

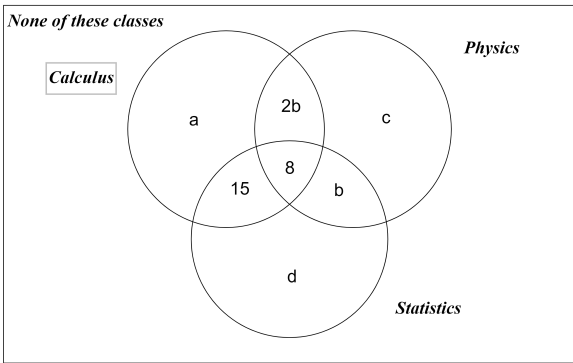
1. The treatments in this experiment are sugar pill, and 0.5g pill, so  $A = 2$ . The blocking factors are just the factors used to separate the randomly chosen participants into groups, so age is the only blocking factor, and  $B = 1$ . Including 54, there are 36 years from 54-89 inclusive, and between 18-89 there are 72 years, so there is a  $\frac{1}{2}$  chance of being in the 54+ age group. There is a  $\frac{1}{3}$  chance of being in one of the three treatment groups, so multiplying the two probabilities gives  $C = \frac{1}{6}$ . The trend described in D is called Simpson's Paradox, and there are 10 distinct letters in both words, so  $D = 10$ .  
Our final answer is  $2 + \frac{1}{6} - 1 + 10 \implies 17$ .
2. Using 1-Var Stats, the mean of this dataset is  $A = 16.7$ . Since this dataset represents a sample of our population (the population being all the members of the baseball team, and our dataset only includes the players with the most home runs) we can also use 1-Var Stats to find the sample standard deviation, with is  $B = 11.0$ . This dataset is already likely right skewed, because the mean, 16.7, is greater than the median, 12, so the value needed to make the distribution right skewed is  $C = 0$ . Of (mean, median, standard deviation, and range). mean is always an unbiased estimator of the population, median is a biased estimator when the population is not normal, and the standard deviation and range are always biased estimators, so  $D = 3$ .  
Our final answer is  $16.7 \times 11.0 \times 0 \times 3 \implies 0$ .
3. For any two independent random variables  $X$  and  $Y$ , with means and variances of  $\mu_X$ ,  $\sigma_X^2$ ,  $\mu_Y$ ,  $\sigma_Y^2$ , respectively,  $X \pm Y$  will result in a mean of  $\mu_X \pm \mu_Y$ , and a variance of  $\sigma_X^2 + \sigma_Y^2$ . Additionally, a linear transformation of  $aX \pm b$  will result in a mean of  $a\mu \pm b$  and a variance of  $a^2\sigma^2$ , for any random variable  $X$ . Finally,  $\mu_{X^2} = \mu_X^2 + \sigma_X^2$ , and  $\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 - 2p\sigma_X\sigma_Y}$  when  $X$  and  $Y$  are not independent and have a correlation coefficient of  $p$ . Given this information, we can solve each part.  $\sqrt{(2^2)(16) + (3^2)(25)}$  gives us that  $A = 17$ .  $16 + 25$  gives us that  $B = 41$ .  $(9^2)(16) + (7^2)(25)$  gives us that  $C = 171$ .  $X^2 + Y^2 + 2XY \implies (X + Y)^2$ . So,  $\mu_{X+Y} = 9 + 7 \implies 16$ , and  $\sigma_{X+Y}^2 = 16 + 25 \implies 41$ . So,  $\mu_{X+Y^2} = (16^2) + 41$ , which gives us that  $D = 297$ .  $\mu_{X+Y} = 9 + 7 \implies 16$ , and  $\sigma_{X+Y} = \sqrt{16 + 25 - 2(0.54)(4)(5)} \implies 4.40$ , and  $\mu_{X+Y} - \sigma_{X+Y}$  gives us  $E = 11.60$ .  
Our final answer is  $17 + 41 - (171 + 297 - 11.60) \implies -398.4$ .
4. The first statement is false - there is either a 0% or 100% chance the mean percentage is within this interval. The second statement is false - the interval is an estimate of the mean not a boundary of population values. The third statement is true. The fourth statement is false - we cannot be sure there are exactly 990. Finally, the fifth statement is also false - we cannot be certain there are exactly 10.  
Since the third statement is the only correct interpretation (true statement), our final answer is  $7$ .
5. The probability of finishing a 10 question team test from a set number of 15 written questions, where each question is independent and has success/fail is a binomial distribution, so  $P(X = 10) = \text{binompdf}(15, 0.65, 15)$ , which gives us  $A = 0.212$ . The probability of finding one question on the 5th trial when continuously going through questions until one is found with the same conditions as in part A is a geometric distribution, so  $P(X = 5) = \text{geometpdf}(0.65, 5)$ , which gives us  $B = 0.010$ . The probability of finishing a 10 question team test on the 13th question written with the same conditions as in part A is a negative binomial distribution, where the probability of failing 3 times before 10 successes  $= \binom{3+10-1}{3} \times 0.65^{10} 0.35^3$ , which gives us  $C = 0.127$ . Part D describes all the possible derangements of a 10 element set, which is given by  $10! \times \sum_{k=0}^{10} \frac{-1^k}{k!}$ , which gives us  $D = 1334961$ .  
Our final answer is  $(0.212 + 0.010 + 0.127) \times 1334961 \implies 465901.389$ .
6. The first 6 triangular numbers are 1, 3, 6, 10, 15, and 21. Students could not have scored a 12, and so,  $A = 0$ . Multiplying each score by its frequency then summing those products results in  $\bar{x} = 8.28 \implies B = 8.28$ . The quartiles can be found easily by looking at the cumulative distribution - the first (at 25%) lies between 0.2414 and 0.3793 so it is a 3. The third quartile (75%) lies between 0.4483 and 0.8276 so it is a 10. Therefore the IQR is 7,  $\implies C = 7$ . 1-Var Stats tells us the standard deviation of the distribution is 6.480, and  $\text{variance} = \text{standard deviation}^2$ . Therefore,  $D = 41.99$ .  
Our final answer is  $0 + 2(8.28) + 3(7) + 4(41.99) \implies 205.52$ .
7. Performing the test for the pitch type data with 3 degrees of freedom gives a  $\chi^2$  value of 56.9085. Performing the test for the pitch speed data with 3 degrees of freedom gives a  $\chi^2$  value of 0.1513.  
By multiplying these two values, we get that the final answer is  $8.6103$ .

