

- The equation you use for jointly proportional variables in this question is $x = k(y^2)\sqrt{z}$, where x = puzzles, y = size of pencil, z = temperature. To solve for k , you plug in $x = 10$, $y = 2$, $z = 108$, and get $k = \frac{5\sqrt{3}}{36}$. Using this k value, you can solve for y by plugging in $x = 5$, and $z = 81$. Using these values you get $y^2 = \boxed{\frac{4\sqrt{3}}{3}}$.
- To find the final price of the statue, you first need to subtract 25% of the starting price, which is 50. 25% of 50 is 12.50, and $50 - 12.50 = 37.50$. Now you need to add 10% of this price. 10% of 37.50 = 3.75, and $37.50 + 3.75 = 41.25$, which is the final price. Now you need to convert this price from CAD to USD, by dividing the final price by 1.5, per the exchange rate. $\frac{41.25}{1.5} = 27.50$. You have 210 USD, so you can buy $\boxed{7}$ statues.
- These exponents can be written as $(2^8)^5$, $(3^6)^5$, $(6^4)^5$, $(7^3)^5$, $(5^4)^5$. Since all of these have the same exponents of 5, we only need to compare the numbers inside the parentheses. $2^8 = 256$, $3^6 = 729$, $6^4 = 1296$, $7^3 = 343$, $5^4 = 625$. Out of all of these, 256 is the least making the answer $\boxed{2^{40}}$.
- The units digit of 81 to any power is 1, the last two digits of 2525 to any power greater than 0 is 25, so the tens digit is 2, and since 2020 ends in 0, it to the power of a large number leads to a long trail of 0s at the end. So, we can assume that the hundreds digit is 0. $1 + 2 + 0 = \boxed{3}$.
- The differences in the pattern are 5, 6, 7, 9, 12, while the differences of the differences are 1, 1, 2, 3. These follow the Fibonacci sequence, so we can assume that the next difference of difference is 5, so that makes the next difference 17, which makes the next number in the pattern $\boxed{64}$.
- The sum of the number of cubes that have 0 sides painted, 1 side painted, 2 sides painted, and 3 sides painted is just all the cubes. The number of cubes in a regular $13 \times 13 \times 13$ is 2197. There are 5 cubes removed on every edge, and there are 12 edges, so 60 cubes in all. There are 4 cubes removed on every face, and there are 6 faces, so 24 cubes in all. This makes 84 cubes removed, leaving $\boxed{2113}$ cubes left.
- $(1+i)^2 = 2i$, so the question can be simplified to $(2i)^2 \cdot (1+i)$. $(2i)^2 = -4$, and $-4(1+i) = \boxed{-4-4i}$.
- The number of pieces of paper is equal to 12, and the number of ways to put 12 pieces of paper into 4 buckets is equal to $12 + 4 - 1$ choose 12, which is equal to $\boxed{455}$.
- $(x + \frac{1}{x})^3 = x^3 + \frac{1}{x^3} + 3(\frac{x+1}{x}) = 5^3$. Since $x + \frac{1}{x} = 5$, $x^3 + \frac{1}{x^3} + 15 = 125$, so $x^3 + \frac{1}{x^3} = \boxed{110}$.
- The area of the big square is x^2 , and the shaded area of the square is $\frac{3}{4}x^2$. The area of the unshaded square is $\frac{1}{4}x^2$, and $\frac{3}{4}$ of that square is shaded, so the total shaded area inside of the smaller square is $\frac{3}{16}x^2$. The total shaded area is then $\frac{15}{16}x^2$ because $\frac{3}{16} + \frac{3}{4} = \frac{15}{16}$. We know that the shaded area is equal to 120, so the total area of the square, or x^2 is 128. This means that x is equal to $\boxed{8\sqrt{2}}$.
- To find the number of factors of a number, the easiest way is to find the prime factorization of it. The prime factorization of 324 is $3^4 \cdot 2^2$, and the number of factors is just the product of 1 more than the exponents of the prime factors. So the number of factors of 324 is $5 \cdot 3 = 15$. The same thing can be done with 2020, prime factorization being $2^2 \cdot 5 \cdot 101$, and the number of factors being $3 \cdot 2 \cdot 2 = 12$. $12 - 15 = \boxed{-3}$.
- If you solve the system of equations, you find that $x = 5$, and $y = 10$. So $x - 2y = \boxed{-15}$.
- To find the perpendicular bisector of a line segment, you have to find the line that is perpendicular to the line segment while passing through the midpoint of the line segment. So the perpendicular bisector of this line segment is $-2x + y - 1 = 0$. Using the coefficients as roots, we get the cubic $x^3 - 3x + 2$. Since there is no x^2 , we can assume that $e = \boxed{0}$.
- 50% of 1000 is 500, and $\frac{3}{4}$ of 500 is 375, and $\frac{3}{5}$ of 375 is $\boxed{225}$.
- We can make an equation: $0.1(x) + 0.85(100) = 0.5(x + 100)$, where x is the amount of the 10% liquid lucky charms solution. Solving this equation, we find x as $\boxed{87.5}$ gallons.

