1. Note that the line $y = 1$ clearly intersects the graph at infinitely many points, and thus the answer is $\boxed{E, \infty}$.

2. Note that $(2+i)^8 = (3+4i)^4 = (-7+24i)^2 = 49 - 576 - 336i = -527 - 336i$, and $(2-i)^8$ is just the conjugate of this, $-527 + 336i$. Therefore, the desired sum is $2(-527) = \boxed{A, -1054}$.

3. Let M be the midpoint of segment AB. Clearly the condition $\angle APB = 90°$ is equivalent to $MP = MA = MB$. This means the desired locus is a sphere with center M and radius $MA = MB$. The radius of this sphere is $0.5AB = 0.5\sqrt{1^2+8^2+4^2} = 4.5$, and thus its volume is $\frac{4}{3}\pi\left(\frac{9}{2}\right)^3 = \boxed{D, \frac{243}{2}\pi}$

4. Using the double and triple angle formulas, we get $f(x) = \sin(x) + 2(1 - 2\sin^2(x)) + 3(3\sin(x) - 4\sin^3(x)) = -12\sin^3(x) - 4\sin^2(x) + 10\sin(x) + 2$. Now, consider the minimum of this function. Note that $f(x) = -4 + (1 - \sin(x))(12\sin^2(x) + 16\sin(x) + 6)$, and using quadratic minimization, we have $12\sin^2(x) + 16\sin(x) + 6 \geq 12 \cdot \frac{4}{9} - 16 \cdot \frac{2}{3} + 6 > 0$. Since $1 - \sin(x) \geq 0$, combining both of these parts gives that $f(x) \geq -4$, with equality at $\sin(x) = 1$.

   Since the sequence of minimums is an arithmetic progression with common difference 2π, the period of the entire function is at least 2π. It is clear that 2π is a valid possible period, and the previous sentence guarantees that it must the minimal period, so our answer is $\boxed{B, 2\pi}$.

5. The principal domain of arctan and arcsin are $\mathbb{R}$ and $[-1, 1]$, respectively. The square root in the numerator gives that $x \leq \frac{1}{2}$. So far, our possible domain is $[-1, 0.5]$. However, this is a fraction, and thus the denominator cannot be 0. If $\arcsin(x) = 0$ or $\arctan(x) = 0$, then $x = 0$, so we must exclude this. Our final domain is $\boxed{A, [-1, 0) \cup (0, 0.5]}$.

6. Consider the function $g(x) = \cos(\arcsin(x))$ (so $f(x) = g(g(x))$). Note that $-0.5\pi \leq \arcsin(x) \leq 0.5\pi \implies \cos(\arcsin(x)) \geq 0$. We also have that $\sin(\arcsin(x)) = x$, so $g(x) = \sqrt{1 - x^2}$. Applying this again gives $g(g(x)) = \sqrt{1 - (1 - x^2)} = \sqrt{x^2} = \boxed{C, |x|}$.

7. The signed area of this parallelogram is $\begin{vmatrix} a & b \\ 1 & 2 \end{vmatrix} = 2a - b$, but we are looking for the unsigned area, so the area is $\boxed{E, |2a - b|}$.

8. Note that $||r||^2 = 1^2 + 2 + 3 = 6$, so $f(r) = \sqrt{6 + \sqrt{6 + \cdots}} = \sqrt{6 + f(r)} \implies f(r) = 3$ (as we always take the positive solution). This means we need to compute $f(< 3, 3, 3 >) = \sqrt{27 + \sqrt{27 + \cdots}} = \sqrt{27 + f(< 3, 3, 3 >)} \implies f(< 3, 3, 3 >) = \frac{1+\sqrt{109}}{2}$. Note that $10 < \sqrt{109} < 11 \implies 5.5 < f(< 3, 3, 3 >) < 6$, and thus the answer is $\boxed{C, 5}$

9. Note that $\sin^2(x) + 2\cos^2(x) + \tan^2(x) = 1 + \cos^2(x) + \tan^2(x) = \cos^2(x) + \sec^2(x)$. The final quantity is equal to $a + \frac{1}{a}$, where $a = \cos^2(x) > 0$, which clearly has a minimum of $\boxed{D, 2}$ by the AM-GM inequality. Equality occurs when $\cos^2(x) = 1 \implies x = k\pi \ \forall \ k \in \mathbb{Z}$.

10. Clearly we can achieve $k = 2$ with $x = \frac{\pi}{4}$. Now, I claim that $k = 3$ is not possible. If $\sin(x) = 0$, then $\tan(x) = 0$, $\csc(x), \cot(x)$ are not defined, and $\sec(x), \cos(x)$ are both 1 or -1, so $k \neq 3$ in this case, and similarly $k = 3$ is not achievable if $\cos(x) = 0$. Therefore, assume $\sin(x), \cos(x) \neq 0 \implies \sin(x), \cos(x) \neq \pm 1$.

