1. A: This is an infinite geometric series with first term $a = 1$ and common ratio $r = \frac{1}{2}$. Using the formula $S = \frac{a}{1-r}$ we find that the sum is $\frac{1}{1/2} = \boxed{2}$.

   B: Using the same formula with common ratio $r = \frac{1}{3}$ and first term $a = \frac{1}{3}$ (note that the lower bound of the summation is $n = 1$ rather than 0) gives $S = \frac{1/3}{2/3} = \boxed{\frac{1}{2}}$

   C: $C = \frac{2}{3} + \frac{4}{9} + \frac{6}{27}...$ To simplify this arithmetico-geometric series, we can multiply the entire thing by 3 and simplify the denominators of each term: $3C = 2 + \frac{4}{3} + \frac{6}{9}....$ Then, subtracting the original from the altered we get $3C - C = 2C = (2-0) + (\frac{4}{3} - \frac{2}{3}) + (\frac{6}{9} - \frac{4}{9})... = 2 + \frac{2}{3} + \frac{2}{9}....$ Dividing by 2, we get $C = 1 + \frac{1}{3} + \frac{1}{9}... = \frac{1}{2/3} = \boxed{\frac{3}{2}}$ by the geometric series formula.

   D: The denominator factors into $(n+3)(n+2)$, so we can use partial fraction decomposition to split the fraction up. Let $\frac{1}{n^2+5n+6} = \frac{A}{n+2} + \frac{B}{n+3}$. Then $A(n+3) + B(n+2) = 1$, so combining like terms we get $(A+B)n + (3A+2B) = 0n + 1$. This means $A + B = 0$ and $3A + 2B = 1$, a system of equations that can be solved to get $A = 1$ and $B = -1$. Now we have $\sum_{n=0}^{\infty} \frac{1}{n^2+5n+6} = \sum_{n=0}^{\infty} \frac{1}{n+2} - \frac{1}{n+3} = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4}....$ Every term except the first cancels, leaving an answer of $\boxed{\frac{1}{2}}$.

   Final Answer: $(2)(\frac{1}{2}) + (\frac{3}{2})(\frac{1}{2}) = \boxed{\frac{7}{4}}$

2. This problem has several steps, but the first is finding the determinant of the 4 x 4 matrix because every element of the inverse is divided by it. We can speed up this process by subtracting twice the first row from the third, which leaves a row with only one 1 and three 0's and does not change the overall determinant. Now the matrix looks like $\begin{vmatrix} 1 & 2 & -3 & 1 \\ 4 & -2 & 1 & 6 \\ 0 & 1 & 0 & 0 \\ 3 & 1 & 7 & 8 \end{vmatrix}$ so the determinant is simply $-\begin{vmatrix} 1 & -3 & 1 \\ 4 & 1 & 6 \\ 3 & 7 & 8 \end{vmatrix} = \boxed{-33}$.

   To find $N_{2,1}$, we actually must work on $M_{1,2}$ because the third step of finding the inverse is to take the transpose of the cofactor matrix. The minor of $M_{1,2}$ is $\begin{vmatrix} 4 & 1 & 6 \\ 2 & -6 & 2 \\ 3 & 7 & 8 \end{vmatrix} = -66$. Then we must negate this because it is in an even position, giving us 66 for the cofactor matrix position and thus also $\boxed{66}$ for the adjugate matrix position.

   To find $N_{3,3}$ we simply work on $M_{3,3}$ because it is on the diagonal so the transpose does not alter its position. The minor of $M_{3,3}$ is $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -2 & 6 \\ 3 & 1 & 8 \end{vmatrix} = -40$. We do not negate this one because it is in an odd position, giving us $\boxed{-40}$ for the adjugate matrix position.

   The sum of the two positions in the adjugate matrix is $66 - 40 = 26$. The final step is to divide each element by the determinant -33, giving an answer of $-\frac{26}{33}$.