16. Since the two people have to sit next to each other, we can count them as one person. The number of ways to arrange 4 people around a table is $3!$, since you can count arrangements as the same if you can rotate the table to get the other arrangement. $3!$ is 6, and you need to multiply by 2 because you can switch the two people. $6 \cdot 2 = \boxed{12}$.
17. To find the y-coordinate point B, you first have to find the point where line X and line Z intersect. One way to find that is by setting both equations equal to each other. This results in $3x = -3x + 12$. Then, you add $3x$ to both sides to get $6x = 12$. Next you divide by 6 on both sides to get $x = 2$. Now that you know $x = 2$ is a solution to the equation, you know that $x = 2$ is the x-coordinate of point B. Point B was defined in the question as the coordinate of the intersection point. From this, you plug in $x = 2$ into either equation. Plugging $x = 2$ gets $y = 6$. The question asks to write the y-coordinate (6) in the form of $\frac{m}{n}$ where $m = 24$. Therefore $\frac{24}{n} = 6$. And $n = \boxed{4}$.
18. This is a question asking you to find the area of a shape in the coordinate plane. Before you graph both line X and Y, you can do a couple things to make it easier for you. First, we know the intersection point (from the previous question) is (2,6). Graph that. Next, by looking at the equations given, you can tell that line Z has a y-intercept at $y = 12$, and line X at $y = 0$. Both lines are written in the form of $y = mx + b$. Where m is the slope and b is the y-intercept. If an equation does not have a b (like $y=3x$), then the b is 0. A proof of the b value being the y-intercept can be shown by plugging in 0 into the x value (the y-intercept is the y-point where $x=0$). Doing so should get you a y-intercept of 0 for X and 12 for Z. Now that you've graphed those three points, find the x-intercept of line Z. We already know the x-intercept of line X is zero, since the y-intercept coordinate was (0,0). The x-intercept similarly is where the x value is zero. This can be found by plugging in 0 for the y value of the equation. Doing so in line Z should get you the following: plug in 0 for y , $0 = -3x + 12$, add $3x$ on both sides, $3x = 12$, divide by 3 on both sides $x = 4$. Connect the dots associated with each respective line! Go from each y-intercept to the intersect. Stop for line X, we don't need to go any further since the question asks for the shape formed with line Z and the X-axis. For line Z, go from the y-intercept to the intersect point and the x-intercept. Now we have a triangle with points (0,0), (2,6), (4,0). The base is from (4,0) to (0,0) giving it a length of 4. And the height is from the middle of the base (2,0) to the intersect (2,6). This gives a height of 6. Plugging this into the equation for area of a triangle we get the following. Area of a triangle: $\frac{1}{2}bh$ (b = length of base, h = height) Plug in our values and get $\frac{1}{2} \cdot 4 \cdot 6 = \boxed{12}$.
19. To find the midpoint of the line X, you plug the start and end coordinates into the midpoint formula. In this case; (0,0) and (2,6). From this you get the point (1,3). Plugging that into the formula given in the question leads to $3^2 - 1 = \boxed{8}$.
20. To find if a third side can form a triangle, take the other sides a and b (where $a > b$). For the unknown side c to be able to form a triangle with a and b , $a-b < c < a+b$. In this case, you have to find a choice that isn't a possible side length. All answer choices fall between $5 < c < 29$, making the answer **NOTA**.
21. The range is the maximum value of a data set subtracted from the minimum. Thus the range is $186,350 - 10 = \boxed{186,340}$.
22. Factoring $18x^2 + 39x + 20$ gets $(6x + 5)(3x + 4)$. This is in the form $(Ax + B)(Cx + D)$. Thus $A = 6$, $B = 5$, $C = 3$, and $D = 4$. Therefore $AB + CD + AC + BD + AD + BC = \boxed{119}$.
23. 10% of 70 is 7, so if Sina is 63 inches tall, then Erin is $\boxed{7}$ inches taller than him.
24. 81 and 243 have common factors of 3. These can thus be written as $\frac{3^4}{3^5}$. Since $(n^a)^x = n^{ax}$ then $\frac{(3^4)^x}{(3^5)^y} = \frac{3^{4x}}{3^{5y}}$. Dividing two exponents gets $3^{4x-5y} = 3^2 = \boxed{9}$.
25. To get $\frac{1}{2}x^2 - 12$ in the form $\frac{1}{2}(x+a)(x-a)$ we must factor out $\frac{1}{2}$. This gives us $\frac{1}{2}(x^2 - 24)$. This is simplified to $\frac{1}{2}(x - 2\sqrt{6})(x + 2\sqrt{6})$. This gives an an value of $\boxed{2\sqrt{6}}$.

