QUESTION 1

Divide each given polynomial and *find the remainder*. The remainder will be the answer for each part. (If there is no remainder, the answer to the part will be 0.)

$$A = \frac{x^4 + 6x^3 + 4x^2 - 6x + 1}{x^2 - 1}$$

$$B = \frac{4x^3 + 6x^2 - 9x + 3}{2x - 1}$$

$$C = \frac{9x^4 - 16x^2 - 13x + 10}{x - 2}$$

$$D = \frac{5x^6 - 25x^5 + x^4 - 10x^3 + 25x^2 - x + 5}{x^3 - 1}$$

Find $\dfrac{D}{B} + C - A$.

QUESTION 2

$A =$ the sum of all terms in the simplification of $\begin{bmatrix} 5 & 8 \\ 9 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 13 \\ -5 & 0 \end{bmatrix}$.

$B =$ the bottom left term in the simplification of $\begin{bmatrix} 14 & 12 & -6 \\ 4 & -19 & -8 \end{bmatrix} + \begin{bmatrix} 0 & -12 & 5 \\ 23 & 35 & -7 \end{bmatrix}$.

$C =$ the value of k such that $\begin{bmatrix} 1 & 3 \\ 7 & k \end{bmatrix}$ does not have an inverse.

$$D = \begin{vmatrix} 5 & 0 & -1 \\ -9 & 4 & 8 \\ 2 & 3 & -7 \end{vmatrix}$$

Find $A + B + C - D$

QUESTION 3

If x, y, and z are integers such that

$$x + y + z = 17$$
$$x^2 + y^2 + z^2 = 105$$
$$x^3 + y^3 + z^3 = 701,$$

Let A be equal to $xy + yz + xz$.
Let B be equal to $x^2y + x^2z + xy^2 + xz^2 + y^2z + yz^2$.
Let C be equal to xyz.
Let D be equal to the largest of x, y, and z.

Find $\dfrac{B - (A + C)}{D}$.

QUESTION 4

Find the type of conic that each equation represents - if the equation is a parabola, give it a value of 1, give an ellipse a value of 2, give a hyperbola a value of -1, and give those that don't fit in a category (i.e. points, lines, or an empty set) a value of 0. Add the numbers that you receive together and add 3 to the final sum - this will be your final answer.

$$xy + 4x - 3y - 12 = 0$$

$$9x^2 - 90x + 212 = -8y^2 + 48y - 84$$

$$50x^2 - 40x + 77 = 3y^2 + 12y + 9$$

$$x^2 + x + 1 = 0$$

$$\frac{(x-5)^2}{6} - \frac{(y+3)^2}{7} = -1$$

$$x^2 + y^2 + 14x - 2y + 51 = 0$$

QUESTION 5

Given the polynomial

$$2x^6 - 7x^5 - 45x^3 - 217x^2 + 352x + 165,$$

with roots a, b, c, d, e, and f,

$$A = \text{the sum of the roots}$$
$$B = \text{the product of the roots}$$
$$C = \text{the sum of the reciprocals of the roots}$$
$$D = \text{the sum of the roots taken 4 at a time } (abcd + abce + abcf + ... + cdef)$$

Find $\dfrac{D}{A} + BC$

QUESTION 6

Find the sum of each series. (Hint: try splitting series into more recognizable ones. Use each others' parts as a guideline.)

$$A = \frac{5}{2} + \frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \frac{5}{216} + \dots$$
$$B = \frac{2}{3} - \frac{4}{15} + \frac{8}{75} - \frac{16}{375} + \dots$$
$$C = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \dots$$
$$D = \sum_{n=3}^{\infty} \frac{2}{n^2 - 2n}$$

Find $ABCD$

QUESTION 7

An architect plans to build an arch right here in Tallahassee, but because she is unoriginal, she decides to ”take inspiration” from the Gateway Arch and make the arch in the shape of a parabola. She plots her blueprint on the coordinate plane **with each unit being 1 foot**, with the axis of symmetry being $x = 3$ and the arch stopping at its x-intercepts (where it hits the x-axis or ”ground”), those being $(-1, 0)$ and $(7, 0)$. The point $(6, 14)$ is on the arch.

$A =$ the y-coordinate of the highest point on the arch
$B =$ the x coordinate of the highest point on the arch

In order to make the arch seem more original, the architect decides to add a single horizontal line segment that passes through $(3, 8)$ and has endpoints on the arch. In order to construct the line, she needs to pay for each foot of metal used (as the bars she uses are measured by length), with a price of \$2/ft.