3. A: By using synthetic division or simply substituting -2 for x in the numerator, we find that the remainder is $(-2)^2 + 5(-2) + 8 = \boxed{2}$.

   B: The numerator factors as $3(x+3)(x-1)$ and the denominator as $(x+3)(x+4)$. The only vertical asymptote is $x = -4$ because the $(x+3)$ factor appears in both the numerator and denominator. Meanwhile, since the degrees of the numerator and denominator are equal, the horizontal asymptote is just the ratio of the leading coefficients: $y = 3$. The two lines intersect at $(-4, 3)$, which is $\boxed{5}$ units from the origin.

C: Using polynomial long division, we find the oblique asymptote to be $y = \frac{60x^3}{30x^2} + \frac{104x^2 - (2x)(7x)}{30x^2} = 2x + 3$. The x-intercept of this line can be found by substituting 0 for y: $0 = 2x + 3$, so $x = \boxed{-\frac{3}{2}}$.

D: By checking possible factors we see that 1 is a root of the numerator, so we can use synthetic division to simplify the numerator to $(x-1)(x^2+5x+6) = (x-1)(x+2)(x+3)$. Meanwhile, the denominator simplifies to $(x+4)(x-1)$. The x-coordinate of the removable discontinuity must be 1 because the $(x-1)$ factor appears in both the numerator and denominator, but to find the ordinate (y-coordinate) we must cancel the factor out and substitute 1 for x. $\frac{(1+2)(1+3)}{1+4} = \boxed{\frac{12}{5}}$.

Final Answer: $(2)(5)\left(-\frac{3}{2}\right)\left(\frac{12}{5}\right) = \boxed{-36}$.

4. A: By change-of-base, $\log_{0.01} 0.000001 = \frac{\log 0.000001}{\log 0.01} = \frac{\log 10^{-6}}{\log 10^{-2}} = \frac{-6}{-2} = \boxed{3}$.

B: Using the common formula, we find that the number of digits is $\lfloor \log 5^{1000} \rfloor + 1 = \lfloor 1000 \log 5 \rfloor + 1$. $\log 2 + \log 5 = \log 10 = 1$, so $\log 5 = 1 - \log 2 = 1 - 0.30103 = 0.69897$. This means the answer must be $\lfloor 1000(0.69897) \rfloor + 1 = \lfloor 6989.7 \rfloor + 1 = 6989 + 1 = \boxed{6990}$.

C: $2 \log 5 = \log 5^2 = \log 25$. Then $\log 6 + \log 25 + \log 4 - \log 3 - \log 2 = \log \frac{(6)(25)(4)}{(3)(2)} = \log 100 = \boxed{2}$.

D: $\frac{1}{\log_a b} = \log_b a$ so the sum is equal to $\log_{xyz} xy + \log_{xyz} yz + \log_{xyz} xz = \log_{xyz}(xy)(yz)(xz) = \log_{xyz}(xyz)^2 = \boxed{2}$

Final Answer: $3 + 6990 + 2 + 2 = \boxed{6997}$

5. A: By completing the square we simplify to $9(x-1)^2 + 25(y-2)^2 = 225$. To put this in ellipse form we divide by 225 on both sides, yielding the equation $\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$. Then $a = \sqrt{25} = 5$, $b = \sqrt{9} = 3$, and $c = \sqrt{a^2 - b^2} = \sqrt{16} = 4$. The eccentricity is $\frac{c}{a} = \boxed{\frac{4}{5}}$

B: To put this in ellipse form we divide by 400 on both sides, yielding the equation $\frac{x^2}{16} + \frac{y^2}{25} = 1$. Then $a = \sqrt{25} = 5$, $b = \sqrt{16} = 4$, and $c = \sqrt{a^2 - b^2} = \sqrt{9} = 3$. The eccentricity is $\frac{c}{a} = \boxed{\frac{3}{5}}$