    If $\sin(x), \cos(x) < 0$, then $\tan(x), \cot(x) > 0$ and $\csc(x), \sec(x) < 0$. This means that if $k = 3$, then 3 of the 4 of $\{\sin(x), \cos(x), \csc(x), \sec(x)\}$ are equal, but then this would mean $\sin(x) = \csc(x)$ or $\cos(x) = \sec(x)$. This contradicts $\sin(x), \cos(x) \neq \pm 1$.

    If $\sin(x) < 0 < \cos(x)$, then $\tan(x), \cot(x) < 0$ and $\csc(x) < 0 < \sec(x)$. This means that if $k = 3$, then 3 of the 4 of $\{\sin(x), \csc(x), \tan(x), \cot(x)\}$ are equal. Since $\sin(x) = \csc(x) \implies \sin(x) = \pm 1$, contradiction, we must have $\tan(x) = \cot(x) \implies \tan(x) = \cot(x) = \pm 1$. However, this means that either $\sin(x)$ or $\csc(x)$ is ± 1, meaning $\sin(x) = \pm 1$, contradiction. A similar contradiction arises when $\cos(x) < 0 < \sin(x)$. Therefore, assume $0 < \sin(x), \cos(x)$.

    If $\sin(x) = \cos(x)$, then $\sin(x) = \cos(x) < 1 = \tan(x) = \cot(x) < \csc(x) = \sec(x)$, not achieving $k = 3$. WLOG $0 < \sin(x) < \cos(x)$. This means that $\sin(x) < \cos(x) < 1 < \cot(x) < \csc(x)$. Therefore, in order for 3 of these to be equal, we must have $\tan(x) = \sec(x) \implies \sin(x) = 1$, contradiction. Therefore, no three of these quantities can be defined and equal $\implies \boxed{A, 2}$.

11. Clearly, we only need to focus on $x \in [0, 2\pi)$. If $\sin(x) = \cos(4x)$, then note that using the sum-to-product formula gives

$$0 = \cos(0.5\pi - x) - \cos(4x) = -2\sin(0.25\pi + 1.5x)\sin(0.25\pi - 2.5x) = 2\sin(0.25\pi + 1.5x)\sin(2.5x - 0.25\pi)$$

This means $0.25\pi + 1.5x = \pi, 2\pi, 3\pi$ or $2.5x - 0.25\pi = 0, \pi, 2\pi, 3\pi, 4\pi$. This means the solutions to this in $[0, 2\pi)$ are $x = \frac{1}{2}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi, \frac{1}{10}\pi, \frac{9}{10}\pi, \frac{13}{10}\pi, \frac{17}{10}\pi$. Now, looking at $\sin(x) = \sin(nx)$, we can use product-to-sum again:

$$0 = \sin(nx) - \sin(x) = 2\cos(0.5((n+1)x))\sin(0.5((n-1)x))$$

If n is even, then both $0.5(n+1)x$ and $0.5(n-1)x$ will be non-zero fractions with denominators divisible by 4, regardless of the value of x chosen. Therefore, the RHS of the equation cannot be 0. This means n must be odd. The least odd composite number is $n = 9$, and this works by choosing $x = \frac{1}{10}\pi$, so our answer is $\boxed{C, 9}$.

12. Note that the center of the original conic is $(0, 0)$, because if (x, y) is a point on the conic, then $(-x, -y)$ is also a point on it. The original counterclockwise rotation angle of this conic satisfies $\cot(2\theta) = \frac{A-C}{B} = \frac{1-1}{1} = 0 \implies \theta = 45°$. This means the axes are $x = y$ and $x + y = 0$. This means that we need to rotate $(0, 0)$ about $(-1, 2)$ 45° clockwise. Shifting the points right 1 and down 2, we need to rotate $(1, -2)$ about the origin 45° clockwise. This is

$$(1\cos(-45°) + 2\sin(-45°), 1\sin(-45°) - 2\cos(-45°)) = \left(-\frac{\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$$

This means $C = \left(-1 - \frac{\sqrt{2}}{2}, 2 - \frac{3\sqrt{2}}{2}\right)$. The desired quantity is $\boxed{D, 3 - \sqrt{2}}$

