

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- A**; Each side of the fountain would be 15 meters. Let the side lengths of the path would be x meters. Subtracting the area of the square fountain and the path, gives that $x^2 - 15^2 = 400 \implies x = 25\text{m}$, and thus the perimeter is 100 meters.
- C**; The coordinates of the centroid of this triangle would be the average of the x-coordinates and y-coordinates, which is $(\frac{31}{3}, 9)$. $D + E = \frac{58}{3}$.
- B**; $R = \frac{abc}{4A}$, where a, b , and c are the side lengths of the inscribed triangle, and A is the area. Heron's formula gives $A = 10\sqrt{3}$, so $R = \frac{7\sqrt{3}}{3}$. The circumference of the circle would then be $\frac{14\sqrt{3}\pi}{3}$. The average of a and b is $\frac{23}{6}$.
- D**; Let h be the desired height. Dropping an altitude from the desired intersection point splits the base into x and y , where x is closer to 45. Similar triangle gives $\frac{x}{x+y} = \frac{h}{10}$, and $\frac{y}{x+y} = \frac{h}{45}$. This means $\frac{h}{10} + \frac{h}{45} = 1 \implies h = \frac{90}{11}$, which is very close to 8.
- E**; $\binom{64}{32}$ indicates the number of ways of choosing 32 squares on the chess board. However, some pieces are identical. These include the 8 white pawns, 2 white horses, 2 white rooks, 2 white bishops and the same with the black pieces. So, taking this into account, the answer is $\frac{64!}{(32!)^2(8!)^2(2!)^2(2!)^2(2!)^2}$
- D**; First, draw the line segments from the centers of both circles to the tangency points of the rope, as well as the line segment between the two centers. Note that the two right trapezoids that form have angles 120° and 60° , since dropping an altitude from the center of the smaller circle to the larger base gives a $6 - 6\sqrt{3} - 12$ triangle. This means that the rope contains $\frac{1}{3}$ of the circumference of the small circle, $\frac{2}{3}$ of the circumference of the large circle, and two segments of length $6\sqrt{3}$. Adding these up gives $\frac{1}{3} \cdot 8\pi + \frac{2}{3} \cdot 20\pi + 12\sqrt{3} = 16\pi + 12\sqrt{3}$
- B**; Using the Law of Cosines, $AC^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8(\sqrt{2}/2) = 113 - 56\sqrt{2} = 113 - \sqrt{6272}$. Note that $57a - b = 57(113) - 6272 = 169 = 13^2$, so our answer is 13.
- E**; The 5 by 6 rectangle is defined by 6 horizontal lines and 7 vertical lines. In order to determine a rectangle, we need to choose 2 of these horizontal lines, and two of these vertical lines, giving $\binom{6}{2}\binom{7}{2} = (15)(21) = 315$.
- C**; The angles of the octagon can be represented like this: $a, a + n, a + 2n, a + 3n, a + 4n, a + 5n, a + 6n, a + 7n$. The average of these angles is 135. Solving in terms of the averages, results in the values of the angles being: $135 - \frac{7n}{2}, 135 - \frac{5n}{2}, 135 - \frac{3n}{2}, 135 - \frac{n}{2}, 135 + \frac{n}{2}, 135 + \frac{3n}{2}, 135 + \frac{5n}{2}, 135 + \frac{7n}{2}$. n must be even, and the only values of n which satisfy the conditions in the problem are 0, 2, 4, 6, 8, 10, 12.
- A**; To calculate the area of the island you need the radius of the circumscribed circle. After creating a diagram, it is evident that the radius of this circle, the apothem, and half the length of the side length form a right triangle. The area of the octagon is $32 + 32\sqrt{2}$. This is equal to $\frac{1}{2}aP$. a is the length of the apothem. The apothem is $2 + \sqrt{2}$, which means the radius is $\sqrt{8 + 4\sqrt{2}}$. The area of the island is $8\pi + 4\pi\sqrt{2}$.
- B**; Both the pentagons are regular, which makes them similar. The square of the ratio of the side lengths of the pentagons is equal to the ratio of their areas. $(\frac{4}{34/5})^2 = \frac{A}{79.55}$. $A \approx 27.5$.
- B**; The contrapositive, converse, and inverse cancel, leaving only the inverse, which is B .
- B**; The table is:

22	17	24
23	21	19
18	25	20

Thus $A + B + C + D = 90$. (If you want to come up with this table, looking at the second row and first column tells you what B is. Then, the top left-bottom right diagonal and the third row tell you what C is. Then, first row and second column give you A , and from here it is easy to fill in the rest.)

