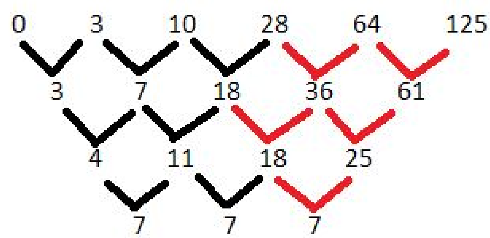
1. The equation of a parabola is $4p(y-k) = (x-h)^2$ where $|p|$ is the focal radius, and (h, k) is the center. The focus can be written as $(h, k+p)$ and the directrix can be written as $y = k - p$. Using the point and line given in the question, we see that $k + p = 6$ and $k - p = 4$. We see that $k = 5$, $p = 1$, and $h = 2$. Plugging these values into the original equation, we get $4(y-5) = (x-2)^2$. Simplifying this, we get $y = \frac{1}{4}x^2 - x + 6$, which is $\boxed{(D)}$.

2. To find the contrapositive of a statement, you switch the hypothesis and conclusion of the statement and then negate both. The contrapositive of the statement given would be "If she does not drink coffee, then Tanusri is not tired." To find the converse of a statement, you switch the hypothesis and conclusion. The converse of the new statement would be "If Tanusri is not tired, then she does not drink coffee." To find the inverse of a statemnt, you negate the hypothesis and conclusion. The inverse of the newest statement would be "If Tanusri is tired, then she drinks coffee", which is $\boxed{(A)}$.

3. We can use the finite differences method for this question. The process is shown below.



From this, we see that $f(2) = 64$ and $f(3) = 125$. $f(2) + f(3)$ is equal to 189 which is $\boxed{(D)}$.

4. According to the British Flag Theorem $AP^2 + CP^2 = BP^2 + DP^2$. Plugging in the values given, we see that $CP^2 = 5^2 + 4^2 - 2^2$. Simplifying and solving for CP, we get $\sqrt{37}$, which is $\boxed{(C)}$.

5. A conic with an eccentricity of 0 is a circle. Since the area of the circle is 72π, the radius would be $6\sqrt{2}$. $\angle ABC = 90^\circ$ signifies that AC is a diameter of the circle and B is on the circumference. The diameter is twice the length of the radius, so AC would be $12\sqrt{2}$. AC^2 is 288, and so the final answer would be 2+8+8 which is equal to 18, which is $\boxed{(D)}$.

6. Every term in this expansion is in the form of $\binom{10}{n}\left(\frac{x^3}{2}\right)^n\left(-\frac{3}{x^4}\right)^{10-n}$. Since the question is asking for the coefficient of x^9, we have to find a value n such that $3n - 4(10-n) = 9$. Solving this equation, we get $n = 7$. Now to find the coefficient, we plug 7 into the given form, and simplify. $\binom{10}{7}$ is equal to 120, the coefficient of $\left(\frac{x^3}{2}\right)^7$ is $\frac{1}{128}$, and the coefficient of $\left(-\frac{3}{x^4}\right)^3$ is 27. We multiply these values and simplify to get $-\frac{405}{16}$, which is $\boxed{(E)}$.

7. Completing the squares, we can rewrite M as $(x+3)^2 + (y-5)^2 = 100$ and see that A would be (-3, 5). We can rewrite the equation of the given line in terms of x and plug that into the equation of M. $x = y - 18$, so our equation of M would be $(y - 18 + 3)^2 + (y-5)^2 = 100$. Expanding, we get $2y^2 - 40y + 150 = 0$ and after solving, we get $y = 15$ and $y = 5$. Plugging these values back into the equation of M, we get $x = -3$ and $x = -13$, respectively. Now that we know all three points, we can find the area of $\triangle ABC$. With a base of 10 and a height of 10, the area of the triangle is 50, which is $\boxed{(A)}$.