13. Let a, b, c be Akhil's, Nitish's, and Eric's numbers. We are asked $\mathbb{E}(a+b+c+abc)$. Note that $\mathbb{E}(a) = 1.5$, $\mathbb{E}(b) = 3.5$, and $\mathbb{E}(c) = 8$. Since a, b, c are independent, linearity of expectation gives

$$\mathbb{E}(a+b+c+abc) = \mathbb{E}(a) + \mathbb{E}(b) + \mathbb{E}(c) + \mathbb{E}(a)\mathbb{E}(b)\mathbb{E}(c) = 1.5 + 3.5 + 8 + (1.5)(3.5)8 = 13 + 42 = \boxed{D, 55}$$

14. The values of $a + b + c$ can be mapped as a 3-D rectangular prism, bounded by the 6 faces $x = 1, x = 2, y = 3, y = 4, z = 6, z = 10$. The desired region is the intersection between this region and $x + y + z \geq 15$. This region is a right triangular prism, with vertices $(2, 4, 10)$, $(1, 4, 10)$, $(2, 3, 10)$, and $(2, 4, 9)$. The volume of this region is $\frac{1}{6}(1)(1)(1) = \frac{1}{6}$. The rectangular prism has volume $(10-6)(4-3)(2-1) = 4$. The answer is thus the quotient of these two, $\boxed{A, \frac{1}{24}}$

15. Under rotation, $A + C$, $B^2 - 4AC$, $D^2 + E^2$, and F are all invariant. This means $A + C = 4$, $0^2 - 4AC = 2^2 - 4(1)(3) \implies AC = 2$, and $D^2 + E^2 = 4^2 + 5^2 = 41$, $F = -6$. This means

$$A^3 + C^3 = (A+C)^3 - 3AC(A+C) = 4^3 - 3(2)(4) = 40 \implies$$

$$A^3 + C^3 + D^2 + E^2 + F = 40 + 41 - 6 = \boxed{B, 75}$$

16. Note that sum-to-product gives that

$$c + 2\cos\theta + 2\sin\theta = c + 2(\cos\theta + \cos(90 - \theta)) = c + 2(2\cos 45°\cos(\theta - 45)) = c + 2\sqrt{2}\cos(\theta - 45°)$$

Now in order to be a cardioid, we must have $c = 2\sqrt{2}$, and thus c^2 is $\boxed{D, 8}$.

17. The solutions of $x^3 = 1$ are $1, \frac{-1 \pm i\sqrt{3}}{2}$, and the solutions of $x^3 = 8$ are $2, -1 \pm i\sqrt{3}$. When 2 is added to each of the roots of $x^3 = 1$, the resulting magnitudes are $3, \sqrt{3}, \sqrt{3}$. By symmetry, the magnitudes should be the same for each of the other solutions to $x^3 = 8$. Thus the total is $3(3 + 2\sqrt{3}) = \boxed{A, 9 + 6\sqrt{3}}$.

18. Clearly Farzan cannot win in 1 or 2 turns. We claim he can do it in 3. Suppose Farzan picks $\sin(x)$ first. If Prabhas picks $\sin(x)$, then Farzan immediately wins. If Prabhas picks $\cos(x)$, then Farzan picks $\tan(x)$ and immediately wins. If Prabhas picks $\tan(x)$, Farzan picks $\cos(x)$ and also wins. Therefore the answer is $\boxed{B, \text{Farzan}, 3}$

19. Converting all of the equations to Cartesian, we have $r = 2 \implies x^2 + y^2 = 4$, $\sin\theta = \cos\theta \implies y = x$, and $r = \cos\theta \implies x^2 + y^2 = x \implies (x - 0.5)^2 + y^2 = 0.5^2$. The line partitions the small circle into two pieces, the smaller one of which will have area B. This area is a quarter circle minus a 45-45-90 triangle, which means $B = \frac{1}{4}\pi(0.5)^2 - \frac{1}{2}(0.5)^2 = \frac{\pi-2}{16}$.
The line also splits the larger circle in half. One of these halves contains the region of area B, and clearly the other part of this half contains the region of area A. This means $A + B = \frac{1}{2}\pi(2^2) = 2\pi \implies A - 15B = 2\pi - 16B$, so we must have $A - 15B = 2\pi - (\pi - 2) = \boxed{B, 2 + \pi}$

20. In order for the graph $n\sin(x)$ to hit the line $y = x$ 5 times, it has to hit the peak at 2.5π as well as the trough at -2.5π, each twice, along with the origin and the two other points on the waves closest to the origin. This means that we would want n, the value at the peak 2.5π, to be greater than $2.5\pi \approx 7.85$ itself. This means we would clearly choose $\boxed{B, 8}$.

