QUESTION 1

Given:

$$\begin{aligned} f(x) &= (x^4 + 2x^2 + 1)^4 \\ g(x) &= \sin(x^4 + 2x^2 + 1) + \cos(x^4 + 2x^2 + 1) \end{aligned}$$

Let:

$$\begin{aligned} A &= f'(2) \\ B &= \text{the number of terms in } f(x) \\ C &= \text{the maximum value of } g(x) \end{aligned}$$

Find $\sqrt{0.00001AB} + C^2$.

QUESTION 2

Let:

$$\begin{aligned} A &= \lim_{x\to 0} \frac{\ln(x^4+2x^2+1)-\ln(x+1)}{x} \\ B &= \lim_{x\to 0} \frac{\ln(x^4+2x^2+1)-\ln(x^2+1)}{x^2} \\ C &= \lim_{x\to 0} \frac{\sin\left(x^4+2x^2+1\right)-\sin 1}{\cos\left(x^4+2x^2+1\right)-\cos 1} \end{aligned}$$

The value $A+B+C$ can be expressed as $\cot(\theta)$ for some $0<\theta<\pi$. Compute θ.

QUESTION 3

Let:

$$\begin{aligned} f(x) &= 2^{x^4+2x^2+1} + 4^{x^4+2x^2+1} + 8^{x^4+2x^2+1} \\ g(x) &= \sin(x^4 + 3x^2 + 2x) \\ A &= f''(0) \\ B &= g'(0) \end{aligned}$$

Find $\dfrac{A}{\ln 2} + 2B$.

QUESTION 4

Let:

$$\begin{aligned} A &= \text{the maximum area of a rectangle with perimeter 36} \\ B &= \text{the maximum volume of a cone with circumradius 3} \\ C &= \text{the maximum volume of a sphere that can be inscribed in a rectangular prism with volume 36} \\ D &= \text{the maximum area of a triangle that can be inscribed in a rectangle with perimeter 36} \end{aligned}$$

Find $A + 3B + C - 2D$.

QUESTION 5

A particle X travels along the x-axis and another particular Y travels along the y-axis. The x-coordinate of particle X is dictated by $f(t) = 3\cos t$ and the y-coordinate of particle Y is dictated by $g(t) = 4\sin t$. Let M be the midpoint of the positions of X and Y.

$$\begin{aligned} A &= \text{the maximum distance between } X \text{ and } Y \\ B &= \text{the maximum distance between } M \text{ and the origin} \\ C &= \text{the rate of change of the distance between } X \text{ and } Y \text{ at } t = 0 \\ D &= \text{the rate of change of the distance between } M \text{ and the origin at } t = 0 \end{aligned}$$

Compute $A + B + C - 2D$.

QUESTION 6

For an infinitely differentiable function $f(x)$, let $f^{@}(x)$ be defined as

$$\int_0^x \int_0^y (f(z) + 3f'(z) + 3f''(z) + f'''(z))e^z \, dz dy$$

Let $g(x) = x^4 + 2x^2 + 1$. Compute $g^{@}(0)$.

QUESTION 7

Let $p_n(x)$ denote the non-constant polynomial with leading coefficient of 1 and minimal degree, such that $p_n(i) = p_n'(i)$ for all $i = 1, 2, \ldots, n$. For example $p_1(x) = x$ because $p_1(1) = p_1'(1) = 1$.

$$\begin{aligned} A &= p_2(3) - p_2'(3) \\ B &= p_3(4) - p_3'(4) \\ C &= p_4(5) - p_4'(5) \\ D &= p_5(6) - p_5'(6) \end{aligned}$$

Compute $A + B + C + D$.

QUESTION 8

f and g are smooth functions on $\mathbb{R}$ satisfying the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	3	1	2
2	5	3	1	5
3	2	1	2	0
4	1	2	2	4
5	2	0	3	1

Also, let $f^n(x) = f(f(\cdots f(x)\cdots))$, where there are n applications of f being applied. Let:

$$A = \text{the derivative of } f(g(f(x))) \text{ at } x = 3.$$

$$B = \text{the derivative of } f(x) + f(f(x)) + f(f(f(x))) \text{ at } x = 1.$$

$$C = \text{the derivative of } \frac{f(x)}{6^1} + \frac{f^2(x)}{6^2} + \frac{f^3(x)}{6^3} + \cdots \text{ at } x = 1 \text{ (assume convergence).}$$

$$D = \text{the derivative of } \frac{g(x)}{6^1} + \frac{g^2(x)}{6^2} + \frac{g^3(x)}{6^3} + \cdots \text{ at } x = 1 \text{ (assume convergence).}$$

Compute $\dfrac{A+B}{D-C}$.

QUESTION 9

Let:

$$\begin{aligned}
A &= \lim_{x\to\infty} (1+x)^{\frac{1}{x}} \\
B &= \lim_{x\to 0} (1+x)^{\frac{1}{x}} \\
C &= \text{maximum of } f(x) = x^{\frac{1}{x}-2e} \text{ for } x > 0 \\
D &= \text{maximum of } g(x) = x^{\frac{1}{x^2}} \text{ for } x > 0
\end{aligned}$$

Compute $\ln(A)\ln(B) + \ln(C)\ln(D)$.

QUESTION 10

For positive integers a, b, c, d, let $f(a, b, c, d) = 1$ if the limit

$$\lim_{x \to 0} \frac{\sin^a(x)(1 - \sin(x))^b}{\cos^c(x)(1 - \cos(x))^d}$$

is well-defined and non-zero, and $f(a, b, c, d) = 0$ otherwise. Compute

$$\sum_{a=1}^{100} \sum_{b=1}^{100} \sum_{c=1}^{100} \sum_{d=1}^{100} f(a, b, c, d).$$

QUESTION 11

Let:

$$A = 1 \text{ if the following series converges or 2 if it diverges: } \sum_{n=1}^{\infty} \frac{n^{10}}{n^{12}-100}$$

$$B = 3 \text{ if the following series converges or 4 if it diverges: } \sum_{n=1}^{\infty} \frac{n^{2}}{\sqrt{n^6+2n^2+1}}$$

$$C = 5 \text{ if the following series converges or 6 if it diverges: } \sum_{n=1}^{\infty} \frac{\ln(n^4+2n^2+1)}{n}$$

$$D = 7 \text{ if the following series converges or 8 if it diverges: } \sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Find $A+B+C+D$.

QUESTION 12

Unscramble the following words having something to do with math. Your answer to each part will be the unscrambled word:

A = ALERTING (there are two mathematical words possible, one is better suited for calculus, choose that one)
B = ALLPEARL
C = BOHRSUM
D = EDISONMIN

The first letters of the answers to the 4 parts rearrange to form a common 4-letter word. Find this word (write your answer in all uppercase letters).

QUESTION 13

A triangle has three angles that are all prime numbers, and the longest side length is 10.

$$\begin{aligned} A &= \text{the perimeter of this triangle, rounded to the nearest integer} \\ B &= \text{the area of this triangle, rounded to the nearest tenth} \end{aligned}$$

Compute AB.

QUESTION 14

For a polynomial $p(x)$, define $p(x)'$ to be $p(x)+p'(x)$. Let $f(x)=x^4+2x^2+1$.

$$\begin{aligned} A &= f(x)'' \text{ evaluated at } x=1 \\ B &= \int_0^1 f(x)dx \\ C &= \int_0^1 f(x)'dx \\ D &= \int_0^1 f(x)'e^x dx \end{aligned}$$

Compute $A-B+C+D$.