8. We can rewrite the conic as $4(y^2 - \frac{1}{2}y) - 7(x^2 - 1) = \frac{25}{4}$. Completing the squares, we are left with $4(y^2 - \frac{1}{2}y + \frac{1}{16}) - 7(x^2 - 1 + \frac{1}{4}) = \frac{19}{4}$. We can divide both sides by $\frac{19}{4}$, and are left with $\frac{(y-\frac{1}{4})^2}{\frac{19}{16}} - \frac{(x-\frac{1}{2})^2}{\frac{19}{28}} = 1$. This is in the form of a hyperbola, where $a^2 = \frac{19}{16}$, $b^2 = \frac{19}{28}$, and the center is $(\frac{1}{2}, \frac{1}{4})$. The asymptotes of the conic would be in the form $y - \frac{1}{4} = \pm\frac{a}{b}(x - \frac{1}{2})$. We know $a = \frac{\sqrt{19}}{4}$ and $b = \frac{\sqrt{19}}{\sqrt{28}}$ (they cannot be negative because $2a$ and $2b$ are the lengths of the axes), so $\frac{a}{b} = \frac{\sqrt{7}}{2}$. The asymptotes of the conic are $y - \frac{1}{4} = \pm\frac{\sqrt{7}}{2}(x - \frac{1}{2})$. The only answer choice equivalent to

one of these equations is $y - \frac{1}{4} = \frac{\sqrt{7}}{2}(x - \frac{1}{2})$, which is $\boxed{(B)}$.

9. Let variable y equal e^{2x}. After subtracting three from both sides and substituting y in, the equation now becomes $y^2 - 4y + 3 = 0$. This can be rewritten as $(y-3)(y-1) = 0$. The only values that ensure this equation holds true are $y = 3$ and $y = 1$. Substituting e^{2x} as y, we now have $e^{2x} = 3$ and $e^{2x} = 1$. Taking the natural logarithm of both sides for both equations, we get $2x = ln3$ and $2x = ln1$. Solving for x in both equations, we get $x = \frac{1}{2}ln3$ and $x = \frac{1}{2}ln1$. These values can be simplified such that $x = ln\sqrt{3}$ and $x = 0$, and so the sum of these values is $ln\sqrt{3}$, which is $\boxed{(D)}$.

10. Since we are given the roots of $g(x)$, we can write the polynomial as $g(x) = a(x-10)(x+1)(x-9)$. If we plug in 1 for x, we get a value of $144a$. The value of $g(1)$ is given as 36, so a must be $\frac{1}{4}$. Now that we have $g(x) = \frac{1}{4}(x-10)(x+1)(x-9)$, we now just have to plug in 2 and 3 as x and then subtract the values. $g(3) = 42$ and $g(2) = 42$, so $g(3) - g(2)$ equals 0, which is $\boxed{(A)}$.

11. We can first rewrite the denominator of the fraction in terms of its factors. $2n^2 - 16n + 30$ becomes $2(n-5)(n-3)$. We can factor out $\frac{1}{2}$ so that our summation becomes $\frac{1}{2}\sum_{n=6}^{\infty} \frac{1}{(n-5)(n-3)}$. Using partial fraction decomposition we can rewrite $\frac{1}{(n-5)(n-3)}$ as $\frac{\frac{1}{2}}{(n-5)} - \frac{\frac{1}{2}}{(n-3)}$. After plugging in the first few values of n, we see that every term cancels out except for $\frac{\frac{1}{2}}{1}$ and $\frac{\frac{1}{2}}{2}$. Adding those two together, we get $\frac{3}{4}$. However, we have to multiply by the $\frac{1}{2}$ we factored out in the beginning. Our final sum is $\frac{3}{8}$, which is $\boxed{(A)}$.