$C =$ the product of the abscissas of the endpoints
$D =$ the amount of money the architect will have to spend to construct the line, rounded to the nearest integer

Find $A + B + C + D$

QUESTION 8

Given the lines

$$L_1 : x - y + 1 = 0$$

$$L_2 : 2x - 3y + 6 = 0$$

$$L_3 : 4x - 5y + 6 = 0$$

$A =$ the x-coordinate of the intersection between L_1 and L_2
$B =$ the y-coordinate of the intersection between L_2 and L_3
$C =$ the sum of the coordinates of the intersection between L_1 and L_3
$D =$ the area of the triangle formed by the points of intersection between the three lines

Find $A + B + C + D$

QUESTION 9

$$A = \text{the number of real solutions to the quadratic } 7x^2 - 14x + 6 = 0$$
$$B = \text{the discriminant of } x^2 - 5x + 8$$
$$C = \text{the maximum value of } f(x) = -\frac{5}{4}(x-5)^2 + 18$$
$$D = \text{the largest x-intercept of } y = 5x^2 - 13x + 6$$

Find $A + B + C + D$

QUESTION 10

Siblings Ashvin and Rishima are trying to go to sleep, but as usual, they end up bickering until past midnight. Both of them realize that in order to stop arguing, they're gonna have to get to the *root* of why they argue and *rationalize* what's going on. Simplify the following expressions with square roots to maybe, quite possibly, get them to sleep earlier. (Unfortunately, this hasn't happened yet and the writer of this test has not gotten enough sleep, but one can hope.)

$$A = \frac{3\sqrt{5} - \sqrt{41}}{\sqrt{3} - \sqrt{2}}$$

$$B = (\sqrt{-6})\left(\frac{\sqrt{-18}}{\sqrt{5}}\right)\left(\frac{\sqrt{405}}{\sqrt{3}}\right)$$

$$C = \frac{(\sqrt{x})y^{2.5}z^{\frac{2}{3}}}{x^2\sqrt{y^7}z^{-\frac{1}{3}}} \text{ where } x = 1, y = 3 \text{ and } z = 2$$

$$D = \frac{\sqrt{3 + 2\sqrt{2}}}{\sqrt{18 + 12\sqrt{2}}}$$

Find $\dfrac{A+B}{\dfrac{D}{C} - \dfrac{\sqrt{6}}{4}}$. If the previous expression is undefined, put 0 as your answer.

QUESTION 11

If

$$f(x) = \frac{5x^2 - 2}{3x - 1}$$

$$g(x) = 2x - 3$$

$$A = f(2)$$
$$B = g(-6)$$
$$C = g(f(3))$$
$$D = \text{the value of } x \text{ for which } f(g(x)) \text{ is not defined}$$

Find $A + B + C + D$

QUESTION 12

If 10 chickens eat 10 identical bags of feed in 5 hours, and each chicken eats the same amount of feed at the same rate, find:

$A =$ the number of hours it'd take for 2 chickens to eat 2 bags of feed

$B =$ the number of bags of feed that 15 chickens could eat in 10 hours

$C =$ the number of chickens that could eat 9 bags of feed in 3 hours

$D =$ the sum of the bags of feed needed to sustain 20 chickens for 5 hours and the number of hours it'd take for 4 chickens to eat 6 bags of feed

Find $A + B + C + D$

QUESTION 13

Suhas *loves* trees. He's spent time measuring the lengths of them and comparing them in multiple ways. He's created a formula to predict the possible amount of trees in a forest (including a fractional amount of trees?) based on the total amount of water in the forest (in gallons) using the function

$$f(g) = -|3g - 6| + 2$$

where $g =$ the number of gallons of water (after all, too much water can cause the forest to decline) and $f(g)$ is the number of trees in thousands. Suhas can only choose g such that $f(g) \geq 0$ (negative trees are not allowed)

$A =$ the positive values of g such that there are 1,000 trees in the forest.
$B =$ the minimum amount of trees that a forest can have.
$C =$ the least number of gallons of water needed to make a forest as small as possible.

Find $A + B + C$, expressed as an improper fraction.

QUESTION 14

Jenna has made a pizza. She is curious about the maximum number of pieces she can divide the pizza into with a specified number of cuts. Let:

$$A = \text{the number of pieces she can have if she makes 3 cuts}$$
$$B = \text{the number of pieces she can have if she makes 5 cuts}$$
$$C = \text{the number of pieces she can have if she makes 9 cuts}$$
$$D = \text{the number of pieces she can have if she makes 1 cut}$$

Find $A + B + C + D$.