*For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!*

1. How many petals does the curve $r = \sin\theta\cos\theta$ have?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

2. Find the largest value of $x$ such that $5^x$ divides $f'(5)$ for the function:

$$f(x) = \prod_{n=1}^{100} x^n$$

   (A) 5049 (B) 5050 (C) 5051 (D) 5052 (E) NOTA

3. Mr. Argand plots all the solutions to the equation $x^{12} = 64$ on the complex plane, and connects them with lines to form a convex polygon. What is the area of this polygon?

   (A) 2 (B) 3 (C) 6 (D) 12 (E) NOTA

4. Convert the following rectangular coordinate to polar form: ($\sqrt{3}$,3)

   (A) $(\sqrt{3}, \frac{\pi}{3})$ (B) $(\frac{\pi}{3}, 2\sqrt{3})$ (C) $(\frac{\pi}{3}, \sqrt{3})$ (D) $(\sqrt{3}, \frac{\pi}{6})$ (E) NOTA

5. Find the amplitude of the periodic function $f(x) = 24\sin x\cos x + 16\sin^2(x) + 9\cos^2(x)$

   (A) 24 (B) 25 (C) 9 (D) 16 (E) NOTA

6. Which of the following answer choices is not a complex number?

   (A) $i + 1$ (B) $e^{i\pi}$ (C) $2i$ (D) 1 (E) NOTA

7. What is the volume of the parallelepiped defined by the vectors $< 2, 4, 1 >, < -2, -1, 2 >, < 3, 2, 5 >$?

   (A) 5 (B) 10 (C) 25 (D) 45 (E) NOTA

8. Evaluate:

$$\lim_{x\to\sqrt[4]{3}} \frac{\sqrt{\arctan(x^2)} - \sqrt{\frac{\pi}{3}}}{x - \sqrt[4]{3}}$$

   (A) $\frac{\sqrt{2\pi}}{2\sqrt[4]{3}}$ (B) $\frac{\sqrt{\pi}}{4\sqrt[4]{3}}$ (C) $\frac{4\sqrt[4]{3}}{\sqrt{2\pi}}$ (D) $\frac{\sqrt[4]{243}}{4\sqrt{\pi}}$ (E) NOTA

9. If $y(t) = (\ln t)^2 e^t + 2^t$, find $y'(5)$.

   (A) $\frac{2\ln 5}{5}e^5 + (\ln 5)^2 e^5 + 32\ln 2$

   (B) $\frac{2\ln 5}{5}e^5 + (\ln 5)^2 e^5 + 32$

   (C) $\frac{2\ln 5}{5}e^5 + (\ln 5)^2 e^5 + 32\ln 5$

   (D) $\frac{2\ln 2}{5}e^5 + (\ln 2)^2 e^5 + 32\ln 5$

(E) NOTA

10. There exists a right triangle with hypotenuse 10 such that the sum of the length of one of its legs and twice the length of the other is minimized. Find the area of this triangle.

(A) 5 (B) 10 (C) 20 (D) 40 (E) NOTA

11. If $f(x) = \sec(\arcsin x)$, evaluate $f''(\sqrt{0.75})$.

(A) -80 (B) -72 (C) 72 (D) 80 (E) NOTA

12. What is the positive difference between the values of $a$ and $b$ that make the following piecewise function continuous?

$$f(x) = \begin{cases} bx^2 - ax & x \leq 2 \\ x^2 + b & 2 < x \leq 3 \\ ax^3 + 2b & x \geq 3 \end{cases}$$

(A) $\frac{1}{23}$ (B) $\frac{2}{23}$ (C) $\frac{3}{23}$ (D) $\frac{4}{23}$ (E) NOTA

13. If $y = x^y$ then which of the following is equivalent to $\frac{dy}{dx}$?

(A) $yx^{y-1}$ (B) $\dfrac{y^2}{x - xy\ln x}$ (C) $\dfrac{y}{x - xy\ln x}$ (D) $\dfrac{y^2}{x - xy\ln y}$ (E) NOTA

14. Use 3 iterations of Newton's method to approximate the root of the equation $y = x^2 - 2x + 1$ with $x_0 = 3$.

(A) $\dfrac{3}{2}$ (B) $\dfrac{5}{4}$ (C) 2 (D) 1 (E) NOTA

15. Vishnav is baking cookies in a room that stays at a constant temperature of 80° F. The temperature of the cookies at the moment he takes them out of the oven is 320° F. After letting them rest for an hour, the temperature of the cookies became 200 ° F. What is the temperature of the cookies in degrees Fahrenheit after an additional hour of cooling? (HINT: Use Newton's Law of Cooling)

(A) 140 (B) 120 (C) 110 (D) 80 (E) NOTA

16. Over which interval is the graph of $y = \dfrac{x^2}{2x+1}$ both increasing and concave up?

(A) $(-\infty, \infty)$ (B) $(0, \infty)$ (C) $\left(-\infty, \dfrac{-1}{2}\right) \cup (0, \infty)$ (D) $\left(-\dfrac{1}{2}, \infty\right)$ (E) NOTA

17. Real numbers $a$ and $b$ are chosen such that the following piecewise function is differentiable:

$$f(x) = \begin{cases} x^3 + x^2 + ax + b & x \leq 4 \\ x^2 + 2 & x \geq 4 \end{cases}$$

For this function, find the sum of all values of $c$ that satisfy the mean value theorem for derivatives on the interval $(0, 32)$.