12. For Karthik's rectangular prism, we know that the product of these areas is the same as the square of the product of the length, width, and height. So, to find the volume of the prism, we must solve $\sqrt{(54)(24)(36)}$, which is 216. For Tanmay's frustrum, we first have to solve for the radii. Solving the quadratic, we see that the roots are 6 and 8. The volume of a frustrum is $\frac{\pi}{3}h(R^2 + r^2 + Rr)$. Now that we have the height and radii, we can just plug in the values. The volume of the frustrum is 148π. For Eric's tetrahedron, the edge length is given, so we just have to plug the value into the formula, $\frac{a^3}{6\sqrt{2}}$. The volume of the tetrahedron is $18\sqrt{2}$. For Shubham's cube, the space diagonal will be $s\sqrt{3}$ (s is the edge length). Equating that to 12, we get that the edge length is $4\sqrt{3}$. The volume of a cube is s^3, so the volume would be $192\sqrt{3}$. Looking at these volumes, we see that 148π is the largest value, so the winner of the competition would be Tanmay, which is $\boxed{(B)}$.

13. We can write $Q(x)$ as $(x-4)P_1(x) + 3$, where $P_1(x)$ is some polynomial. Since the remainder is 6 when $Q(x)$ is divided by $x-7$, we can say that $Q(7) = 6$ (Remainder Theorem). We can now say that $Q(7) = (7-4)P_1(7) + 3 = 6$, and solving, we get $P_1(7) = 1$. If we complete the same process for $P_1(x)$, we can say that $P_1(x) = (x-7)P_2(x) + 1$, where $P_2(x)$ is some polynomial. We can substitute this value of $P_1(x)$ into $Q(x) = (x-4)P_1(x) + 3$, resulting in $Q(x) = (x-4)((x-7)P_2(x) + 1) + 3$. Expanding, we get $Q(x) = (x-4)(x-7)P_2(x) + x - 4 + 3$, which simplifies to $Q(x) = (x-4)(x-7)P_2(x) + [x-1]$. From this, we can see that $x - 1$ is the remainder when $Q(x)$ is divided by $(x-4)(x-7)$, making the answer $\boxed{(D)}$.

14. The formula for the area of a hexagon is $\frac{3s^2\sqrt{3}}{2}$, where s is the side length. Since the area of the hexagon is given as $108\sqrt{3}$, we can equate the two, and get s^2 as 72 (not simplified for easier computation later on). The area of the circumscribed circle is $s^2\pi$ because the diameter of the circle is equivalent to the diagonal of the hexagon, which has a length of $2s$. The area of the inscribed circle is $\frac{3}{4}s^2\pi$ because the radius of the inscribed circle is $\frac{s\sqrt{3}}{2}$. Plugging is 72 for s^2, we get the total area as $72\pi + 54\pi$ which is equal to 126π, which is $\boxed{(B)}$.

15. We can use Newton's Sums to solve this question. Let $S_k = r_1^k + r_2^k + r_3^k$ where the roots of the cubic are r_1, r_2, r_3. Newton's Sums states that $aS_1 + b = 0$, $aS_2 + bS_1 + 2c = 0$, and $aS_3 + bS_2 + cS_1 + 3d = 0$ where a, b, c, d are the coefficients of the cubic. Solving the first equation, we get $S_1 = 0$. Solving the second equation, we get $S_2 = -2$. Solving the third equation, we get $S_3 = -30$, which is the sum of the cubes of the roots, so our answer is $\boxed{(A)}$.

16. We can use mass points for this question. Based on the ratios of the segments, different values can be assigned to each point to "balance" the triangle out. We assign point A a "mass" of 2 and D a "mass" of 3. Point F would be assigned as 5, point C would be assigned as 1, and E would be assigned as 4. Based on these values, we can determine that the ratio of $BD : DC$ is $\frac{1}{2}$, which is $\boxed{(C)}$.

17. Using the formula $rs = A$, where r is the inradius, s is the semiperimeter of the triangle, and A is the area of the triangle, we can solve for the inradius. The semiperimeter of the triangle is 21 and the area of the triangle is 84, so the inradius is 4. We are given the center of the circle, so the equation for the incircle would be $(x-5)^2+(y+3)^2 = 16$. Using the answers choices, and plugging in the respective x and y values, we see that $(3, 2\sqrt{3}+3)$ does not lie on the circle's circumference, which makes the answer $\boxed{(C)}$.

