For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Compute the first derivative of $f(x) = x^4 + 2x^2 + 1$ at $x = 3$.

   (A) 112 (B) 115 (C) 120 (D) 125 (E) NOTA

2. Compute the second derivative of $f(x) = x^4 + 2x^2 + 1$ at $x = 3$.

   (A) 112 (B) 115 (C) 120 (D) 125 (E) NOTA

3. Compute the value of a such that for the function $f(x) = x^4 + ax^2 + 1$, we have $f'(3) = f''(3)$.

   (A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA

4. Fill in the two blanks to make a true statement: The function $f(x) = \sin(x^4 + 2x^2 + 1)$ has a local ________ that is not absolute, and the function $g(x) = \cos(x^4 + 2x^2 + 1)$ has a local ________ that is not absolute.

   (A) maximum, maximum
   (B) maximum, minimum
   (C) minimum, maximum
   (D) minimum, minimum
   (E) NOTA

5. Compute the maximum **slope** of a line tangent to $f(x) = \ln(x^4 + 2x^2 + 1)$.

   (A) 1 (B) $\frac{e}{2}$ (C) 2 (D) e (E) NOTA

6. If $\lim_{x\to\infty} \frac{x^4 + 2x^2 + 1}{x^a} = b$, where a and b are positive integers, compute $a + b$.

   (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

7. Compute the following limit

$$\lim_{x\to 0} \frac{\sin(x^4 + 2x^2 + 1) - \sin(1)}{x^2}$$

   (A) $\sin(1)$ (B) $\cos(1)$ (C) $2\sin(1)$ (D) $2\cos(1)$ (E) NOTA

8. The derivative of $y = (x^4 + 2x^2 + 1)^2$ at $x = a$ is a perfect cube, and a is an integer. Which of the following could not be a?

   (A) 1 (B) 8 (C) 36 (D) 216 (E) NOTA

9. Compute the product of the x-coordinates of the points of inflection of $f(x) = (x^4 + 2x^2 + 1)^{-1}$.

   (A) -3^{-1} (B) -5^{-1} (C) -7^{-1} (D) -9^{-1} (E) NOTA

10. Compute the slope of the line tangent to $x = y^4 + 2y^2 + 1$ at $(100, 3)$.

    (A) 100^{-1} (B) 112^{-1} (C) 120^{-1} (D) 150^{-1} (E) NOTA

11. Let $f(x)$ be a differentiable function defined on all real numbers such that the function $g(x) = f(x^4 + 2x^2 + 1)$ is non-decreasing. Compute $f(2) - f(0)$.

    (A) 0 (B) 1 (C) 2 (D) Not enough info (E) NOTA

12. Compute the maximum value of $f(x) = (x^4 + 2x^2 + 1)^{1/2} - 4x$, where the function has domain $[-1, 2]$.
(A) 1 (B) 2 (C) 4 (D) 6 (E) NOTA

13. What is the value of c guaranteed by the Mean Value Theorem for Derivatives on the function $f(x) = (x^4 + 2x^2 + 1)^{1/2} - 4x$ on the interval $[-1, 2]$?
(A) 0 (B) 0.5 (C) 1 (D) Does Not Exist (E) NOTA

14. Let $f(x) = x^4 + 2x^2 + 1$. Compute the sum of the squares of all real values of x such that $f(x) = f'(x)$.
(A) 10 (B) 12 (C) 14 (D) 15 (E) NOTA

15. Let $f(x) = x^4 + 2x^2 + 1$ and $g(x) = f(x) + f'(x) + f''(x) + f'''(x) + \cdots$. Compute $g(3) - g'(3)$.
(A) 96 (B) 100 (C) 120 (D) 144 (E) NOTA

16. Compute the maximum y-coordinate of an inflection point of $y = \ln(x^4 + 2x^2 + 1)$.
(A) $\ln 2$ (B) 1 (C) $\ln 3$ (D) $\ln 4$ (E) NOTA

