

1. A: The numerator and denominator both evaluate to 0 at 4, so we can use L'hospital's rule to get  $\frac{3x^2}{1/2\sqrt{x}} = 6x^2\sqrt{x}$ .

When  $x = 4$  this evaluates to 192.

B: Multiplying and dividing by the conjugate, we get a numerator of  $(x^2 - 46x + 2020) - (x^2 + 46x + 2021) = -92x - 1$  and a denominator of  $\sqrt{x^2 - 46x + 2020} + \sqrt{x^2 + 46x + 2021}$ . However, as  $x$  tends to infinity, both of the roots on the bottom tend to  $x$  as the lower order terms vanish, resulting in  $2x$  on the bottom and  $-92x$  on the top (asymptotically). These divide to give  $-46$

C: Numerator and denominator both evaluate to 0 at  $x = 0$ , so we can use L'hospital's again. The derivative of the numerator is  $\frac{3}{2}\sqrt{x+4} + e^x \rightarrow 4$  while the derivative of the denominator is just 1, so the limit is 4

D: This is equivalent to the limit of  $\ln\left(\left(\frac{x^3 + 9x^2 + 27x + 27}{x^3}\right)^x\right)$ . The numerator of the fraction is equivalent to  $(x+3)^3$ , so the fraction can be rewritten as  $(1+3/x)^3$ . Taking out the logarithm, we have  $\ln(\lim_{x \rightarrow \infty} (1+3/x)^{3x}) = \ln(e^9) = 9$

FINAL: 159

2. A: Let  $m = y/x$ . Then we are looking for a locus of  $m$  values for which  $m = \ln(1/m) = -\ln(m)$ . However, sketching the graphs  $y = m$  and  $y = -\ln(m)$  (a different  $y$  than before) shows that they should only intersect at one point in the first quadrant, as  $-\ln(m)$  decreases monotonically from infinity to negative infinity with  $m > 0$  and  $m$  increases monotonically from 0 to infinity with  $m > 0$ . Thus, there is exactly one value of  $m$ , or  $y/x$ , that satisfies the equation, which we will denote as  $\lambda$  (as it cannot be found analytically). Then  $y/x = \lambda$ , so  $y = \lambda x$  and our graph is equivalent to a line of slope  $\lambda$  passing through the origin. Thus, the slope of the normal line at any point, including  $x = 21$ , is  $-1/\lambda$

B: Sketching these lines we see they form a trapezoid with bases at  $x = 1$  and  $x = 21$ . The length of one base is  $y(1) = \lambda$  and the length of the other base is  $y(21) = 21\lambda$ . Also, the height is just  $21 - 1 = 20$ . Thus, the area of the trapezoid is  $(1/2)(22\lambda)(20) = 220\lambda$

FINAL: -220

3. A: The formula for a step of Newton's method is  $x_{i+1} = x_i - f(x_i)/f'(x_i)$ , which in this case simplifies to  $x_{i+1} = x_i - (x_i/2) = x_i/2$ . Thus if  $x_0 = 80$ ,  $x_1 = 40$ ,  $x_2 = 20$ , and so on. The actual root is 0 of course, so we want the

least  $n$  such that  $80/2^n < 1/100 \rightarrow 2^n > 8000$ .  $2^{13} = 8192 > 8000$ , so the answer for this part is  $n = 13$ .

B: The formula for a step of Euler's method is  $f(x_{i+1}) = f(x_i) + (x_{i+1} - x_i)f'(x_i)$ , which in this case simplifies to  $f(x_{i+1}) = f(x_i) + (1)(2x_i)$  (where the step length is 1 because we are trying to reach  $n$  in  $n$  steps). Since we start out with  $f(x_0) = 0$ ,  $f(x_1) = 0 + 2(0) = 0$ ,  $f(x_2) = 0 + 2(1) = 2$ ,  $f(x_3) = 2 + 2(2) = 6$ , and so on, adding  $2i$  for each  $i$  less than  $n$ . Thus the approximation of  $f(x_n)$  is  $2 \sum_{i=0}^{n-1} i = (n-1)(n) = n^2 - n$ . Then the absolute error is  $n$ , so the percent error is  $n/n^2 = 1/n$ . For this to be less than 1%, we must have  $1/n < 0.01$ , so  $n > 100$  and the least integer value of  $n$  is 101.