18. We can use Stewart's Theorem for this question. If we assign AC as a, BC as b, AB as c, BD as d, AD as m, and DC as n, then Stewart's Theorem states $man + dad = bmb + cnc$. Since AD is three times the length of DC, we can say $AD = 3x$, $DC = x$, and $AC = 4x$. Plugging in the values given and the expression we just found, the equation becomes $(3x)(4x)(x) + (6)(4x)(6) = (14)(3x)(14) + (12)(x)(12)$. Simplifying, we get $12x^3 + 144x = 588x + 144x$, and solving for x, we get 7. AC would have a length of 28, but since $AB + BC < AC$, a triangle with these side lengths cannot exist, making the answer $\boxed{(E)}$.

19. By Law of Cosines, $\cos B = \frac{12^2+5^2-x^2}{2\cdot5\cdot12}$ and $\cos D = \frac{2^2+10^2-x^2}{2\cdot2\cdot10}$, where $x = AC$. Since these angles are supplementary, their cosines are negatives of each other, so we have $\frac{12^2+5^2-x^2}{2\cdot5\cdot12} = \frac{2^2+10^2-x^2}{2\cdot2\cdot10} \implies 169-x^2 = 3(104-x^2) \implies 2x^2 = 143 \implies 2x = \sqrt{286} \implies \boxed{(B)}$.

20. We can say that $\arccos(\frac{9}{15}) = \theta$. We can rewrite this equation as $\cos(\arccos(\frac{9}{15})) = \cos(\theta)$, which simplifies to $\frac{9}{15} = \cos(\theta)$. If we draw a right triangle with an angle of θ, we see that the adjacent side has a length of 9, the opposite side has a length of 12, and the hypotenuse has a length of 15. The question is asking for $\sin(\theta) + \tan(\theta)$, which is $\frac{12}{15} + \frac{12}{9}$. This simplifies to $\frac{32}{15}$, which is $\boxed{(D)}$.

21. $\log 1^1 + \log 2^2 + \log 3^3 + \cdots + \log 100^{100}$ can be rewritten as $\log\left((1^1)(2^2)(3^3)\cdots(100^{100})\right)$ and $a_2 \log 2 + a_3 \log 3 + a_5 \log 5 + \cdots + a_{97} \log 97$ can be rewritten as $\log\left((2^{a_2})(3^{a_3})(5^{a_5})\cdots(97^{a_{97}})\right)$. Since the values are equal, we realize that $(2^{a_2})(3^{a_3})(5^{a_5})\cdots(97^{a_{97}})$ is the prime factorization of $(1^1)(2^2)(3^3)\cdots(100^{100})$. We want $a_7 + a_{13}$ so we only have to find the powers of 7 and 13 in the prime factorization. The exponent of 13, a_{13}, would be $13+26+39+52+65+78+91$, or 364. These are the only multiples of 13 between 1 and 100 and 13 only appears once in their individual prime factorizations. The exponent of 7, a_7, would be $7+14+21+28+35+42+2(49)+56+63+70+77+84+91+2(98)$, or 882. These are the only multiples of 7 between 1 and 100 and 7 only appears once in their individual prime factorizations, excluding 49 and 98. Since 49 and 98 have 7^2 within their individual prime factorizations, we must multiple these numbers by 2. $a_7 + a_{13}$ would be $882 + 364$, or 1246, which is $\boxed{(C)}$.

22. We can rewrite the expression as a single fraction, which would be $\frac{(r^2+1)(1-s)+(s^2+1)(1-r)}{1-r-s+rs}$. Expanding the numerator, we get $\frac{r^2+s^2-rs(r+s)-(r+s)+2}{1-(r+s)+rs}$. We know that $r^2+s^2 = (r+s)^2 - 2rs$, $r+s = -7$, and $rs = -10$ (Vieta's Formula). Plugging these values in, we get $\frac{49+20-70+7+2}{1+7-10}$. Simplyfing, we get $\frac{8}{-2}$, or -4, which is $\boxed{(A)}$.