26. The system of equations:

$$\begin{aligned}x + 2y - 3z &= 4 \\4x - 3y + 2z &= 8 \\2x + 2y + 2z &= 12\end{aligned}$$

Can be solved using Gaussian elimination (and Cramer's rule if you want to get really fancy). Starting with the first equation, multiplying by -4 and adding the result gets

$$\begin{aligned}x + 2y - 3z &= 4 \\-11y + 14z &= -8 \\2x + 2y + 2z &= 12\end{aligned}$$

Then multiplying the first equation by -2 and adding the result to the third equation results in

$$\begin{aligned}x + 2y - 3z &= 4 \\-11y + 14z &= -8 \\-2y + 8z &= 4\end{aligned}$$

Next multiply the second equation by $-\frac{2}{11}$ and add the result to the third equation. This results in:

$$\begin{aligned}x + 2y - 3z &= 4 \\-11y + 14z &= -8 \\60/11z &= 60/11\end{aligned}$$

Solving for z

$$\begin{aligned}60/11z &= 60/11 \\z &= 1\end{aligned}$$

Solve for y

$$\begin{aligned}-11y + 14z &= -8 \\-11y + 14 * 1 &= -8 \\y &= 2\end{aligned}$$

Solve for x by substituting $y = 2$ and $z = 1$ into the first equation, and you get $x = 3$, $y = 2$, $z = 1$. The question then asks for $2000 - abc + cba$. Where $a = x$, $b = y$, and $c = z$. Using this, we get $2000 - 321 + 123 = \boxed{1802}$.

27. To simplify $\frac{(13! - 12!)}{(6! - 5!)}$ we first take out $12!$ from the numerator and $5!$ from the denominator. Here we get $\frac{12!(13 - 2)}{5!(6 - 1)}$. Furthermore this becomes $\frac{12 * 12!}{5 * 5!}$. Cancelling out the $5!$ from the $12!$ gets $12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$ left over. This, alongside the $\frac{12}{5}$ in the fraction, gets an answer of $\frac{12}{5} \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = \boxed{120 \times 42 \times 99 \times \frac{96}{5}}$ is a simplified version of this answer.
28. i to the power of anything is either i , $-i$, 1 , or -1 . This becomes a cyclic pattern of the i to whatever exponent having a solution of i to the power of the remainder of said exponent divided by 4. i^{768} can be written as $i^{192(4)}$. 768 divides evenly into 4, with a remainder of 0. Thus i^{768} can be written as i^0 which is equal to $\boxed{1}$.
29. The discriminant is $b^2 - 4ac$. $3x^2 + 9x + 4$ is in $ax^2 + bx + c$ form, thus $9^2 - 4 * 3 * 4 = \boxed{33}$.
30. $\frac{1}{2}$ of anything involves dividing it by two. One way to do this is write the numbers as 0.5. Divide that by two and you get 0.25, then 0.125, then 0.625, then 0.03125, then 0.015625 then 0.0078125. Although it is quite repetitive the answer is \boxed{NOTA} .