C: We know that the actual value of the integral is  $n^3$ . The approximated value is  $1(1/2)(f(0) + (2 \sum_{i=1}^{n-1} f(i)) + f(n))$  by the formula for trapezoidal sum. Then plugging in  $f(i) = 3i^2$  we get  $(3/2)(0 + 2(1^2 + 2^2 + \dots + (n-1)^2) + n^2) = (3/2)(\frac{(n-1)(n)(2n-1)}{3} + n^2) = (3/2)\left(\frac{2n^3 + n}{3}\right) = \frac{2n^3 + n}{2} = n^3 + n/2$ . Then the absolute error is  $n^3 + n/2 - n^3 = n/2$ , so the percent error is  $(n/2)/n^3 = 1/(2n^2)$ . For percent error less than 1%, we must have  $\frac{1}{2n^2} < 0.01$  so  $2n^2 > 100$  and the least integer  $n$  is 8.

FINAL: 122

4. A: Velocity of  $X$  is  $x'(t) = 12t^2 - 28$  and speed is magnitude of velocity or  $|12t^2 - 28|$ . We have to test 3 points for maximums - the two endpoints of the interval, and the local minimum of the velocity (which may become a maximum when the absolute value is taken). The local minimum is clearly at  $t = 0$ , because this is the vertex. Testing we get  $|x'(-1)| = 16$ ,  $|x'(0)| = 28$ , and  $|x'(2)| = 20$ , so the maximum speed is 28.

B: It's tempting to simply take the difference between the two positions, but we must remember that particle Y might switch directions. To find where it switches directions, we check where the velocity changes signs.  $y'(t) = 4t^3 - 28t = 4t(t^2 - 7)$ , so velocity changes signs at  $t = 0$  and  $t = \pm\sqrt{7}$ . The only one of those roots within the interval is  $t = \sqrt{7}$ , so we take  $|y(\sqrt{7}) - y(1)| + |y(3) - y(\sqrt{7})| = |-24 - (12)| + |(-20) - (-24)| = 36 + 4 = 40$ .

C: Let  $L = \sqrt{x^2 + y^2}$  be the distance between the two points. Then to minimize  $L$  we want to find the points where its derivative switches from positive to negative on the interval.  $L'(t) = \frac{x(t)x'(t) + y(t)y'(t)}{L}$ , and since  $L$  is always positive only the numerator matters. Plugging in  $x(t)$  and  $y(t)$  the numerator becomes  $(4t^3 - 28t)(12t^2 - 28) + (t^4 - 14t^2 + 25)(4t^3 - 28t) = (4t^3 - 28t)(t^4 - 2t^2 - 3) = 4t(t^2 - 7)(t^2 - 3)(t^2 + 1)$ . Thus the roots are  $t = 0, \pm\sqrt{3}, \pm\sqrt{7}$ . Tracking the sign of the derivative, we see it is positive between  $-\sqrt{7}$  and  $-\sqrt{3}$ , negative between  $-\sqrt{3}$  and 0, positive between 0 and  $\sqrt{3}$ , and negative between  $\sqrt{3}$  and  $\sqrt{7}$ . This implies that the only possible candidates for

maxima are  $t = \pm\sqrt{3}$ . Trying both we get  $y(t) = -8$  and  $x(t) = \pm 16\sqrt{3}$ . Thus, both give the same maximum  $L$  of  $8\sqrt{13}$ .

FINAL: 900

5. A: Horizontal asymptote occurs when  $f'(x) = 0$  and thus  $f(x)$  is constant as  $x$  goes to infinity or negative infinity. Since the right-hand side simplifies to  $(f(x) + 4)(2 - 3x)$ , the value of  $y = f(x)$  for which  $f'(x) = 0$  is just  $y = -4$ . (This same answer can be found by using separation of variables and then finding the limit as  $x$  goes to infinity).