C: To put this in ellipse form we divide by 4 on both sides, yielding the equation $\frac{x^2}{4} + \frac{y^2}{1} = 1$. Then $a = \sqrt{4} = 2$, $b = \sqrt{1} = 1$, and $c = \sqrt{a^2 - b^2} = \sqrt{3}$. The eccentricity is $\frac{c}{a} = \boxed{\frac{\sqrt{3}}{2}}$

D: To put this in ellipse form we divide by 36 on both sides, yielding the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Then $a = \sqrt{9} = 3$, $b = \sqrt{4} = 2$, and $c = \sqrt{a^2 - b^2} = \sqrt{5}$. The eccentricity is $\frac{c}{a} = \boxed{\frac{\sqrt{5}}{3}}$

Final Answer: $\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{5}}{3}\right) = \boxed{\frac{2\sqrt{15}}{25}}$.

6. A: The first term contains x^8, the second x^7, so the third must involve x^6. Using the binomial theorem, we can find this term as $\binom{8}{6}x^6(-3)^2$, which has a coefficient of $(28)(9) = \boxed{252}$

B: The constant term is the one where $(x^k)(x^{-2(6-k)}) = x^0$, so $k - 12 + 2k = 3k - 12 = 0$, meaning $k = 4$. Then, by binomial theorem, the constant term is $\binom{6}{4}(2^4) = (15)(16) = \boxed{240}$.

C: The sum of the coefficients can be found by substituting 1 for each variable and solving, just as you would in an already expanded expression. $(1 + 4 + 1)^3 = 6^3 = \boxed{216}$.

D: This question is essentially asking for the number of ways to distribute 8 "powers" among 3 variables. Using the stars-and-bars formula, this is equal to $\binom{8+3-1}{8} = \boxed{45}$

Final Answer: $252 - 240 + 216 - 45 = \boxed{183}$.

7. A: By Vieta's formulas, the sum of the roots of an n-degree polynomial in the form $a_nx^n + a_{n-1}x^{n-1}...$ taken m at a time is $(-1)^m\left(\frac{a_n}{a_{n-m}}\right)$. In this case, $n = 5$ and $m = 1$ so the sum is $(-1)^1(\frac{a_4}{a_5}) = -\frac{7}{13}$, and the reciprocal is $\boxed{-\frac{13}{7}}$.

B: Let the roots be v, w, x, y, and z. $\frac{1}{v} + \frac{1}{w} + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{wxyz + vxyz + vwyz + vwxz + vwxy}{vwxyz} = \frac{4 \text{ at a time}}{5 \text{ at a time}}$. Using Vieta's formula with $m = 4$, we get that the numerator is $(-1)^4(\frac{a_1}{a_5}) = \frac{2}{13}$. Using Vieta's formula with $m = 5$, we get that the numerator is $(-1)^5(\frac{a_0}{a_5}) = \frac{10}{13}$. The fraction simplifies to $\frac{2}{10} = \boxed{\frac{1}{5}}$

C: Using the same formula with $m = 3$ for 3 at a time, we get $(-1)^3(\frac{a_2}{a_5}) = \boxed{-\frac{9}{13}}$.

D: Letting the roots be v, w, x, y, z again, we see that $(v + w + x + y + z)^2 = v^2 + w^2 + x^2 + y^2 + z^2 + 2vw + 2vx + 2vy... + 2yz$. In other words, the sum of the squares of the roots is equal to their sum squared minus twice their sum two at a time. We already know that the sum is $-\frac{7}{13}$ from A. The sum two at a time can be found using Vieta's formula with $m = 2$, resulting in $(-1)^2(\frac{a_3}{a_5}) = -\frac{10}{13}$. Therefore the sum of the squares of the roots is $(-\frac{7}{13})^2 - 2(-\frac{10}{13}) = \boxed{\frac{309}{169}}$.