8. Performing linear regression on this data tells us that the slope = 55.050  $\implies$   $A = 55.050$ . Performing a *LinReg* T-Test on this data tells us that  $r = 0.976 \implies B = 0.976$ . It is a known rule within bivariate statistics that the sum of all the residuals of a least squares regression line is 0. Therefore  $C = 0$ . Graphing the data or trying to approximate a nonlinear relation equation between the length and the amount of times, gives that the amount of likes is always approximately  $2(\text{length})^2$ . This shows a power model, and the amount of letters in power, is 5  $\implies D = 5$ .  
 Our final answer is  $5 + 0.976 + \frac{0}{55.050} \implies 5.976$ .

9. Using *normalcdf*, we calculate *normalcdf*(0, 35, 65, 15), which gives us that  $A = 0.02$ . Next, calculating *normalcdf*(100, 120, 65, 15) gives us that  $B = 0.01$ . The Z-Score is calculated as the difference between the observed value and the mean, divided by the standard deviation. Plugging in those values here results in  $\frac{-22-65}{15} \implies C = -5.80$ . Finding the proportion of students who scored from a 90 to a 120 can be found using *normalcdf*(90, 120, 65, 15), which is 0.0477. Multiplying this proportion by the number of students, 200, yields 9.5334, and by rounding,  $D = 10$ .  
 Our final answer is  $0.02(-5.8) + 0.01(10) \implies -0.016$ .

10. Since, by definition, the area under a pdf function is equal to 1, the equation  $y = 2x$  creates a triangle with area 1 on the interval  $[0, A]$ , with base A, and length 2A. Using  $\frac{1}{2}bh$ ,  $A = 1$ . Since X is a continuous random variable, the probability of X taking on a single value is 0, so  $P(X = 0) + P(X = 0.5) = 0 \implies B = 0$ . The median can be found by splitting the triangle in half, where the left side is a smaller triangle and the right side is a trapezoid. This smaller triangle has base x, and height 2x (from the equation  $y = 2x$ ), so area  $x^2$ . The trapezoid has bases 2x and 2, and height  $1 - x$ , so area  $1 - x^2$ . Since the two shapes have to have the same area,  $x = 0.707 \implies C = 0.707$ .  
 $P(X > 0.75 | X < 0.99) = \frac{P(0.75 < X < 0.99)}{P(X < 0.99)}$ .  $P(0.75 < X < 0.99)$  creates a trapezoid with bases 1.5 and 1.98, with height 0.24, which gives an area of 0.4176.  $P(X < 0.99)$  creates a triangle with base 0.99 and height 1.98, which gives an area of 0.9801.  $\frac{0.4176}{0.9801} = 0.4261$ .  $P(X < 0.88 | X > 0.3) = \frac{P(0.3 < X < 0.88)}{P(X > 0.3)}$ .  $P(0.3 < X < 0.88)$  creates a trapezoid with bases 0.6 and 1.76, and height 0.58, which gives an area of 0.6844.  $P(X > 0.3)$  creates a trapezoid with bases 2 and 0.6, and height 0.7, which gives an area of 0.91.  $\frac{0.6844}{0.91} = 0.7521$ .  $0.4261 \times 0.7521 = 0.320 \implies D = 0.320$ .  
 Our final answer is  $1^{0+0.707} \times 0.320 \implies 0.320$ .

