For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Consider the graph of the equation $y = \csc(x)$. A line is tangent to this graph at r points. Find the maximum value of r.
   (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

2. Evaluate $(2-i)^8 + (2+i)^8$.
   (A) -1054 (B) -1024 (C) 512 (D) 540 (E) NOTA

3. Compute the volume enclosed by the locus of points P which satisfy $\angle APB = 90^\circ$ or $P = A, B$, where $A = (-1, 2, 6)$ and $B = (0, 10, 10)$.
   (A) $\frac{256}{3}\pi$ (B) 96π (C) 108π (D) $\frac{243}{2}\pi$ (E) NOTA

4. Compute the period of $f(x) = \sin(x) + 2\cos(2x) + 3\sin(3x)$.
   (A) 4π (B) 2π (C) π (D) 0.5π (E) NOTA

5. Find the domain of $g(x) = \frac{\sqrt{1-2x}}{\arctan(x)\arcsin(x)}$.
   (A) $[-1, 0) \cup (0, 0.5]$ (B) $[-1, 0.5]$ (C) $[-1, 0) \cup (0, 1]$ (D) $(-\infty, 0)$ (E) NOTA

6. For all values of x in its domain, simplify $f(x) = \cos(\arcsin(\cos(\arcsin(x))))$.
   (A) $\sqrt{1-x^2}$ (B) x (C) $|x|$ (D) $-x$ (E) NOTA

7. Compute the area of a parallelogram defined by the vectors $< a, b >$ and $< 1, 2 >$.
   (A) $2a - b$ (B) $b - 2a$ (C) $4a - 2b$ (D) $2b - 4a$ (E) NOTA

8. Shreyas' favorite villain is Vector from Despicable Me. In honor of him, Shreyas defines that for a vector v, $f(v) = \sqrt{||v||^2 + \sqrt{||v||^2 + \sqrt{||v||^2 + \cdots}}}$. Compute the largest positive integer that is at most $f(< f(r), f(r), f(r) >)$, given that $r =< 1, \sqrt{2}, \sqrt{3} >$.
   (A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

9. Compute the minimum possible (defined) value of $\sin^2(x) + 2\cos^2(x) + \tan^2(x)$.
   (A) 0 (B) 0.5 (C) 1 (D) 2 (E) NOTA

10. The trigonometric transformers are tantalizing to Tanmay. What is the maximum value of k such that in the list $(\sin(x), \cos(x), \tan(x), \csc(x), \sec(x), \cot(x))$, there exists a real x such that k of these quantities have the same value, and this value is defined?
    (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

11. What is the smallest positive composite integer n such that $\sin(x) = \cos(4x) = \sin(nx)$ has a real solution for x?
    (A) 6 (B) 8 (C) 9 (D) 10 (E) NOTA

12. The conic $x^2 + xy + y^2 = 3$ is rotated clockwise by θ° about $(-1, 2)$ such that the rotated image has axes that are parallel to the x-axis and y-axis $(0 < \theta < 90)$. Let $C = (r, s)$ be the coordinates of the center of the rotated conic. Compute $s - r$.
    (A) $-1 - \sqrt{2}$ (B) $1 - \sqrt{2}$ (C) $2 - \sqrt{2}$ (D) $3 - \sqrt{2}$ (E) NOTA

13. Akhil, Nitish, and Eric love choosing random numbers for no reason. Akhil chooses a random real number in the interval $[1, 2]$, Nitish chooses a random real number in the interval $[3, 4]$, and Eric chooses a random real number in the interval $[6, 10]$. What is the expected value of the sum of the sum of their numbers and the product of their numbers?

(A) 48 (B) 50 (C) 54 (D) 55 (E) NOTA

14. Based on the information of the previous question, what is the probability that the sum of the three numbers they choose exceeds 15?

(A) $\frac{1}{24}$ (B) $\frac{1}{36}$ (C) $\frac{1}{48}$ (D) $\frac{1}{50}$ (E) NOTA

15. Deekshita's favorite conic, $x^2+2xy+3y^2+4x+5y = 6$ is rotated θ° clockwise (by rotation and relabeling of the axes), and the new conic is $Ax^2+Cy^2+Dx+Ey+F = 0$. Compute the maximum possible value of $A^3+C^3+D^2+E^2+F$.

(A) 63 (B) 75 (C) 96 (D) 108 (E) NOTA

16. Karthik's favorite polar graph is $r = 1 + 2\cos\theta$. Shubham wants to spice it up a bit, and he turns it into $r = c + 2\cos\theta + 2\sin\theta$, for some positive constant c. Shubham tells you that his new curve is not conventionally called a limacon, but rather it is now a cardioid! Compute c^2.