Final Answer: $(-\frac{13}{7})(\frac{1}{5})(-\frac{13}{9})(\frac{309}{169}) = \boxed{\frac{103}{105}}$.

8. C: We first solve C because it requires no outside information. The sum simplifies to $\log_C 2 + \log_C 3 + \log_C 4 = 1$, so $\log_C 24 = 1$. This means $C^1 = 24$ so $C = \boxed{24}$.

D: $2^{12} = 4096$, so $\log_2 4096 = 12$, meaning $\log_{4096} 2 = \frac{1}{12}$. Then using our answer from Part C we get $D = \frac{24}{12} = \boxed{2}$.

B: Since $8^2 = 64$, $\log_8 64 = 2$. Then $B = D^{\log_8 64} = D^2 = 2^2 = \boxed{4}$.

A: Using the hint from question 4 that $\log 2 \approx 0.301$, $A = \log B = \log 4 = \log 2^2 = 2\log 2 \approx 2(0.301) = \boxed{0.602}$.

Final Answer: Rounding A to the nearest hundredth, we get $\boxed{0.60}$.

9. A: First we must simplify the conic to hyperbola form by dividing both sides by 144, yielding $\frac{x^2}{16} - \frac{y^2}{9} = 1$. This means $a = \sqrt{16} = 4$ and $b = \sqrt{9} = 3$, so $c = \sqrt{a^2 + b^2} = \sqrt{16 + 9} = \sqrt{25} = 5$. The eccentricity is $\frac{c}{a} = \boxed{\frac{5}{4}}$.

   B: Since c is the distance from each focus to the center, the distance between the foci is just $2c = 2(5) = \boxed{10}$.

   C: The length of the conjugate axis is $2b = 2(3) = \boxed{6}$.

   D: The length of the latus rectum is $\frac{2b^2}{a} = \frac{2(9)}{4} = \boxed{\frac{9}{2}}$.

   Final Answer: $BCD = (10)(6)\left(\frac{9}{2}\right) = 270$, so $\frac{BCD}{A} = 270\left(\frac{4}{5}\right) = \boxed{216}$.

10. Since the parabola's directrix is vertical, it must be horizontal, because the directrix is perpendicular to the axis of symmetry. Also, the focus is left of the directrix, which indicates that the parabola opens leftward because the opening is always in the same direction as the focus. Therefore, the equation is in the form $x = -a(y + b)^2 + c$. We know that $(c, -b)$ is the vertex of the parabola, and $a = \frac{1}{4p}$ where p is the focal radius. Since the vertex is halfway between the focus and directrix, we can use the midpoint formula to find that the parabola's vertex is at $(\frac{5}{2}, 0)$, and that its focal radius is $\frac{5}{2}$. Then $a = \frac{1}{4(\frac{5}{2})} = \frac{1}{10}$, $b = 0$, and $c = \frac{5}{2}$. Thus the equation of the parabola is $x = -\frac{1}{10}y^2 + \frac{5}{2}$. Multiplying by 10 and rearranging to the form given by the question we find that the original conic is $0x^2 + 0xy + 1y^2 + 10x + 0y - 25 = 0$. Then the altered conic is $1x^2 + 0xy + 2y^2 + 20x + 0y - 75 = 0$. To find the intersection point we can substitute the y^2 in this equation with $25 - 10x$ because the original parabola simplifies to $y^2 = 25 - 10x$. Then we have $x^2 + 50 - 20x + 20x - 75 = 0$ so $x^2 = 25$ and $x = \pm 5$. $x = 5$ was not in the domain of the original parabola, so we must use $x = -5$. Then $y^2 = 25 - 10(-5) = 75$, $y = \pm 5\sqrt{3}$. The two intersection points are $(-5, 5\sqrt{3})$ and $(-5, -5\sqrt{3})$, with a distance between them of $\boxed{10\sqrt{3}}$.