11. To start solving each of the parts, one must find out how many people take each combination of Calculus, Statistics, and Physics. Drawing a Venn diagram using the information found in the question gives the following:



Using this, we can create a system of equations:

$$\begin{aligned} a + c + d &= 72 \\ c + 3b + 8 &= 57, \text{ so } c + 3b = 49 \\ b + d + 15 + 8 &= 53, \text{ so } b + d = 30 \\ a + 2b + 23 &= 40, \text{ so } a + 2b = 17 \end{aligned}$$

Adding equation 2 and 3 gives  $c + 4b + d = 79$ , and subtracting that from equation 1, gives  $a - 4b = -7$ . Adding that to 2 times equation 4 gives  $3a = 27$ , and  $a = 9$ .  $b = 4$ ,  $2b = 8$ ,  $c = 37$ , and  $d = 26$  by

using the value of  $a$  in all the equations in the system above.  $a = 9 \implies \boxed{A = 9}$ . Adding all the values in the Venn diagram gives 107, and subtracting that from the 112 total seniors gives  $5 \implies \boxed{B = 5}$ .  $P(3 \text{ classes} \mid \text{takes Calculus}) = \frac{8}{40} \implies \boxed{C = 0.20}$ , because everyone who takes 3 classes takes Calculus.  $P((\text{takes Calculus} \cup \text{takes Physics and Statistics only}) \mid 2 \text{ classes}) = 1 \implies \boxed{D = 1}$ , because the union contains everyone who takes 2 classes, according to the Venn diagram. Our final answer is  $9 + 5 + 0.20 + 1 \implies \boxed{15.20}$ .

12. The  $Z$  value for a 95% confidence interval is 1.96. Using  $z \frac{\sigma}{\sqrt{n}}$  to find the uncertainty and multiplying that by 2, we get that  $\boxed{A = 1.26}$ . Using *InvNorm*, we find that the  $Z$  value for a 99.9% confidence interval is 3.2905. Using  $z \frac{\sigma}{\sqrt{n}}$  to find the uncertainty and multiplying that by 2, we get that  $\boxed{B = 2.11}$ . Solving  $z \frac{2.27}{\sqrt{50}} = 0.4$  gives us a  $z$  value of 1.246, which we find corresponds to a confidence % of 78.72, so  $\boxed{C = 78.72}$ . Solving  $0.2 = 1.96 \frac{2.27}{\sqrt{n}}$  results in a  $n$  value of 494.8845, which, when rounded to a whole numbered sample size, yields  $\boxed{D = 495}$ . Our final answer is  $1.2584 + 2.1127 + 78.72 + 495 \implies \boxed{577.09}$ .

13.  $\frac{2}{9} \cdot \frac{1}{5} \cdot \frac{7}{10} \cdot \frac{4}{15} = \frac{28}{3375} \implies \boxed{A = 0.008}$ .  $\frac{7}{9} \cdot \frac{4}{5} \cdot \frac{7}{10} \cdot \frac{11}{15} = \frac{1078}{3375} \implies \boxed{B = 0.319}$ .  $\frac{2}{9} \cdot \frac{4}{5} \cdot \frac{3}{10} \cdot \frac{11}{15} = \frac{44}{1125} \implies \boxed{C = 0.039}$ .  $\frac{7}{9} \cdot \frac{4}{5} \cdot \frac{7}{10} \cdot \frac{4}{15} + \frac{7}{9} \cdot \frac{4}{5} \cdot \frac{3}{10} \cdot \frac{11}{15} + \frac{7}{9} \cdot \frac{1}{5} \cdot \frac{7}{10} \cdot \frac{11}{15} + \frac{2}{9} \cdot \frac{4}{5} \cdot \frac{7}{10} \cdot \frac{11}{15} = \frac{2863}{6750} \implies \boxed{D = 0.424}$ . Our final answer is  $\frac{0.008(0.319)}{0.039(0.424)} \implies \boxed{0.154}$ .

14. Statement A is false. A Bernoulli trial is one where the probability of success (and failure) is constant, each and every time the experiment is conducted. Statement B is false. The equation that represents Bayes' theorem is slightly different from what is written in the problem; it actually states that  $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ . However, this does indeed give us the probability of event A occurring given that event B occurs, as said in the statement. Statement C is true. It is impossible to completely remove bias from all aspects of an experiment. Statement D is false. The standard deviation and interquartile range are measures of spread, but the median is a measure of center, and the upper quartile is neither a measure of spread or measure of center. Finally, the number of assumptions/conditions for a Bernoulli trial is 3, which are the existence of only two possible outcomes, a fixed probability of each outcome occurring ( $p$  and  $1 - p$ ), and complete independence between each trial. Thus, the final answer would be  $(\frac{3^4}{2})^2$ , which is equal to  $\boxed{1640.250}$ .

15.  $\frac{13}{52} \cdot \frac{12}{51}$  gives us  $\boxed{A = \frac{1}{17}}$ .  $\frac{12}{52} \cdot \frac{40}{51}$  gives us  $\boxed{B = \frac{40}{221}}$ . Since there is only one King of Hearts in a normal deck,  $\boxed{C = 0}$ .  $\frac{26}{52} \cdot \frac{2}{51}$  gives us  $\boxed{D = \frac{1}{51}}$ . Our final answer is  $\frac{1}{17} \cdot \frac{40}{221} \cdot \frac{1}{51} + 0 = \boxed{\frac{120}{221}}$ .