14. \boxed{A} ; $7 - 24 - 25$ and $15 - 20 - 25$ are Pythagorean triples. 25 is the length of one of the diagonals. Using Ptolemy's theorem we get $\frac{117}{5}$.
15. \boxed{B} ; $(BD)^2 = CD \cdot AD$ by similar triangles, so $DC = 9$.
16. \boxed{B} ; If s is the side length of the original triangle, then $s^2\sqrt{3}/4 = 4\sqrt{3} \implies s = 4$. This means the hexagon has perimeter $4(3) = 12$, and side length 2, meaning its apothem is $\sqrt{3}$.
17. \boxed{C} ; $\frac{60H-11M}{2}$ is the formula to calculate the angle between the hour hand and minute hand. Plugging in the values from the answer choices, results in $4 : 42$.
18. \boxed{D} ; The ostrich can roam the area of the hexagon, plus two 240 degree sectors outside the hexagon, with radius 10. The area of the hexagon can be found by multiplying the area of an equilateral triangle by 6, which gives you $150\sqrt{3}$. The circular area that the cow can be represented by two 240 degree arcs with a radius of 10, which gives an area of $\frac{400\pi}{3} + 150\sqrt{3}$.
19. \boxed{E} ; Using Euler's formula, $F + V - E = 2$, the number of vertices this polyhedron has is 62.
20. \boxed{B} ; The pool has been draining for 5 hours at a rate of 2π cubic meters per hour so it has drained a total of 10π cubic meters of water into the cylindrical tank. The cylinder tank currently holds 10π cubic meters of water and has a radius of 2 meters. $10\pi = \pi(2^2)h$ because 10π is the volume and 2 is the radius of the base of the tank. By solving the equation you get the height is 2.5 meters. However, the answer asks for the height in centimeters so multiple 2.5 by 100 to convert the height from meters to centimeters.
21. \boxed{D} ; The cylinder's height is 5 times the radius of the hemisphere, meaning that its height is 15. The volume of a cylinder is $\pi r^2 h$, so $\pi(3^2)15 = 135\pi$. The volume of a hemisphere is $\pi r^3 \cdot \frac{2}{3}$, so $\pi(3^3)\frac{2}{3} = 18\pi$. $18\pi + 135\pi = 153\pi$. $\frac{153\pi}{\pi/2} = 306$. 306 minutes after 1 : 05 is 6 : 11.
22. \boxed{C} ; Abbreviate home, Rickards, Tanmay's House, and supermarket as H, R, T, S . The taxicab distances between the 4 locations are $HT = 5, HS = 11, HR = 25, RS = 14, ST = 6, RT = 20$. Note that the path $H - T - S - R$ is the same as $H - R - S - T$, so we only need to consider half the paths. $H - R - S - T$ gives $25 + 14 + 6 + 5 = 50$. $H - R - T - S$ gives $25 + 20 + 6 + 11 > 50$. $H - T - R - S$ gives $5 + 20 + 14 + 11 = 50$. Therefore, $H - R - T - S$ or $H - S - T - R$ are the longest paths, and the only one in the answer choices is C .
23. \boxed{B} ; The total amount of steps to the right and up choose the number of steps to the right calculates the number of ways to get from 2 different points. So, the number of ways to get from A to B is $\binom{11}{8} = 165$. Repeat for B to $C = \binom{11}{7} = 330$. $165 \cdot 330 = 54450$.
24. \boxed{A} ; Power of a point states that $AE \cdot CE = BE \cdot DE$, so $5 * 9 = 7 * BE$. $BE = \frac{45}{7}$.
25. \boxed{E} ; B lies on the perpendicular bisector of AC , $y = 0.5x + 1$, and the x-axis, meaning $B = (-2, 0)$. D is the reflection of B over the midpoint of AC , which is $(10, 6)$, meaning that $D = (22, 12)$. The sum of coordinates is 34.
26. \boxed{D} ; Why reflect 3, when you can reflect one? Reflecting the centroid of ABC , $(\frac{2}{3}, \frac{22}{3})$, over $y = 3x + 2$ gives $(\frac{8}{3}, \frac{20}{3})$, and reflecting this over the y-axis gives $(-\frac{8}{3}, \frac{20}{3})$. This is the centroid of $A''B''C''$, and so the sum of the coordinates of this triangle is $20 - 8 = 12$.
27. \boxed{C} ; The volume of his sandwich is 96 inches cubed. Dividing this by the average volume of his pieces results in 92, which is the number of pieces Akhil cut. It can be shown that with n cuts, at most $\frac{1}{6}(n^3 + 5n + 6)$ cuts can be made, the n -th Cake Number. For $n = 8$, we get 93 pieces, just above the desired 92 pieces. We can clearly merge two of the cuts together by tilting the plane just right, so 8 cuts suffices.
28. \boxed{A} ; The circle's equation can be manipulated to $(x - 2)^2 + (y + 3)^2 = 16$ by completing the squares and simplifying. This means the center of the circle is $(2, -3)$. Then use the point to line formula to get $\frac{12\sqrt{5}}{5}$.
29. \boxed{D} ; Let F be the foot of the altitude from B onto AC . $BE = \frac{13}{2}$. We also have that triangles CFB and CBA are similar, so $BF = \frac{60}{13}$. $\sin(\angle AED) = \sin(\angle AEB) = \frac{120}{169}$.
30. \boxed{A} ; Using the Pythagorean Theorem, we can solve for 3 sides of the triangle, which are $(AB, BC, AC) = (8, 15, 17)$. The angle bisector theorem gives $BD = \frac{24}{5}$, and using the Pythagorean Theorem, we solve for AD , which is $\frac{8\sqrt{34}}{5}$.