21. Clearly the points $(-1, 0), (1, 0), (0, 1), (0, -1)$ satisfy the conditions. Suppose that r, s are fractions. Note that we can group up the desired points into clusters of 4, because if (r, s) works, then so does $(-r, s), (r, -s), (-r, -s)$. Hence, WLOG assume $r, s > 0$. Let $r = \frac{a}{b}, s = \frac{c}{d}$ be fully simplified fractions. This means

$$1 = r^2 + s^2 = \frac{a^2}{b^2} + \frac{c^2}{d^2} \implies b^2d^2 = a^2d^2 + b^2c^2 \implies d^2(b^2 - a^2) = b^2c^2, b^2(d^2 - c^2) = a^2d^2$$

This means d divides bc and b divides ad. However, $\gcd(c, d) = 1$ and $\gcd(a, b) = 1$, so we must have d divides b and b divides d. This means $b = d$. Since $25rs = \frac{25ac}{b^2}$ is an integer, then b^2 divides 25, and since we assumed $b \neq 1$, we must have $b = 5$. This means $(a, c) = (3, 4), (4, 3)$, and for each pair, we can choose signs for each coordinate, this case gives $2 \cdot 4 = 8$ pairs, comining with the 4 original pairs to get a total of $\boxed{B, 12}$

22. The number of permutations with $ANANYA$ as a substring is counted by simply considering $ANANYA$ as a distinct letter, leaving 6 letters with 2 of them identical, for a total of $0.5(6!) = 360$. The number of permutations with $NAVYA$ as a substring is counted by assuming it as a distinct letter, leaving 7 letters with 3 A's and 2 N's, for a total of $\frac{7!}{3!2!} = \frac{5040}{12} = 420$. This gives 780 permutations in total, but we are double counting $ANANYANAVYA$, $NAVYAANANYA$, $ANAVYANANYA$, and $NAVYANANYAA$, giving an actual count of $780 - 4 = \boxed{E, 776}$.

23. Note that

$$\lim_{x\to1}\left(\frac{x^2 + \frac{1}{x} - 2}{x^3 - 1}\right) = \lim_{x\to1}\left(\frac{x^3 - 2x + 1}{x^4 - x}\right) = \lim_{x\to1}\left(\frac{x^2 + x - 1}{x^3 + x^2 + x}\right) = \frac{1 + 1 - 1}{1 + 1 + 1} = \frac{1}{3}$$

and we also have

$$\lim_{x\to1}\left(\frac{x^2 - 2x + 1}{x - 1}\right) = \lim_{x\to1}(x - 1) = 0$$

Combining these together gives $\boxed{D, \frac{1}{3}}$

24. Note that $a \times b = -b \times a$, so if θ is the angle between a, b, we have $||a \times b|| = ||b \times a|| = ||a||||b|| \sin\theta = \sin\theta$. Therefore

$$||a \times b||^2 - 2((a \times b) \cdot (b \times a)) + ||b \times a||^2 = 2\sin^2\theta - 2||a \times b||^2 \cos(\pi) = 4\sin^2\theta$$

The maximum is clearly $\boxed{D, 4}$.

25. The given information implies that the equation below has exactly one solution in t:

$$c(5 - 6t)^2 = (1 + 2t)^2 + (3 - 4t)^2 = 20t^2 - 20t + 10 \implies (36c - 20)t^2 + (20 - 60c)t + (25c - 10) = 0$$

This means the discriminant of the quadratic is equal to 0, so we have

$$(20 - 60c)^2 = 4(36c - 20)(25c - 10) = 400(3c - 1)^2 \implies (9c - 5)(5c - 2) = 5(3c - 1)^2$$

$$\implies 45c^2 - 43c + 10 = 45c^2 - 30c + 5 \implies c = \frac{5}{13} \implies \boxed{B, 18}$$

26. Note that the matrix quantity is equal to twice the signed area of a triangle with vertices $(\sin(a), \cos(a))$, $(\sin(b), \cos(b))$, and $(\sin(c), \cos(c))$. Note that all three points lie on the unit circle, and thus the question becomes "What is the max and min of twice the signed area of a triangle inscribed in a unit circle?". Clearly the maximum signed area is an equilateral triangle, with side length $\sqrt{3}$, and thus area $0.25s^2\sqrt{3} = \frac{3\sqrt{3}}{4}$. Therefore, the minimum signed area is simply the negative of this, $-\frac{3\sqrt{3}}{4}$. Therefore, we have $(M, m) = \left(\frac{3\sqrt{3}}{2}, -\frac{3\sqrt{3}}{2}\right)$, and

$$M^2 + m^2 = 2 \cdot \frac{27}{4} = \boxed{C, 13.5}$$

27. Note that this graph is x-periodic and y-periodic modulo 2π, so we must take this into consideration. This means if the x-axis is tangent to $c + \sin(x) = c^{\sin(y)}$, then $c + \sin(x) = c^{\sin(0)} = 1$ has exactly one solution in $[0, 2\pi)$. This clearly occurs when $c = 2$, as $\sin(x) = -1$ has exactly one solution in this interval. This means that $c^{10} = 1024$, and the greatest integer less than this is 1023, so the answer is $\boxed{D, 023}$