(A) 1 (B) 2 (C) 4 (D) 8 (E) NOTA

17. Let A be the set of solutions of $x^3 = 1$ and let B be the set of solutions of $x^3 = 8$. Compute

$$\sum_{a\in A}\sum_{b\in B}|a+b|$$

(A) $9+6\sqrt{3}$ (B) $9+9\sqrt{3}$ (C) $10+10\sqrt{3}$ (D) $10+9\sqrt{3}$ (E) NOTA

18. Let $P = 1$. Farzan and Prabhas play a game, where Farzan goes first and turns are alternated. In the game, the person who goes next chooses a function from the set $\{\sin(x), \cos(x), \tan(x)\}$, and the value of P gets multiplied by the function that the person chose. If at any point in the game, past the first turn, the value of P can be simplified to the quantity $\sin^m(x)\cos^n(x)$, where m, n are even integers (not necessarily positive), Farzan wins. If Prabhas can always find a way to avoid this forever, then he wins. Who wins this game with perfect play, and if Farzan can win, what is the minimum amount of turns (of both players) in which he can guarantee a win?

(A) Prabhas (B) Farzan, 3 (C) Farzan, 4 (D) Farzan, 5 (E) NOTA

19. The polar equations $r = 2$, $\sin\theta = \cos\theta$, and $r = \cos\theta$ partition the polar plane into finite and infinite regions. Out of the finite regions, let A be the largest area of a finite region and let B be the smallest area of a finite region. Compute $A - 15B$.

(A) $1+\pi$ (B) $2+\pi$ (C) $2\pi-3$ (D) $3\pi-5$ (E) NOTA

20. Find the smallest positive integer n such that $n\sin(x) = x$ has 7 real solutions.

(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

21. Dylan's favorite conic is $x^2 + y^2 = 1$, the simplistic unit circle. His favorite point is on this circle, but he wont tell you what it is! Ms. Cross wants to know what this point is, and she know exactly the same information you do. She asks him for a clue, and you overhear the following statement:

"Both coordinates of this point are rational numbers, and if the point is (r, s), then $25rs$ is an integer."

How many possible values are there for this point, based on the information you know, assuming Dylan never lies?

(A) 8 (B) 12 (C) 16 (D) 20 (E) NOTA

22. Ananya and Navya notice that their names are really similar. How many permutations of $ANANYANAVYA$ contain either $ANANYA$ or $NAVYA$ as a continuous substring?

(A) 418 (B) 420 (C) 778 (D) 780 (E) NOTA

23. Compute

$$\lim_{x\to 1}\left(\frac{x^2+\frac{1}{x}-2}{x^3-1}+\frac{x^2-2x+1}{x-1}\right)$$

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) NOTA

24. Given that a and b are two real, 3-D vectors, both with with magnitude 1, compute the maximum possible value of $||a\times b||^2-2((a\times b)\cdot(b\times a))+||b\times a||^2$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

25. 3-D line ℓ is defined by the parametric equation $(x,y,z)=(1+2t,3-4t,5-6t)$. Let c be a constant such that $cz^2=x^2+y^2$ is tangent to ℓ. If $c=\frac{m}{n}$, where m,n are relatively prime, positive integers, compute $m+n$.

(A) 16 (B) 18 (C) 25 (D) 27 (E) NOTA

26. Let a,b,c be real numbers. If m and M are the minimum and maximum value of the expression below, compute M^2+m^2.

$$\begin{vmatrix}\sin(a) & \cos(a) & 1\\ \sin(b) & \cos(b) & 1\\ \sin(c) & \cos(c) & 1\end{vmatrix}$$

(A) 10.5 (B) 12 (C) 13.5 (D) 18 (E) NOTA

27. Let c denote the unique positive constant such that the x-axis is tangent to the graph of $c+\sin(x)=c^{\sin(y)}$. Compute the last three digits of the greatest integer less than c^{10}.

(A) 049 (B) 048 (C) 024 (D) 023 (E) NOTA

28. If x is a real number such that $\sin(x)+\cos(2x)+\sin(3x)=1$, let m denote the sum of all possible values of $\cos(4x)$. Compute $m+\frac{1}{m}$.

(A) 10.5 (B) 12 (C) 13.5 (D) 15 (E) NOTA

29. When graphed, the equation $\sin x+\cos y=3(\sin y+\cos x)$ partitions the Cartesian plane into an infinite amount of closed regions, each with area m. Compute m.

(A) $1.5\pi^2$ (B) $2\pi^2$ (C) $3\pi^2$ (D) $4\pi^2$ (E) NOTA

30. Consider the family of functions $f_n(x)=2\sin(2x)+\cos(nx)$ for all positive integers n. Let R be a continuous region which satisfies the following: For any positive integer n, any point P on the graph of $f_n(x)$ lies inside or on the boundary of R. Compute the minimum (finite) possible value of the area of the region bounded by R, $x=1$, and $x=3$, over all possible regions R.

(A) 4 (B) 2π (C) 8 (D) 16 (E) NOTA