QUESTION 1

Let:

$$\begin{aligned} A &= \arccos\left(\frac{-1}{2}\right) \\ B &= \tan\left(\frac{\pi}{12}\right) \\ C &= \csc\left(\frac{-5\pi}{6}\right) \\ D &= \arctan\left(\frac{-\sqrt{3}}{3}\right) \end{aligned}$$

Calculate $A + B + C + D$

QUESTION 2

Let:

$$\begin{aligned} A &= \cos(15^\circ)\cos(75^\circ) \\ B &= \cos(15^\circ)+\cos(75^\circ) \\ C &= \sin^2(37.5^\circ)+\cos^2(37.5^\circ) \\ D &= \text{Triangle } ABC \text{ is a right triangle with all integer side lengths, with } AB=37 \text{ and } AC=12. \text{ Find } \cot(2B). \end{aligned}$$

Find $2A+B^2+C+840D$

QUESTION 3

Let:

$$A = \text{the determinant of} \begin{bmatrix} 3 & -13 & 5 & 4 \\ 0 & 9 & 1 & -10 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

B = the volume of the parallelepiped defined by the vectors $(4, 7, 3), (-9, 5, 1), (6, -2, -6)$

C = the volume of the tetrahedron with vertices $(2, 2, 0), (-8, 6, 12), (-2, 0, 6), (4, 8, -10)$

D = the volume of the tetrahedron defined by the vectors $(1, 3, 5), (6, -2, 4), (5, 0, 7)$

Calculate $\dfrac{A+B}{DC}$

QUESTION 4

Let:

$$
\begin{aligned}
A &= \text{The eccentricity of the conic described by the formula } (y-3)^2 = 8x - 16 \\
B &= \text{The area of the conic described by the formula } 4x^2 - 12x + y^2 + 8y = 0 \\
C &= \text{The length of the conjugate axis of the conic described by the formula } 9x^2 - 16y^2 - 54x + 64y = 127. \\
D &= \text{The length of the transverse axis of the conic described by the formula } 9x^2 - 16y^2 - 54x + 64y = 127.
\end{aligned}
$$

Calculate $\frac{ABCD}{\pi}$.

QUESTION 5

Given $f(x) = 11\sin(4x - 8\pi) + 5$, let:

$$\begin{aligned} A &= \text{the period of } f(x) \\ B &= \text{the frequency of } f(x) \\ C &= \text{the absolute value of the difference between the maximum and minimum values of } f(x) \\ D &= \frac{\text{phase shift of } f(x)}{\text{vertical shift of } f(x)} \end{aligned}$$

Calculate $\dfrac{ABC}{D}\pi$

QUESTION 6

Let:

$$\begin{aligned} A &= \text{The dot product of} < 7, 11, 12 > \text{and} < 8, 10, 9 > . \\ B &= \text{The sum of the components of the cross product of} < 16, 5, 7 > \text{and} < 3, 9, 13 > . \\ C &= \text{The cosine of the angle formed by the vectors} < 3, 8, 5 > \text{and} < 10, 2, 7 > . \\ D &= \text{The area of triangle } ABC \text{ given that } AB = 10,\ BC = 18, \text{ and } \angle B = 30^\circ. \end{aligned}$$

Calculate $A + B - 7C\sqrt{34} - D$.

QUESTION 7

Let:

$$\begin{aligned} A &= \lim_{x \to -6} \frac{x^2 - 36}{x^2 + 3x - 18} \\ B &= \lim_{x \to \frac{\pi}{4}} \frac{\sin^2 x + \cos^2 x}{\cos x} \\ C &= \lim_{x \to \infty} x \ln(x) \end{aligned}$$

(NOTE: if the answer is ∞ let C = 1, if answer is $-\infty$ let C = -1, otherwise use the exact answer)

$$D = \lim_{x \to e} \frac{\ln(x) - 1}{x - e}$$

Find $3e \cdot AB^2CD$

QUESTION 8

Let:

A = The remainder when $-x^4 + 7x^3 + 5x^2 - 23x + 107$ is divided by $x - 2$.

B = The sum of the possible numbers of non-real roots of $-x^4 + 7x^3 + 5x^2 - 23x + k$, given k is a real number.