23. The height of the triangle is the distance between the vertex and the line, which can be rewritten as $x+y\sqrt{3}-1 = 0$. Using the formula $\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$ where a is the coefficient of the x-term in the line, b is the coefficient of the y-term in the line, c is the constant of the line, x is the x-coordinate of the point, and y is the y-coordinate of the point. $\frac{|3+4\sqrt{3}-1|}{\sqrt{1^2+\sqrt{3}^2}}$ simplifies to $1+2\sqrt{3}$. The height of an equilateral triangle with length s is $\frac{s\sqrt{3}}{2}$, so to find the side length, we equate the two and get $s = \frac{2+4\sqrt{3}}{\sqrt{3}}$. The perimeter would be $3s$, or $12+2\sqrt{3}$, which is $\boxed{(E)}$.

24. The series can be rewritten as $\frac{1}{2}+\frac{2}{6}+\frac{5}{36}+\frac{1}{4}+\frac{8}{216}+\frac{11}{1296}+\frac{1}{8}+\ldots = \frac{m}{n}$. From this, we divide the series into two, the first being $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ (a geometric series) and the second being $\frac{2}{6}+\frac{5}{36}+\frac{8}{216}+\frac{11}{1296}+\ldots$ (an arithmetico-geometric

series). For the geometric series, we can use the formula $\frac{a}{1-r}$, where a is the first term and r is the common ratio. The sum would be $\frac{\frac{1}{2}}{1-\frac{1}{2}}$, or 1. For the arithmetico-geometric series, we set it equal to S, the total sum. The denominators are in a geometric sequence with a common ratio of 6, so if we multiply the entire series by 6, we are left with $2 + \frac{5}{6} + \frac{8}{36} + \frac{11}{216} + \ldots = 6S$. If we subtract the two series, we get $2 + \frac{3}{6} + \frac{3}{36} + \frac{3}{216} + \ldots = 5S$. Now that we have a normal geometric series, we can use the same formula above to get $5S = 2 + \frac{3}{5}$, and after solving, we get $S = \frac{13}{25}$. The sum of the entire original series is $1 + \frac{13}{25}$, or $\frac{38}{25}$. Since 38 and 25 are relatively prime positive integers, $m + n = 38 + 25$, or 63, which is $\boxed{(D)}$.

25. To solve the expression, we have to essentially work backwards with the summations. Starting with the summation where $a_{10} = 1$, we find that the sum is (10)(0.000001), which is 0.00001. As we continue to calculate the value of each summation, we notice that we continue to multiply by 10. This would make our final sum equal to $(10^{10})(0.000001)$, which is 10000. log(10000) is 4, which is $\boxed{(C)}$.

26. x, y, z must have the same remainder when divided by 3 to have a sum of 60. The two possible cases are when the remainder is 1 and the remainder is 2. Looking at the first case, if the remainder of each of the numbers is 1 when divided by 3, we can rewrite each in the following way: $x = 3a + 1$, $y = 3b + 1$, and $z = 3c + 1$. Adding these values together, we get $3(a + b + c) + 3 = 60$, which simplifies to $a + b + c = 19$. Using stars and bars, we see that the number of combinations is $\binom{21}{2}$, or 210. Looking at the second case, if the remainder of each of the number is 2 when divided by 3, we can rewrite each in the following way: $x = 3a + 2$, $y = 3b + 2$, and $z = 3c + 2$. Adding these values together, we get $3(a + b + c) + 6 = 60$, which simplifies to $a + b + c = 18$. Using stars and bars, we see that the number of combinations is $\binom{20}{2}$, or 190. Overall, the total number of combinations would be $210 + 190$, or 400, which is $\boxed{C}$.