B: Since  $g(x)$  is a quartic, anything past  $g^{(4)}(x)$  will be 0, so we just add derivatives from 0 to 4. Let  $g(x) = x^4 + ax^3 + bx^2 + cx + d$  (we know that the leading coefficient is 1 because it must be the same as in the summation). Then  $g'(x) = 4x^3 + 3ax^2 + 2bx + c$ ,  $g''(x) = 12x^2 + 6ax + 2b$ ,  $g'''(x) = 24x + 6a$ , and  $g^{(4)}(x) = 24$ . The coefficient of  $x^3$  in the summation should thus be  $a + 4 = 0$ , so  $a = -4$ . The coefficient of  $x^2$  should be  $b + 3a + 12 = 0$ , so  $b = 0$ . The coefficient of  $x$  should be  $c + 2b + 6a + 24 = 16$ , so  $c = 16$ . The constant term should be  $d + c + 2b + 6a + 24 = 0$ , so  $d = -16$ . Altogether this gives  $g(x) = x^4 - 4x^3 + 16x - 16$ . Testing roots and using synthetic division, this simplifies to  $(x - 2)^3(x + 2)$ , so the only distinct roots are 2 and  $-2$ , and their product is  $-4$ .

C:  $V = \frac{4}{3}\pi r^3$  and  $A = 4\pi r^2$ , so we have  $\frac{dV}{dt} = -kA \rightarrow 4\pi r^2 \frac{dr}{dt} = -k(4\pi r^2)$  and thus  $r'(t) = -k$  for some positive constant  $k$ . Integrating, or just recognizing this is linear, gives  $r(t) = r_0 - kt$  for some original radius  $r_0$ . Then  $V(t) = \frac{4}{3}\pi(r_0 - kt)^3$ . To have  $V(3)/V(0) = 1/8$ , we must have  $(r_0 - 3k)^3/(r_0)^3 = 1/8$ . This simplifies to  $1 - 3k/r_0 = 1/2$ , so  $k = r_0/6$ . Then for the volume to be 0 we must have  $r_0 - kt = 0 \rightarrow t = 6$ . Discounting the time that has already been taken, it takes 3 additional minutes for the rest of the snowball to melt.

FINAL: 192

6. C: Sketching a coordinate plane with  $x = d$  and  $y = f$ , we see that the sample space is a rectangle of width 9 ( $1 < x < 10$ ) and height 8 ( $0 < y < 8$ ). Rearranging the inequality we see that it is just a parabola:  $y > -x^2 + 14x - 40 = -(x - 7)^2 + 9$ . The parabola is downward-sloping and crosses the sample space at  $(4, 0)$ ,  $(6, 8)$ ,  $(8, 8)$ , and  $(10, 0)$ . To find the probability that  $y$  is below the parabola, we must find the area bounded under the parabola and within the rectangle. This can be found as  $\int_4^6 -x^2 + 14x - 40 dx + \int_6^8 8 dx + \int_8^{10} -x^2 + 14x - 40 dx = 28/3 + 16 + 28/3 = \frac{104}{3}$ . Then we divide this by the total area of the rectangle,  $(9)(9) = 72$ , to get  $13/27$ . But this is the probability that  $y$  is below the parabola, when we are actually looking for the probability that it is above, so we take the complement

to get  $14/27$ .

D: Use the same trick of converting to coordinates, with Alex's number being  $x$ , Vishnav's being  $y$ , and Akash's being  $z$ . The sample space is a unit cube with one vertex at the origin (sides of  $0 < x < 1$ ,  $0 < y < 1$ ,  $0 < z < 1$ ). We are looking for the probability that the inequality  $x^2 + 4y^2 + 4z^2 < 4$ . Dividing by 4 we get  $\frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{1} > 1$ , the equation for an ellipsoid with  $x$ -axis of  $2(2) = 4$ ,  $y$ -axis of  $2(1) = 2$ , and  $z$ -axis of  $2(1) = 2$ . Sketching it out, we see that this ellipsoid must be the figure formed when an ellipse with horizontal axis 4 and vertical axis 2 is revolved around the  $x$ -axis. Thus, to find the volume of the region enclosed in a cube with  $0 < x < 1$ , we integrate. The ellipse that is being revolved has equation  $x^2/4 + y^2/1 = 1$ , so  $y = \sqrt{1 - x^2/4}$ . Then by disc method, the volume is  $\pi \int_0^1 1 - x^2/4 dx = \frac{11}{12}\pi$ . But this is the volume of the entire ellipsoid in  $0 < x < 1$ , and we are only looking for the portion with positive  $y$  and  $z$ , so we must divide by 4 to get  $\frac{11}{48}\pi$ . Then to get the probability we divide by the volume of the unit cube, which is just 1, and our answer is  $\frac{11}{48}\pi$ .

