangle. Eric comes across  $ERIC$  and chooses a random point in the rectangle. Let  $X$  be that randomly chosen point. Given that  $RX = 3$ ,  $IX = 4$ , and  $CX = 6$ , find the length of  $EX$ .
- (A)  $\sqrt{31}$  (B)  $\sqrt{27}$  (C)  $\sqrt{33}$  (D)  $\sqrt{35}$  (E) NOTA

21. Given that a parabola has focus  $(2, 3)$  and vertex  $(2, 7)$ , find the equation of this parabola.  
(A)  $y = \frac{1}{16}(x + 2)^2 - 7$  (B)  $y = \frac{1}{16}(x - 2)^2 + 7$  (C)  $y = \frac{1}{16}(x + 2)^2 - 3$  (D)  $y = \frac{1}{16}(x - 2)^2 + 3$  (E) NOTA
22. Evaluate the remainder when  $37^{38}$  is divided by 10.  
(A) 7 (B) 9 (C) 3 (D) 1 (E) NOTA
23. On the Buck Lake bus to Rickards, everyone noticed that Sid and Nitish were singing the song "Umbrella" by Rihanna surprisingly well. Karthik was taken aback and said "You guys are singing in perfect harmony!". When Sid hears Karthik say this, he became angry for some reason and yelled at Karthik to calculate the harmonic mean of Nitish's birthday digits. Given that Nitish's birthday digits are 5 and 24, find the harmonic mean of these two numbers.  
(A)  $\frac{190}{29}$  (B)  $\frac{230}{29}$  (C)  $\frac{260}{29}$  (D)  $\frac{270}{29}$  (E) NOTA
24. Calculate the determinant of  $\begin{bmatrix} 2 & -4 & 7 \\ -3 & 1 & 5 \\ 8 & -9 & 2 \end{bmatrix}$ .  
(A) 29 (B) 32 (C) 23 (D) 43 (E) NOTA
25. Given the following equation  $f(x) = 9x^2 - 3x + 4$ , find the minimum real value of  $f(x)$ .  
(A)  $\frac{15}{4}$  (B)  $\frac{13}{4}$  (C)  $\frac{11}{4}$  (D)  $\frac{9}{4}$  (E) NOTA
26. When Tanmay and Akhil were biking on the Southwood trail, Tanmay got the sudden urge to annoy Akhil, so he listed odd facts about bicycles for an hour straight. Akhil finally snapped, and screamed at Tanmay to find the area of a cyclic quadrilateral with side lengths 5, 7, 2, and 4. Calculate the area of this cyclic quadrilateral to assist Tanmay.  
(A)  $2\sqrt{30}$  (B)  $2\sqrt{70}$  (C)  $3\sqrt{30}$  (D)  $3\sqrt{70}$  (E) NOTA
27. A regular hexagon's side length is equivalent to the following equation  $20 + \frac{4(\sqrt{169})}{(6+5+2)} - 2\sqrt{\left(\frac{500}{5}\right)}$ . Evaluate the length of this hexagon's apothem.  
(A)  $2\sqrt{5}$  (B)  $2\sqrt{3}$  (C)  $4\sqrt{5}$  (D)  $4\sqrt{3}$  (E) NOTA
28. Suhas tells Sukeerth that there exists a regular polygon that has an exterior angle of  $36^\circ$ . Find how many sides this polygon has.  
(A) 10 (B) 12 (C) 8 (D) 6 (E) NOTA
29. Given that  $i = \sqrt{-1}$  and  $(i - 3)(x + yi) = 12i$ , evaluate  $y$ .  
(A)  $\frac{18}{5}$  (B)  $\frac{-18}{5}$  (C)  $\frac{21}{5}$  (D)  $\frac{-21}{5}$  (E) NOTA
30. What is the area of a triangle with side lengths 3, 4, and 5?  
(A) 6 (B) 12 (C) 10 (D) 20 (E) NOTA