17. Compute the following limit:

$$\lim_{x \to 0} (x^4 + 2x^2 + 1)^{\frac{1}{x}}.$$

(A) 0.5 (B) 1 (C) 2 (D) 4 (E) NOTA

18. Let $f(x) = x^4 + 2x^2 + 1$. Compute $f'(1) + f''(1) + f'''(1) + \cdots$.
(A) 24 (B) 48 (C) 72 (D) 84 (E) NOTA

19. Let $f(x) = x^4 + 2x^2 + 1$. The unique line passing through the origin that is tangent to $f(x)$, that has positive slope, has slope r. Compute the nearest integer to r^2.
(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

20. Let $F(x) = (x^4 + 2x^2 + 1)^5$. Compute the constant term of $F''(x)$.
(A) 5 (B) 10 (C) 15 (D) 20 (E) NOTA

21. Rolle's theorem can be applied for the function $f(x) = x^4 + 2x^2 + 1$ on the values $x = a$ and $x = b$, where $a \neq b$. Compute $a + b$.
(A) -1 (B) 0 (C) 1 (D) Not enough info (E) NOTA

22. Navya and Haasini are walking on the number line. Both start at the origin, and after t seconds, Navya's position is modeled by $N(t) = t^4 + 2t^2 + 1$, while Haasini's position is modeled by $H(t) = t^8 + 2t^4 + 1$. Compute the rate in which the distance between them is increasing at time $t = 2$.
(A) 1048 (B) 1060 (C) 1096 (D) 1200 (E) NOTA

23. Let $f(x) = x^4 + 2x^2 + 1$. Let a, b be distinct integers. Which of the following could be the slope of the line through $(a, f(a))$ and $(b, f(b))$?
(A) 20 (B) 25 (C) 26 (D) 27 (E) NOTA

24. Sukeerth's favorite line is $y = x$. Let $f(x) = x^4 + 2x^2 + 1$. Let s be the x-coordinate of the closest point on $y = f(x)$ to Sukeerth's favorite line. Compute $(s^3 + s)^{-2}$.

(A) 16 (B) 18 (C) 20 (D) 24 (E) NOTA

25. Let $f(x) = \sqrt{x^4 + 2x^2 + 1}$. Compute the following sum:

$$\frac{f(0)}{2^0} + \frac{f(1)}{2^1} + \frac{f(2)}{2^2} + \frac{f(3)}{2^3} + \cdots$$

(A) 6 (B) 8 (C) 10 (D) 12 (E) NOTA

26. Compute the following limit

$$\lim_{x \to 0} \frac{(x^4 + 2x^2 + 1)^5 - (x^4 + 2x^2 + 1)^3}{(x^4 + 2x^2 + 1)^4 - 1}$$

(A) 0.125 (B) 0.25 (C) 0.5 (D) 1 (E) NOTA

27. The slope of the line tangent to $y = (x^4 + 2x^2 + 1)^{x^4+2x^2+1}$ at $x = 1$ can be expressed as $a(1 + \ln b)$ for integers a, b. Compute $a + b$.

(A) 1032 (B) 1033 (C) 2052 (D) 2053 (E) NOTA

28. The function $f(x) = x^4 + 2x^2 + 1$ is too boring for Akhil. He changes it to $A(x) = x^a + 2x^b + 1$, where a and b are Akhil's favorite positive integers. He notes that both functions are tangent to each other at $x = 1$, but $A(x)$ is not the same function as $f(x)$. Compute ab.

(A) 4 (B) 6 (C) 8 (D) Not enough info (E) NOTA

29. Eric the estimator loves using the tangent-line approximation, but sometimes he uses it too much! Help him approximate $2^4 + 2(2)^2 + 1$ using the tangent-line approximation to the graph of $y = x^4 + 2x^2 + 1$ at $(3, 100)$.

(A) -20 (B) -16 (C) 16 (D) 20 (E) NOTA

30. Tanmay has a special cone, which changes its dimensions over time. Its radius is defined by $r(t) = t^4 + 2t^2 + 1$, and its height is defined by $h(t) = r(t)t^{-4}$, where $t > 0$ is the number of seconds that have elapsed. The minimum volume that Tanmay's cone can attain is $\frac{a}{b}\pi$, where a, b are coprime, positive integers. Compute $a + b$.

(A) 256 (B) 257 (C) 258 (D) 259 (E) NOTA