FINAL:  $\boxed{42 + 22\pi}$

7. A: Let Josh's location along the  $x$ -axis be  $j(t)$ , Nihar's location along the  $y$ -axis be  $n(t)$ , and Rayyan's location along the  $z$ -axis be  $r(t)$ . Same naming conventions as previous, with  $j(t) = 10t$ ,  $n(t) = 20t$ , and  $r(t) = 30t$  along the  $z$ -axis. Then we have the three points  $(10t, 0, 0)$ ,  $(0, 20t, 0)$ , and  $(0, 0, 30t)$ . We can find the area of the triangle with these vertices by considering vectors. The vector from Josh to Nihar is  $\langle -10t, 20t, 0 \rangle$  and the vector from Josh to Rayyan is  $\langle -10t, 0, 30t \rangle$ . It is well-known that the area of a triangle with two vector sides is equal to half the magnitude of the vector cross-product. We can easily evaluate this cross-product as  $\langle 600t^2, 300t^2, 200t^2 \rangle$ . Then the magnitude is  $\sqrt{360000t^4 + 90000t^4 + 40000t^4} = t^2\sqrt{490000} = 700t^2$  and the area is  $A(t) = 350t^2$ . Thus  $A'(t) = 700t$  so  $A'(3) = 2100$ .

B:  $h = d = 2r$  so  $V(t) = \frac{1}{3}\pi r(t)^2 h(t) = \frac{2}{3}\pi r(t)^3$ . Then  $V'(t) = 2\pi r(t)^2 r'(t) = 30$  and thus  $r'(t) = 15/(\pi r(t)^2)$ . If  $h(t) = 10$  then  $r(t) = 5$ , so  $r'(t) = 3/(5\pi)$  and thus  $h'(t) = \frac{6}{5\pi}$ .

C: Let the distance from the shadow's tip to Tanvi be  $a$  and the distance from Tanvi to the streetlight be  $b$ . By similar triangles,  $6/a = 15/(a + b)$ , so we can simplify to get  $a = 2b/3$ . Assume the streetlight is at  $x = 0$ . So if Tanvi's position is  $x = b(t)$ , then the tip of her shadow is at  $x = b(t) + a(t) = \frac{5}{3}b(t)$ . Thus the speed of the shadow's tip is  $\frac{5}{3}b'(t) = 25/3$ .

FINAL:  $\boxed{210\pi}$

8. A: Using integration by parts  $\int \ln(x)dx = x \ln(x) - \int \frac{1}{x}(x)dx = x \ln(x) - x$ . The upper bound evaluates to  $-1$ . The lower bound evaluates to  $\lim_{x \rightarrow 0} x \ln(x) = \lim_{x \rightarrow 0} \frac{\ln(x)}{1/x}$ . In this limit both the numerator and denominator go to infinity, so we can use L'hospital's. The derivative of the numerator is  $1/x$  and the derivative of the denominator is  $-1/x^2$ , so the overall limit becomes  $\lim_{x \rightarrow 0} -x = 0$ . Thus the definite integral is  $-1 - 0 = -1$ .

B:  $\int x^{-1/2}dx = 2\sqrt{x}$ . The upper bound evaluates to  $2\sqrt{3}$  and the lower bound to  $\lim_{x \rightarrow 0} 2\sqrt{x} = 0$ . So the definite integral is  $2\sqrt{3} - 0 = 2\sqrt{3}$

C: This is just the limit Riemann Sum equivalent of  $\int_0^{e-1} \ln(x+1)dx$ . Using the same IBP as in part A, we get  $(x+1)\ln(x+1) - (x+1)$ . Upper bound evaluates to  $e - e = 0$ . Lower bound evaluates to  $0 - 1 = -1$ . Definite integral is then  $0 - (-1) = 1$ .

