For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. D. $r = \sin\theta\cos\theta$

   $= \dfrac{2\sin\theta\cos\theta}{2}$

   $= \dfrac{1}{2}\sin 2\theta$

   Since 2 is even, there are $2(2) = \boxed{4}$ petals.

2. C. $f(x) = x^{5050}$, so $f'(x) = 5050x^{5049}$ and $f'(5) = 5050(5^{5049}) = 202(5^{5051})$. Thus the largest possible value of x is $\boxed{5050}$.

3. C. Solutions are roots of unity so they form a regular dodecagon with circumradius $\sqrt[12]{64} = \sqrt{2}$. This dodecagon can be split into 12 isosceles triangles, each with vertex angle of 30° and leg length $\sqrt{2}$. Thus each of the triangles has area $\frac{1}{2}(\sqrt{2})(\sqrt{2})\sin(30°) = \frac{1}{2}$ so the total area is $\boxed{6}$.

4. A. To convert $(\sqrt{3}, 3)$ into polar form we need to find the angle and length of the point with respect to the origin.

   $Angle = \arctan(\dfrac{3}{\sqrt{3}}) = \dfrac{\pi}{3}$

   $Length = \sqrt{(\sqrt{3})^2 + 3^2} = 2\sqrt{3}$,

   $polar form = \boxed{(2\sqrt{3}, \dfrac{\pi}{3})}$

5. E. $y = 24\sin\cos x + 16\sin^2(x) + 9\cos^2(x)$ can be factored into $y = (3\cos x + 4\sin x)^2$

   The amplitude of $y = 3\cos x + 4\sin x$ is $\sqrt{9+16} = 5$. Thus the maximum value is $5^2 = 25$, but the minimum value is just 0 because the squared value cannot be negative. The amplitude of a periodic function is half the distance from maximum to minimum, so it is $\frac{25-0}{2} = \boxed{12.5}$.

6. $\boxed{E}$. All the listed numbers are complex numbers.

7. D. Volume of parallelepiped $= \begin{vmatrix} 2 & 4 & 1 \\ -2 & -1 & 2 \\ 3 & 2 & 5 \end{vmatrix} = \boxed{45}$.

8. B. This limit is the limit definition of the derivative of $\sqrt{\arctan(x^2)}$ when $x = \sqrt[4]{3}$. So it is equivalent to finding $f'(\sqrt[4]{3})$ given that $f(x) = \sqrt{\arctan(x^2)}$. $f'(x) = \dfrac{1}{2}(\arctan(x^2))^{-1/2}\left(\dfrac{2x}{x^4+1}\right)$

   $f'(\sqrt[4]{3}) = \boxed{\dfrac{\sqrt[4]{243}}{4\sqrt{\pi}}}$

9. A. By simple derivative rules, $y'(t) = \boxed{\dfrac{2\ln 5}{5} * e^5 + (\ln 5)^2 e^5 + 32\ln 2}$

10. C. If one of the angles in the triangle is θ, the two legs are $10\sin\theta$ and $10\cos\theta$. Then we wish to minimize $10\sin\theta + 20\cos\theta$. We can do this with trigonometric simplifications, or just by setting the derivative equal to 0: $10\cos\theta - 20\sin\theta = 0$, $\tan\theta = \frac{1}{2}$. If this is the case, then we can sketch out a triangle to see that $\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$. This means the two side lengths are $2\sqrt{5}$ and $4\sqrt{5}$, for a total area of $\boxed{20}$.

11. D. By drawing out a triangle, we can see that $\sec(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2}$. The derivative of this, by power and chain rule, is $f'(x) = -x(1-x^2)^{-3/2}$. Then by product rule, $f''(x) = -(1-x^2)^{-3/2} - 3x^2(1-x^2)^{-5/2}$. Plugging in $x = \sqrt{3}/2$ gives $-8 - 72 = \boxed{-80}$.

12. C. We can create 2 systems of equations from the two equality conditions. Equality at 2 means $4b - 2a = 4 + b \to 3b - 2a = 4$. Equality at 3 means $9 + b = 27a + 2b \to 27a + b = 9$. Subtracting the former from 3 times the latter, we get $83a = 23 \to a = \frac{83}{23}$. Plugging this back into $3b - 2a = 4$, we get $b = \frac{86}{23}$. The positive difference is $\boxed{\frac{3}{23}}$

13. B. $y = x^y \to \ln y = \ln x^y \to \ln y = y \ln x$
$\frac{d}{dx}(\ln y = y \ln x)$
$= \frac{\frac{dy}{dx}}{y} = \frac{dy}{dx} \ln x + \frac{y}{x}$
$\frac{dy}{dx} = \boxed{\frac{y^2}{x - xy\ln x}}$

14. B. $x_1 = 3 - \frac{4}{4} = 2$
$x_2 = 2 - \frac{1}{2} = \frac{3}{2}$
$x_3 = \frac{3}{2} - \frac{\frac{1}{4}}{1} = \boxed{\frac{5}{4}}$