(A) 8 (B) 14 (C) 24 (D) 28 (E) NOTA

18. Evaluate:

$$\lim_{x\to-\infty}\left(1-\frac{x}{4}\right)^x$$

(A) $e^4$ (B) $e^{-4}$ (C) $e^{-\frac{1}{4}}$ (D) 0 (E) NOTA

19. Evaluate:

$$\lim_{x\to-1}\sqrt[4]{x+1}$$

(A) 0 (B) 1 (C) $\infty$ (D) DNE (E) NOTA

20. Vishnav's hair looks a bit like the function $y=\sin(x)+5$. Approximate the area bounded by $y=\sin(x)+5$, $x=2\pi$, and the x- and y-axes, using a trapezoidal sum with 4 equal sub-intervals.

(A) 10 (B) $10\pi$ (C) 20 (D) $20\pi$ (E) NOTA

21. To collect lake water, Rohan has a hollow, empty cylindrical can with the top circular base removed. The can has radius 6 cm and height 10 cm. Akash secretly glues a solid cone with the same radius and height to the inside of Rohan's can. The cone's circular base is attached to the bottom circular base of the can, and its axis of symmetry is the same as that of the can. When Sina comes by, he pours lake water into the contraption through its open top at a constant rate of $15\pi$ cm$^3$/s. After 5 seconds of this, what is the instantaneous rate of change of the water's height along the walls of the can? Express your answer in cm/s.

(A) 1/9 (B) 2/9 (C) 4/9 (D) 5/9 (E) NOTA

22. Rohan, Varun, Hitesh, and Anurag often argue about integral approximations. Rohan only uses the Trapezoidal Rule, Varun only uses Left-hand Riemann Sums, Hitesh only uses Right-hand Riemann Sums, and Anurag only uses Midpoint Riemann Sums. The table below shows the number of arguments, $a$, they have as a function of time. Using this table, find the range of the answers that Rohan, Varun, Hitesh, and Anurag calculate when they are asked to use 4 subdivisions of equal length to approximate $f(8)$ such that

$$f(x)=\int_0^x a(t)dx$$

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $a(t)$ | 7 | 78 | 11 | 20 | 32 | 40 | 43 | 32 | 44 |

(A) 74 (B) 37 (C) 154 (D) 165 (E) NOTA

23. Vishnav is obsessed with Ancient Egypt, and wants to find the volume of one of its famous pyramids. Given the pyramid is a tetrahedron with vertices at $(6,3,12)$, $(2,6,4)$, $(5,12,13)$, $(17,22,-3)$, find its volume.

(A) 1548 (B) 256 (C) 516 (D) 258 (E) NOTA

24. Aniketh gets triggered by trigonometry. To provoke him, Vishal tells him to find the product of the maximum, minimum, and period of $3\sin(4x+12)=2y(\cos(2x+6))+10\cos(2x+6)$. Solve Vishal's problem so Aniketh does not get riled up.

(A) $4\pi$ (B) $8\pi$ (C) $16\pi$ (D) $2\pi$ (E) NOTA

25. Farzan and Dylan are best friends, but they went to different Elementary Schools. Dylan's elementary school was located at $(-7,3)$, while Farzan's was located at $(3,4)$. Given Dylan and Farzan live at $(2,2)$ and $(-1,1)$ respectively, and both travelled in a straight line from their home to elementary school, let $\theta$ be the smaller angle at which

their paths intersected. Which of the following intervals contains $\theta$?

(A) $(15°, 30°]$ (B) $(30°, 45°]$ (C) $(45°, 60°]$ (D) $(60°, 75°]$ (E) NOTA

26. Jeromy's favorite food is a doughnut. Because we do not have our usual doughnut stand at the competition, he has decided to draw doughnuts and imagine they are real during practices. To keep him working on math, Vishal decides to use this to his advantage. He tells Jeromy to rotate the ellipse

$$16x^2 + 9y^2 - 128x - 18y + 121 = 0$$

around the y-axis to create a doughnut. Help Jeromy by finding the volume of the rotated conic to satisfy his doughnut needs.

(A) $48\pi$ (B) $96\pi$ (C) $48\pi^2$ (D) $96\pi^2$ (E) NOTA

27. Evaluate

$$\int_3^6 \frac{\sqrt{16x^2 - 9}}{3x} dx$$

(A) $\sqrt{3} - 1 - \frac{\pi}{3}$
(B) $3\sqrt{7} - \sqrt{15} - \arccos\frac{1}{8} + \arccos\frac{1}{4}$
(C) $2\sqrt{15} - \arccos\frac{1}{8} + \arccos\frac{1}{4}$.
(D) $\sqrt{3} - \frac{\pi}{3}$
(E) NOTA

**Use the following information for questions 28 - 29**
Tanmay's favorite conic is $xy = 1$, but Karthik keeps bugging him with questions about it. Help him answer them quickly so he can get back to thinking about dinosaurs!

28. Karthik's first question is "What is the eccentricity of the conic?" Find the eccentricity for Tanmay.

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) NOTA

29. Karthik's second question is "What is the area bound by the conic's first-quadrant portion and its first-quadrant latus rectum?" Tanmay has been so exhausted with the previous question that he has no clue how to even start this one, so help him out by answering it for him.

(A) $\sqrt{2} - \ln(1 + 2\sqrt{2})$ (B) $2\sqrt{2} - \ln(1 + \sqrt{2})$ (C) $2\sqrt{2} - \ln(3 + 2\sqrt{2})$ (D) $\sqrt{2} - \ln(3 + \sqrt{2})$ (E) NOTA

30. Given

$$f(x) = \frac{d}{dx} \int_{\sin x}^{2x^2 + \frac{\pi}{3}} \sin x dx$$

Which of the following is equivalent to $f(x)$?
(A) 0
(B) $-\cos(x) + C$
(C) $4x \sin(2x^2 + \frac{\pi}{3}) - \cos x(\sin(\sin x))$
(D) $4x \sin(2x^2 + \frac{\pi}{3}) + \cos x(\sin(\sin x))$
(E) NOTA