D: First we factor an  $n^2$  out of the denominator to get  $\frac{1}{n} \sum_{i=1}^{3n} \sqrt{1 + 2\sqrt{i/n} + (i/n)}$ , which is clearly the limit Riemann Sum equivalent of  $\int_0^3 \sqrt{x + 2\sqrt{x} + 1}dx$ . Looking carefully, the expression in the radical is actually equivalent to  $(\sqrt{x} + 1)^2$ , so we get  $\int \sqrt{x} + 1dx = \frac{2}{3}x^{3/2} + x$ . Upper bound evaluates to  $2\sqrt{3} + 3$  and lower evaluates to 0, so this part is just  $2\sqrt{3} + 3$ .

FINAL: 3

9. A: This forms a circle, but the circumference can be found more easily with the polar arc length formula.  $L = \int \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta = \int_0^\pi \sqrt{(6 \cos \theta - 3 \sin \theta)^2 + (-6 \sin \theta - 3 \cos \theta)^2}d\theta = \int_0^\pi \sqrt{36 + 9}d\theta = 3\pi\sqrt{5}$ .

B: This forms a frustum, but the surface area can be found more easily by using the revolution surface area formula:  $A = 2\pi \int y \sqrt{1 + (\frac{dy}{dx})^2}dx = 2\pi \int_1^4 (2x + 6)\sqrt{1 + 2^2}dx = 2\pi\sqrt{5} \int_1^4 2x + 6dx = 2\pi\sqrt{5}(4^2 + 6(4) - 1^2 - 6(1)) = 66\pi\sqrt{5}$ .

C: This is just a figure with a semicircle base and cross-sections perpendicular to the diameter that are 30-60-90 triangles. Let the semicircle be centered at the origin. For any  $x$ ,  $y = \sqrt{r^2 - x^2} = \sqrt{12 - x^2}$ . Then the other leg of the 30-60-90 triangle has length  $y/\sqrt{3}$ , so the area is  $\frac{y^2}{2\sqrt{3}} = \frac{12-x^2}{2\sqrt{3}}$ . To find the volume we integrate along all  $x$  values and add up all the areas of these triangles:  $\frac{1}{2\sqrt{3}} \int_{-2\sqrt{3}}^{2\sqrt{3}} 12 - x^2 dx = \frac{1}{2\sqrt{3}}(12x - \frac{1}{3}x^3) \rightarrow 16$ .

D: Consider two circles of radius 2, one centered at  $(-1, 0)$  and the other at  $(1, 0)$ . Since the spherical situation is axially symmetric, we can obtain the volume of intersection by simply revolving the area of intersection about the  $x$ -axis. Thus we integrate from 0 to 1 using disk method, and double our answer due to symmetry. At each  $x$  value

from 0 to 1, the distance from the center at  $(-1, 0)$  is  $x + 1$ , so the corresponding  $y$  value is  $y = \sqrt{r^2 - (x + 1)^2} = \sqrt{4 - (x + 1)^2}$ . Then we have  $2\pi \int_0^1 y^2 dx = 2\pi \int_0^1 (4 - (x + 1)^2) dx = 2\pi(4x - \frac{1}{3}(x + 1)^3) = \frac{10\pi}{3}$

FINAL:  $\boxed{20\pi}$

10. A: In the limit, this series can be compared to  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ . The latter series diverges because it is a power series with  $p < 1$ , so therefore the former must diverge.

B: We use the ratio test:  $a_{n+1} = \frac{(2n+2)!}{((n+1)!)^2 6^{n+1}}$ , so the ratio  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{(2n+2)(2n+1)}{6(n+1)^2} = 4/6 < 1$ .

Thus, the series converges.

C: We use the ratio test:  $\lim_{n \rightarrow \infty} |\frac{a_{n+1}}{a_n}| = x^3$ , so the series will only converge if  $|x| < 1$ . Therefore the radius of convergence is 1.