15. A. $200 = 80 + (320 - 80)e^{-k}$
$k = \ln 2$
$T_2 = 80 + (320 - 80)e^{-2\ln 2}$
$T_2 = \boxed{140}$

16. B. To find where the graph is increasing, find where the first derivative is positive:

$\frac{dy}{dx} = \frac{2x^2 + 2x}{(2x+1)^2}$
$\frac{dy}{dx} > 0$ on $(-\infty, -1) \cup (0, \infty)$
To find where the graph is concave up, find where the second derivative is positive:

$\frac{d^2y}{dx^2} = \frac{4x+2}{(2x+1)^4}$
$\frac{d^2y}{dx^2} > 0$ on $\left(-\frac{1}{2}, \infty\right)$
Looking at the intervals we can see that the line is both increasing and concave up on the interval $\boxed{(0, \infty)}$

17. B. We use the same strategy as in 12 to find two equations: $4^3 + 4^2 + 4a + b = 4^2 + 2$, and $3(4)^2 + 2(4) + a = 2(4)$. The latter gives $a = -48$. Then the former gives $b = 18 - 80 + 4(48) = 130$. The conditions for mean value theorem are satisfied because the function is differentiable across the entire interval. The average rate of change can be found by evaluating $f(0) = 130$, $f(32) = 1026$, and then $\frac{1026-130}{32} = 28$. There are two options: either $3c^2 + 2c - 48 = 28$ or $2c = 28$. For the former, $c > 4$, so it is not possible. For the latter $c = 14$, which falls between 4 and 32 so it is a viable answer. Thus the only answer is $\boxed{14}$.

18. D. When you plug in large negative numbers you can notice that $\left(1 - \frac{x}{4}\right)^x$ approaches $\boxed{0}$.

19. D. The limit $\boxed{\text{does not exist}}$, since the one sided limits are not equal to each other.

20. B. $\frac{1}{2}(\frac{\pi}{2})((5+6) + (6+5) + (5+4) + (4+5))$
$= \boxed{10\pi}$

21. C. Let the height of the water at a given time be h. If the cone was not present, the volume of water in the cylinder with this height would be $\pi r^2 h = 36\pi h$. However, to find the actual volume we must subtract the volume of the frustum with height h from this. By similarity, the smaller cone above the water level has height $10 - h$ and radius $\frac{3}{5}(10-h)$, so its volume is $\frac{3}{25}\pi(10-h)^3$. The total cone has volume $\frac{1}{3}\pi(6)^2(10) = 120\pi$. Then the volume of the frustum is $120\pi - \frac{3}{25}\pi(10-h)^3$, and the total volume of water is $V = 36\pi h - 120\pi + \frac{3}{25}\pi(10-h)^3$. After 5 seconds of pouring, $V = 75\pi$, so $75 = 36h - 120 + \frac{3}{25}(10-h)^3$, and simple trial-and-error shows that $h = 5$. Then we take the derivative to get $V'(t) = h'(t)(36\pi - \frac{9}{25}\pi(10-h)^2)$. Plugging in $h = 5$, we get $15\pi = h'(t)(27\pi)$ so $h'(t) = \boxed{5/9}$.

22. C. Left Riemann Sum: $2(7 + 11 + 32 + 43) = 186$
Right Riemann Sum: $2(11 + 32 + 43 + 44) = 260$
Midpoint Riemann Sum: $2(78 + 20 + 40 + 32) = 340$
Trapezoidal Sum: Average of LRS and RRS = 223
So the range is $340 - 186 = \boxed{154}$.

23. D. Area of triangular pyramid $= \dfrac{1}{6}\begin{vmatrix} 4 & -3 & 8 \\ 3 & 6 & 9 \\ 15 & 17 & -7 \end{vmatrix} = \boxed{258}$.

24. E (DNE). $y = \dfrac{10\cos(2x+6) - 3\sin(4x+12)}{-2\cos(2x+6)}$

$= \dfrac{10\cos(2x+6) - 6\sin(2x+6)\cos(2x+6)}{-2\cos(2x+6)}$

$= 3\sin(2x+6) - 5?$

It is tempting to believe that the two functions are completely equivalent and so the former must have maximum -2 and minimum -8. However, the latter function is maximized/minimized only when $2x + 6 = \frac{\pi}{2} + \pi n$ for integer n, and in this case $\cos(2x+6) = 0$ so the former function has a removable discontinuity. In other words, the function is undefined wherever it should have a maximum or minimum, so there is actually no defined maximum or minimum! Regardless of the period, the answer must be $\boxed{E}$. Poor Aniketh):