C = The sum of the roots taken 2 at a time of $x^3 + 37x^2 + 23x - 403 = 0$.

D = The eighth roots of unity can all be expressed in the form $a + bi$. Find the sum of the eighth roots of unity where a is positive.

Calculate $C - \sqrt{\frac{2AB}{3}} + (D-1)^2$.

QUESTION 9

Starting with the value of -1, add 2 for every true statement and subtract 6 for each false statement below to compute the final answer.

1. For a matrix A, $\det(A^T) = \det(A)$.
2. The slope of the tangent line to the function $y = x^3 - 1$ when $x = 1$ is 2.
3. The eccentricity of the conic section $r = \dfrac{9}{7 - 3\cos\theta}$ is $\dfrac{7}{3}$.
4. The graph given by the parametric equations $x(t) = 3 + 4\sec(t)$ and $y(t) = 6 + 5\tan(t)$ is a hyperbola.
5. $\tan(3x) = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$.
6. -1 is a primitive 4th root of unity.
7. The determinant of a rotation matrix is always 1.

QUESTION 10

Let:

$$\begin{aligned}
A &= \text{The sum of the first 10 positive perfect cubes.}\\
B &= \text{The sum of the terms in the sequence } 1,\ 1,\ \frac{3}{4},\ \frac{4}{8},\ \frac{5}{16},\ \ldots\\
C &= \text{the positive value that satisfies the equation } \sum_{n=3}^{\infty}\left(5x^{n-2}+35x^{n-1}\right)=75.\\
D &= \text{The minimum positive integer } x \text{ such that the series } \frac{x}{12}+\frac{x^2}{123}+\frac{x^3}{1234}+\frac{x^4}{12345}+\ldots \text{ does not converge.}
\end{aligned}$$

Evaluate $\sqrt{A}-(B+D)C$.

QUESTION 11

Let:

$$\begin{aligned}
A &= \text{the modulus of } \frac{9+3i}{1+3i} \\
B &= \text{the product of the coordinates of } 8e^{\frac{-4\pi i}{3}} \cdot 5e^{\frac{7\pi i}{6}} \text{ in rectangular form} \\
C &= \text{the distance between } \left(6, \frac{-3\pi}{4}\right) \text{ and } \left(4, \frac{-7\pi}{4}\right)\text{, where both points are polar.} \\
D &= \text{the sum of the elements of } \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{18}
\end{aligned}$$

Calculate $\dfrac{B\sqrt{3}}{ACD}$

QUESTION 12

Let:

A = the sum of the possible values of the angular coordinate in the range $[0, 2\pi]$ that represent the point $(1, -\frac{\sqrt{3}}{3})$ in its polar form.

B = the number of petals formed by the graph expressed as $r = 20\cos(22\theta)$.

C = the number of ways to arrange the letters in the name of the shape expressed by the equation $r = \frac{17}{5 + 3\sin\theta}$.

D = the number of intersections between the graphs $x^2 + y^2 = 2022$ and $r = \frac{10}{5 + 6\cos\theta}$.

Calculate $\frac{ACD}{B\pi}$.

QUESTION 13

Let:

A = the area of the circumcircle of $\triangle$ABC where $BC = 8$ and $\angle A$ measures 135°

B = the cosine of the largest angle in a triangle with side lengths 17, 15, 21

C = 40 N of force is placed on a box at an angle of 30° and another 60 N is applied at an angle of 210°. Find the magnitude of the total force applied to the box.

Find $\dfrac{51ABC}{64\pi}$

QUESTION 14

Akhil is making a stone path that will surround two adjacent rectangular gardens. He is also making an additional stone path in between the two gardens that is equal to the width of the garden. Let $A =$ the maximum area, in yd^2, he can make one of the gardens if he can make 360 ft of stone path in total and the gardens have identical dimensions.

$B =$ The maximum volume, in ft^3, a closed box can have if it is made from a total of 156 ft^2 of cardboard
$C =$ The minimum distance between $(10, 6)$ and $y = 5x - 18$.

A soda company wants to create cylindrical cans that each use 120π cm^2 of aluminum. Let $D =$ the maximum volume, in cm^3, the cans can be, assuming that all sides of the can are made with aluminum.

Solve $\dfrac{1000\pi BC}{AD}$