28. Let $a = \sin(x)$. Using the double and triple-angle identities gives

$$a + (1 - 2a^2) + (3a - 4a^3) = 1 \implies 4a^3 + 2a^2 - 4a = 0 \implies 2a(2a^2 + a - 2) = 0$$
$$\implies a = 0, \frac{-1 \pm \sqrt{17}}{4}$$

However, $\frac{-1-\sqrt{17}}{4} < -1$ is outside the range of sine. This means that $a = 0$ or $\frac{\sqrt{17}-1}{4}$, so we have

$$\cos(2x) = 1 - 2a^2 = 1 - 2(0)^2 \text{ or } 1 - 2\left(\frac{\sqrt{17}-1}{4}\right)^2 = 1 \text{ or } 1 - \frac{18 - 2\sqrt{17}}{8} = 1 \text{ or } \frac{\sqrt{17}-5}{4}$$
$$\implies \cos(4x) = 2\cos^2(2x) - 1 = 2(1^2) - 1 \text{ or } 2\left(\frac{\sqrt{17}-5}{4}\right)^2 - 1$$

The first value is equal to 1, while the second value is equal to $\frac{42-10\sqrt{17}}{8} - 1 = \frac{17-5\sqrt{17}}{4}$. This sum of these two values is $m = \frac{21-5\sqrt{17}}{4}$, and so our answer is

$$\frac{21 - \sqrt{441 - 16}}{4} + \frac{4}{21 - \sqrt{441 - 16}} = \frac{21 - \sqrt{441 - 16}}{4} + \frac{21 + \sqrt{441 - 16}}{4} = \frac{21}{2} = \boxed{A, 10.5}$$

29. Note that we can rewrite the equation as follows:

$$\sin(x) - 3\cos(x) = 3\sin(y) - \cos(y) \implies \frac{1}{\sqrt{10}}\sin(x) - \frac{3}{\sqrt{10}}\cos(x) = \frac{3}{\sqrt{10}}\sin(y) - \frac{1}{\sqrt{10}}\cos(y)$$

Let θ be the acute angle that satisfies $\sin\theta = \frac{3}{\sqrt{10}}$ and $\cos\theta = \frac{1}{\sqrt{10}}$. This means we have that

$$\cos\theta\sin(x) - \sin\theta\cos(x) = \sin\theta\sin(y) - \cos\theta\cos(y) \implies \sin(x - \theta) = -\cos(y + \theta)$$
$$\implies \sin(x - \theta) + \sin(0.5\pi - y - \theta) = 0$$

Using sum-to-product (again) we see that

$$2\sin\left(\frac{0.5\pi - 2\theta + x - y}{2}\right)\cos\left(\frac{x + y - 0.5\pi}{2}\right) = 0$$

This means that either $\frac{0.5\pi - 2\theta + x - y}{2} = k\pi \implies y - x = -2k\pi + 2\theta - 0.5\pi$ $(k \in \mathbb{Z})$ or $\frac{x+y-0.5\pi}{2} = \frac{(2m+1)\pi}{2} \implies x + y = 2m\pi + 1.5\pi$ $(m \in \mathbb{Z})$. These two equations are families of parallel lines, which are equally spaced at a distance of $\frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}$ apart. This means the graph partitions the coordinate plane into squares with side length $\pi\sqrt{2}$, and thus with area $\boxed{B, 2\pi^2}$

30. Note that regardless of the value of n, $2\sin(2x) - 1 \le f_n(x) \le 2\sin(2x) + 1$. In fact, as n approaches infinitely, the graph of f_n has a frequency that approaches infinity as well. Therefore, for any region R that satisfies the conditions, it must contain all points in the interval $[2\sin(2x) - 1, 2\sin(2x) + 1]$ for all $x \in \mathbb{R}$, in order to account for the infinitely increasing frequency of $f_n(x)$. Therefore, the width of R at any $x \in \mathbb{R}$ has to be at least $2 = (2\sin(2x) + 1) - (2\sin(2x) - 1)$. This means that by Cavalieri's Principle, the area bounded by R, $x = 1$, and $x = 3$ has to be at least $2(2) = \boxed{A, 4}$.