

Name: \_\_\_\_\_

School: \_\_\_\_\_

1. \_\_\_\_\_ The probability that Wesley takes speed math instead of mental math is  $\frac{1}{2}$ . The probability that Wesley gets any odd numbered question correct on this test is  $\frac{1}{2}$ , and the probability he gets any even numbered question correct on this test is  $\frac{1}{16}$ . What is the base 2 logarithm of the probability that he gets a perfect on this test?
2. \_\_\_\_\_ At the Greek Sweet Shop, Tanmay buys a falafel wrap which totals to \$6.49 after taxes. Tanmay is known to be a very meticulous person, so he pays with exact change. Let  $D$  be the number of paper bills he pays with (1-dollar bills or 5-dollar bills). Let  $C$  be the number of coins he pays with (pennies, nickels, dimes, quarters). Find the positive difference between the maximum and minimum value of  $D + C$ .
3. \_\_\_\_\_ Find  $17^3$ .
4. \_\_\_\_\_ Evaluate  $895 + 462 - 221 * 3$
5. \_\_\_\_\_ The Southgate common room can be represented as a rectangular prism with dimensions 9 feet by 12 feet by 112 feet. Ananya, who stands at 5 feet 2 inches, wonders how many Ananya clones could be suspended lengthwise in the space diagonal of the room. Help her find how many clones she needs to generate to have abundant resources to execute the task at hand.
6. \_\_\_\_\_ Find the total number of distinct prime factors of  $675 \cdot 406$ .
7. \_\_\_\_\_ Nitish finds an atrocious carrot slice (which tastes terrible) in his dinner. Rather than eating it, he chooses to keep it in his wallet because it slightly resembles a coin (it is cylindrical with a smaller height than its radius). Given that the volume is 96 cube units and the radius and height are integers, how many possible radii could the carrot slice have?
8. \_\_\_\_\_ If the equation  $x^2 - 2022x + k = 0$  only has one solution, what is the value of  $k$ ?
9. \_\_\_\_\_ Find the value of the constant term when  $(18x - 2y + 3z)^4$  is expanded.
10. \_\_\_\_\_ Karthik refuses to shuffle cards in a normal manner, because he believes it isn't truly random. He only uses a truly random shuffling method called "washing," which involves mixing the cards as chaotically as possible on a table. Given that the card on the top of the deck before shuffling was the Queen of Spades, what is the probability that the Queen of Diamonds is on the bottom of the deck after shuffling? Express your answer as a decimal to the nearest hundredth.
11. \_\_\_\_\_ How many trailing zeroes are in  $\frac{2022!}{22!}$ ?
12. \_\_\_\_\_ A fair coin is tossed 19 times and lands on heads every time. What is the probability that the next toss is heads?
13. \_\_\_\_\_ Simplify  $\sqrt{7} + \sqrt{28} + \sqrt{8} + \sqrt{32}$ .
14. \_\_\_\_\_ Sruthi, Himal, and Tanmay bring up numbers often when they text one another. However, Sruthi only uses base 7, while Himal and Tanmay only use base 10. Find the sum in base 10 of the number Tanmay and Sruthi would interpret when Himal tells them that she is 12 years old.
15. \_\_\_\_\_ Find the slope of the tangent line of  $y = \frac{1}{x}$  when evaluated at  $e$ .
16. \_\_\_\_\_ Akhil is a big Meghan Trainor fan and identifies with the "Megatronz." He's a little basic so his favorite Meghan Trainor song is "All About that Base." How many trailing zeroes does  $2022!$  have when expressed in base 17?
17. \_\_\_\_\_ Evaluate  $0!$
18. \_\_\_\_\_ Sharvaa only wears socks for gamers! He has 6 distinct pairs of gamer socks. One day, Saathvik mixes up Sharvaa's sock drawer. If Sharvaa picks socks at random from the drawer, how many socks does he need to pick to ensure that he has a matching pair to wear?
19. \_\_\_\_\_ Calculate  $\binom{10}{5}$ .

20. ----- Find the 6<sup>th</sup> triangular number.
21. ----- Find the 4<sup>th</sup> pentagonal number.
22. ----- Find the sum of the numbers in the 5<sup>th</sup> and 8<sup>th</sup> rows of Pascal's triangle (assuming row 0 has only 1 number).
23. ----- What day of the week will the Rickards Invitational be, assuming it will be on November 20<sup>th</sup>, 2027 (not confirmed) and we are reasonable people?
24. ----- Dylan picks a random integer from the bound  $[11, 38]$ . What is the probability that when the number is enumerated (i.e. 1 would become 1<sup>st</sup>, 5 would become 5<sup>th</sup>), neither of the two letters in the superscript contain the letter 't'?
25. ----- A roll of tape's center hole has a radius of 2 inches. The tape itself extends another 2 inches outwards. The height of the roll of tape is 1.5 inches. The volume of the tape can be expressed as  $k\pi$ . Find  $k$ .
26. ----- Find the positive difference between the number of permutations of the letters of *WEEKND* and *WEEKEND*.
27. ----- Let  $A$  be the LCM of 2022 and 1001. Compute the sum of the distinct prime factors of  $A$ .
28. ----- Find the harmonic mean of the following set of numbers: 6, 12, 15, 5, 3. Express your answer as an improper fraction.
29. ----- Find the arithmetic mean of the following set of numbers: 29, 30, 2, 18, 92, 57, 52, 27, 11, -1. Express your answer as a decimal.
30. ----- Find the eccentricity of the following conic:  $x^2 - y^2 + 9 = 10$ .
31. ----- What is the area of the largest hexagon that can fit in a circle with area  $36\pi$ ?
32. ----- Simplify  $\sqrt{8800}$  into simplest radical form.
33. ----- Find the minimum value of  $f(x) = 6x^2 + 24x - 17$ .
34. ----- Ms. Cross works 8 hours 5 days a week. How many minutes does she work for over the course of a 9-week period?
35. ----- Find the number of positive integers that are coprime to, and less than 2024.
36. ----- Himal, Sharvaa, Tanmay, Srijan, and Jesse make dinner reservations for FAMAT States 2023. How many different ways can they sit on a round dinner table (permutations that are rotations of one another are not distinct)?
37. ----- Mr. Reedy (who is alive today, November 12, 2022) notices that in the future, when he turns age  $X$ , the year will be  $X^2$ . Find  $X$ .
38. ----- There are 4 points on a circle. When each pair of points is connected by a line segment, what is the maximum number of different regions created within the circle?
39. ----- It takes Ananya  $\frac{9}{17}$  of an hour to eat a dish of some devious mac and cheese. How many whole dishes of mac and cheese can she eat in 89 hours?
40. ----- Evaluate  $\langle 8, 1, 7, 2, 5 \rangle \cdot \langle 1, -3, 6, 9, -4 \rangle$