27. $5 + 2\sqrt{6}$ and $5 - 2\sqrt{6}$ are conjugates and reciprocals. Using this, we can say that $y = (5 - 2\sqrt{6})^{x^2-6x+5}$ and we can rewrite the equation as $y + \frac{1}{y} = 10$. This can be written as $y^2 - 10y + 1 = 0$, and after solving using the quadratic formula, we see that $y = 5 \pm 2\sqrt{6}$. Based on the values of y and that $5 + 2\sqrt{6}$ and $5 - 2\sqrt{6}$ are reciprocals, we can determine that $x^2 - 6x + 5 = \pm 1$. Looking at the two equations $x^2 - 6x + 4 = 0$ and $x^2 - 6x + 6 = 0$, we know the solutions for each are two distinct, real numbers (the discriminant for both equations is greater than 0). Thus, we can just add the sums of the roots of both equations without actually finding them. Using Vieta's Formula, the sum of the roots of the first equation would be 6 and the sum of the roots of the second equation would be 6. The total sum of all the real solutions is 12, which is $\boxed{(C)}$.

28. We must first find the determinant of the matrix. Since the second row has a 0 in it, finding the determinant will be a bit easier if we switch the first and second row (and then negate the final value). With $0, x-5, x$ in our top row now, the determinant would be $0-(x-5)((x+2)(3x-1)-(1)(2x-5))+(x)(0-(2x-5)(x))$. After expanding and simplifying, we are left with $-5x^3+17x^2+12x+15$. However, since we switched the rows, we must negate this value to get the determinant of the original matrix, $5x^3-17x^2-12x-15$. We know that this must equal $(x-A)(Bx^2+Cx+D)-69$, so we first add 69 to both sides. Now our equation is $5x^3 - 17x^2 - 12x + 54 = (x - A)(Bx^2 + Cx + D)$. $x - 3$ is a factor of $5x^3-17x^2-12x+54$, so A must be 3. To solve for the remaining variables, we divide $5x^3-17x^2-12x+54$ by $x - 3$, and get $5x^2 - 2x - 18$. This means that $B = 5$, $C = -2$, and $D = -18$. The question is asking for $ABC - D$, which is -12, so the answer is $\boxed{(B)}$.

29. For xyz to be divisible by 10, at least one of them must be divisible by 5 and at least one of them must be even. Since the range of the values is from 1 to 7, at least one of x, y, z must be 5. We can use complementary counting for this question. The total number of combinations is 7^3, or 343. The number of combinations without any 5's is 6^3, or 216, since there are only 6 remaining possible values for each of x, y, z. The number of combinations without any even numbers is 4^3, or 64, since there are only 4 remaining possible values for each of x, y, z. However, within these cases, we've overcounted and subtracted the number of cases without any even numbers or 5's twice, so we must add that back. The number of cases with no even numbers or 5's is 3^3. Overall, the total number of combinations would be $7^3 - 6^3 - 4^3 + 3^3$, or 90, which is $\boxed{(C)}$.

30. We have to rationalize the denominator, so we multiply the numerator and denominator by the "conjugate", $\sqrt{2}+\sqrt{7}+\sqrt{5}$. The fraction now becomes $\frac{\sqrt{2}+\sqrt{7}+\sqrt{5}}{(\sqrt{2}+\sqrt{7}-\sqrt{5})(\sqrt{2}+\sqrt{7}+\sqrt{5})}$, and after expanding and simplifying, we get $\frac{\sqrt{2}+\sqrt{7}+\sqrt{5}}{4+2\sqrt{14}}$. We have to rationalize the denominator again, so we multiply the numerator and denominator by the conjugate, which is now $4-2\sqrt{14}$. We now have $\frac{(\sqrt{2}+\sqrt{7}+\sqrt{5})(4-2\sqrt{14})}{(4+2\sqrt{14})(4-2\sqrt{14})}$, and after expanding and simplifying, we get $\frac{4\sqrt{5}-10\sqrt{2}-2\sqrt{70}}{-40}$. We can rewrite this as $\frac{\sqrt{70}+5\sqrt{2}-2\sqrt{5}}{20}$, which is $\boxed{(B)}$.