$A + B + C = 2 + 3 + 1 = 6$ .  $\zeta(6) = \frac{1}{1^6} + \frac{1}{2^6} + \frac{1}{3^6} + \dots$  while  $\eta(6) = \frac{1}{1^6} - \frac{1}{2^6} + \frac{1}{3^6} + \dots$ . If we can subtract just the even terms from  $\zeta(s)$  we can get  $\eta(s)$ . Fortunately, we can isolate the even terms by multiplying  $\zeta_2(6) = \frac{1}{2^6}\zeta(6) = \frac{1}{2^6} + \frac{1}{4^6} + \dots$ . Then  $\zeta(6) - 2\zeta_2(6) = \frac{31}{32}\zeta(6) = \eta(6)$ , so the ratio is  $\boxed{\frac{31}{32}}$ .

11. We have six conditions on  $x(t)$  and  $y(t)$ :  $x(0) = 4$ ,  $y(0) = 0$ ,  $x'(0) = 0$ ,  $y'(0) = v_0$ ,  $x''(t) = -4x$ , and  $y''(t) = -4y$ .

This type of differential equation is quite common and it is well-known that it results in a sinusoidal oscillation about an equilibrium, of the sort  $x(t) = \sin(t)$ . But to make sure the other initial conditions are satisfied, we must modify the function with phase shift, amplitude, and period to get  $x(t) = 4 \cos(2t)$ . Similarly, we find that  $y(t) = 0.5v_0 \sin(2t)$ . Differentiating these two functions shows that they indeed satisfy all the necessary conditions.

A: We find the shape of the path using the normal trick for solving parametric trig equations.  $x/4 = \cos(2t)$  and  $y/(0.5v_0) = \sin(2t)$ , so  $\frac{x^2}{16} + \frac{y^2}{0.25v_0^2} = 1$ . This is only a circle when  $0.25v_0^2 = 16$ , so  $|v_0| = 8$ .

B: The two differential equations are completely separate, so since we are only looking at the  $x$ -coordinate we only have to consider  $x(t) = 4 \cos(2t)$ .  $x(t) = 0$  when  $t = \frac{\pi}{4} + n\pi$  for integer  $n$ . Since  $\pi \approx 3.14$ ,  $31.4 \approx 10\pi$  and thus  $28.26 \approx 9\pi$ . There should be 2 roots within every interval of  $\pi$ , for a total of 18 up to  $9\pi$ . There should be one additional root for  $9\pi + \pi/4 \approx 28.26 + 0.78 < 30$ . But  $9\pi + 3\pi/4$  would not be less than 30 so it is not a root. Thus there are 19 crossings.

C: Using the equations of motion, found earlier,  $x(\pi/6) = 2$ ,  $x'(\pi/6) = -4\sqrt{3}$ ,  $y(\pi/6) = 3\sqrt{3}$ ,  $y'(\pi/6) = 6$ . Since

velocities are constant after this point, the time it takes to reach the  $y$ -axis is  $t = \frac{\Delta x}{v} = \frac{2}{4\sqrt{3}} = \frac{\sqrt{3}}{6}$ . Then the change in  $y$ -value is  $\Delta y = v_y t = \sqrt{3}$ . The  $y$ -coordinate at crossing is  $y(\pi/6) + \Delta y = 4\sqrt{3}$ .

FINAL:  $\boxed{108\sqrt{3}}$

12. Given the  $x$ -coordinate of  $P_i$ , we want a formula for the  $x$ -coordinate of  $P_{i-1}$ . Let the  $x$ -coordinate of point  $P_i$  be  $a_i$ , and assume it is positive (since the function is odd, answer should be the same if it is negative). The derivative at  $x = a_i$  is  $3a_i^2$ , so the tangent line equation is  $y - a_i^3 = 3a_i^2(x - a_i)$ . Then to find the other intersection point, we plug in  $y = x^3$ :  $x^3 - a_i^3 = 3a_i^2(x - a_i)$ , so assuming  $x - a_i \neq 0$  then we can divide on both sides to get  $x^2 + a_i x + a_i^2 = 3a_i^2 \rightarrow x^2 + a_i x - 2a_i^2 = (x + 2a_i)(x - a_i) = 0$ . Thus  $P_{i-1}$  has  $x$ -coordinate  $-2a_i$ , and by the same logic,  $P_{i+1}$  has  $x$ -coordinate  $-\frac{1}{2}a_i$ . From this we can find a simple explicit formula:  $a_i = (-\frac{1}{2})^i a_0$ .