25. First, we graph the points and see that the two paths do indeed intersect. The slope of Dylan's path is $-\frac{1}{9}$ while the slope of Farzan's path is $\frac{3}{4}$. If Dylan's path makes an angle a with the horizontal, and Farzan's path makes an angle b, $b - a = \theta$. We take the tangent of both sides to get $\tan\theta = \tan(b-a) = \frac{\tan(b)-\tan(a)}{1+\tan(b)\tan(a)}$. However, by the definition of slope, we know that $\tan(b) = \frac{3}{4}$ and $\tan(a) = -\frac{1}{9}$. Plugging these in, we get $\tan\theta = \frac{31}{33}$. $\tan 45° = 1$, so $\tan\theta < \tan 45°$ and $\theta < 45°$. Also, $\tan 30° = \frac{\sqrt{3}}{3}$. By squaring both sides, we can see that $\frac{31}{33} > \frac{\sqrt{3}}{3}$, so $\theta > 30°$. Thus the answer is $\boxed{B}$.

26. D. First, we complete the square: $16x^2 + 9y^2 - 128x - 18y + 121 = 16(x-4)^2 - 256 + 9(y-1)^2 - 9 + 121 \to 16(x-4)^2 + 9(y-1)^2 = 144 \to \frac{(x-4)^2}{9} + \frac{(y-1)^2}{16} = 1$. The area of the ellipse is $(3)(4)\pi = 12\pi$. The center of the ellipse is $(4, 1)$, so its distance from the axis of rotation is 4. By Pappus's theorem, the volume of the donut is $2\pi(4)(12\pi) = \boxed{96\pi^2}$.

27. B. Using trig substitution, when we set $x = \dfrac{3}{4}sec\theta$, $\int \frac{\sqrt{16x^2-9}}{3x}dx$ becomes $\int \tan^2(\theta)d\theta$.
$\int \tan^2(\theta)dx = \int \sec^2(\theta)d\theta - \int 1 d\theta$
$= tan\theta - \theta = \dfrac{\sqrt{16x^2-9}}{3} - \arccos\dfrac{3}{4x}$.
Solving this from 6 to 3 we get the answer as $\boxed{3\sqrt{7} - \sqrt{15} - \arccos\dfrac{1}{8} + \arccos\dfrac{1}{4}}$.

28. C. To eliminate the xy term, the conic must be rotated. To find the angle we solve:
$\cot(2\theta) = 0 \to 2\theta = 90° \to \theta = 45°$
To rotate $xy = 1$ by 45° we need to replace x with $x\cos 45° + y\sin 45°$ and y with $-x\sin 45° + y\cos 45°$:
$$\left(x\frac{\sqrt{2}}{2} + y\frac{\sqrt{2}}{2}\right)\left(-x\frac{\sqrt{2}}{2} + y\frac{\sqrt{2}}{2}\right) = 1$$

$\rightarrow \dfrac{y^2}{2} - \dfrac{x^2}{2} = 1$. The conic has transverse axis $a = \sqrt{2}$ and conjugate axis $b = \sqrt{2}$, so $c = \sqrt{a^2+b^2} = 2$. The eccentricity is always c/a, so it is equal to $\boxed{\sqrt{2}}$.

29. C. Using the fact that $c = 2$, we know that the hyperbola's focus is a distance of 2 from the origin and that it is on the line $y = x$ (by symmetry of $y = \frac{1}{x}$), so the focus must be at $(\sqrt{2}, \sqrt{2})$. The latus rectum lies on a line perpendicular to the transverse axis which has slope 1, so it must have slope -1. The latus rectum is then on the line passing through $(\sqrt{2}, \sqrt{2})$ with slope -1, which is $y = -x + 2\sqrt{2}$. To find the bounds of integration, we see where this line intersects the hyperbola $y = \frac{1}{x}$: $-x + 2\sqrt{2} = \frac{1}{x} \rightarrow x^2 - 2x\sqrt{2} + 1 = 0$. By quadratic formula, $x = \sqrt{2} \pm 1$. To find the bound area, we find the area of the trapezoid bound by the latus rectum and the lines $x = \sqrt{2} - 1$ and $x = \sqrt{2} + 1$, then subtract the area under $y = \frac{1}{x}$ between those lines. The trapezoid has one base of length $\frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$, and another of length $\frac{1}{\sqrt{2}+1} = \sqrt{2} - 1$, with a height of $(\sqrt{2} + 1) - (\sqrt{2} - 1) = 2$. Thus the area is $(\frac{b_1+b_2}{2})(h) = 2\sqrt{2}$. The area under the curve is $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{1}{x} dx = \ln(\frac{\sqrt{2}+1}{\sqrt{2}-1}) = \ln(3+2\sqrt{2})$. Thus the total bound area is $\boxed{2\sqrt{2} - \ln(3 + 2\sqrt{2})}$.

30. D. Using the second fundamental theorem of calculus we find that $\dfrac{d}{dx} \int_{\sin x}^{2x^2+\frac{\pi}{3}} \sin x dx = \boxed{4x \sin(2x^2 + \dfrac{\pi}{3}) + \cos x(\sin(\sin x))}$.