For all  $i > 0$ ,  $A_i$  is the area bound by the line connecting  $(a_i, a_i^3)$  and  $(-2a_i, -8a_i^3)$ , and the curve  $y = x^3$ . We also know by sketching concavity that the tangent line will always fall below the curve if  $a_i > 0$ . Thus  $A_i = \int_{-2a_i}^{a_i} x^3 - (a_i^3 + 3a_i^2(x - a_i)) dx = \int_{-2a_i}^{a_i} x^3 - 3a_i^2 x + 2a_i^3 dx = \frac{1}{4}(a_i^4 - 16a_i^4) - \frac{3}{2}a_i^2(a_i^2 - 4a_i^2) + 2a_i^3(a_i - (-2a_i)) = -\frac{15}{4}a_i^4 + \frac{9}{2}a_i^4 + 6a_i^4 = \frac{27}{4}a_i^4$ . Since  $A_i = \frac{27}{4}a_i^4$  and  $a_i = (-\frac{1}{2})^i a_0$ ,  $A_i = (\frac{1}{16})^i (\frac{27}{4})a_0^4$ . Thus the sum of the  $A_i$  is a geometric series with initial term  $\frac{27}{64}a_0^4$  and common ratio  $\frac{1}{16}$ , for a total sum of  $\frac{9}{20}a_0^4$ . Plugging in  $a_0 = 2\sqrt{5}$ , the answer is  $\boxed{180}$ .

13. B:  $x^3 + 6x^2 + 12x + 18 = (x + 2)^3 + 10$ , so  $\int x^3 + 6x^2 + 12x + 18 dx = \frac{1}{4}(x + 2)^4 + 10x$ . The upper bound evaluates to  $\frac{100^4}{4} + 980 = 25000980$ , and the lower bound evaluates to 4, so in total it is 25000976. The sum of the digits is 29.

C: Factoring out the 30, integrand simplifies to  $(1 - \cos^2(x))^2(\sin x)$ , so we can use the u-sub  $u = \cos(x)$  to get  $\int_0^1 (1 - u^2)^2 du = \int_0^1 u^4 - 2u^2 + 1 du = \frac{1}{5} - \frac{2}{3} + 1 = \frac{8}{15}$ . Then we factor back in the 30 to get 16.

$A - D = -\int_0^\pi e^x \cos x - e^x \sin x dx$ . But the integrand is just the derivative of  $e^x \cos x$ , so it all evaluates to  $1 + e^\pi$ .

FINAL:  $\boxed{46 + e^\pi}$

14. A: Taking the derivative we get  $\frac{5(\log_2 x)^4}{x \ln 2} - \sin\left(\frac{\pi}{x-4}\right) \frac{-1}{(x-4)^2}$ . Then plug in  $x = 5$  to get  $\frac{(\log_2 5)^4}{\ln 2}$

B: By fundamental theorem of calculus,  $g'(x) = 3x^2(\cos(x) + \sin(x))$  so  $g'(\pi) = -3\pi^2$

C:  $g''(x) = 6x(\cos x + \sin x) + 3x^2(\cos x - \sin x)$ , so  $g''(\pi) = -6\pi - 3\pi^2$

D: Let numerator be  $n(x)$  and denominator be  $d(x)$ . When  $2 < x < 4$ ,  $x^2 - 6x + 8 < 0$ , so  $n(x)$  is equivalent to  $-x^2 + 6x - 8$ .  $n(3) = 1$ ,  $n'(3) = 0$ ,  $d(3) = 10$ ,  $d'(3) = 1$ . By quotient rule,  $h'(3) = -1/100$ .

FINAL:  $\boxed{106 + \ln(5)}$