11. A: If the sequence is arithmetic then there is some common difference d such that $1026 = 38 + d(31 - 10)$, so $988 = 21d$ and $d = \frac{988}{21}$. The 17^{th} element must be $a_{10} + d(17 - 10) = a_{10} + 7d = \boxed{38 + \frac{988}{3}}$.

    B: If the sequence is geometric then we know there is a common ratio r such that $1026 = 38r^{31-10}$ so $r = \left(\frac{1026}{38}\right)^{\frac{1}{21}} = \sqrt[21]{27}$. The 17^{th} element must be $a_{10}r^{17-10} = a_{10}r^7 = 38\sqrt[3]{27} = 38(3) = \boxed{114}$

    C: Using our common difference from Part A we find that $a_{24} = a_{10} + d(24 - 10) = a_{10} + 14d = 38 + 14(\frac{988}{21}) = \boxed{38 + 2\left(\frac{988}{3}\right)}$

    D: Using our common ratio from Part B we find that $a_{24} = a_{10}r^{24-10} = a_{10}r^{14} = 38(27^{\frac{14}{21}}) = 38(27^{\frac{2}{3}}) = 38(3^2) = 38(9) = \boxed{342}$

    Final Answer: $(A + C) - (B + D) = (38 + \frac{988}{3} + 38 + 2\left(\frac{988}{3}\right)) - (114 + 342) = (76 + 988) - (456) = \boxed{608}$

12. If the discriminant of a quadratic is nonnegative it has real roots. Thus, for each polynomial we can write an inequality regarding its discriminant: The first becomes $3^2 - 4(a)(-9) = 9 + 36a \geq 0$, so $a \geq -\frac{1}{4}$.
    The second becomes $(-a)^2 - 4(9)(1) = a^2 - 36 \geq 0$, so $|a| \geq 6$.
    The third becomes $5^2 - 4(-1)(a) = 25 + 4a \geq 0$, so $a \geq -\frac{25}{4}$.

The fourth becomes $a^2 - 4(1)(4) = a^2 - 16 \geq 0$, so $|a| \geq 4$.
The binary Key must be between 0000 and 1111 or 0 and 16 in base 10, so there are 4 possible positive perfect squares: 1, 4, 9, and 16. In binary, these are 0001, 0100, 1001, and 1111. We must check each of these Keys to see which values of a make them possible.
0001 means the first 3 inequalities are false and the fourth is true. $a < -\frac{1}{4}$, $|a| < 6$, $a < -\frac{25}{4}$, and $|a| \geq 4$. However, it is impossible that $a < -\frac{25}{4}$ and $|a| < 6$, so there are no values of a for which the Key is 0001.
0100 means the second inequality is true but all others are false. $a < -\frac{1}{4}$, $|a| \geq 6$, $a < -\frac{25}{4}$, and $|a| < 4$. However, it is impossible that $|a| < 4$ and $|a| \geq 6$, so there are no values of a for which the Key is 0100.
1001 means the first and last inequalities are true but the others are false. $a \geq -\frac{1}{4}$, $|a| < 6$, $a < -\frac{25}{4}$, and $|a| \geq 4$. However, it is impossible that $a \geq -\frac{1}{4}$ and $a < -\frac{25}{4}$, so there are no values of a for which the Key is 1001.
1111 means all 4 inequalities are true. $a \geq -\frac{1}{4}$, $|a| \geq 6$, $a \geq -\frac{25}{4}$, and $|a| \geq 4$. This implies that $a \geq -\frac{1}{4}$ and $|a| \geq 6$, so $a \geq 6$. All values of a that satisfy this inequality and $|a| \leq 30$ result in a perfect square Key of 1111. The sum of these values is $(30 - 6 + 1)(\frac{30 + 6}{2}) = 450$.
Adding up the possible values of a for each Key value we get $0 + 0 + 0 + 450 = \boxed{450}$.

13. Using Cramer's Rule...
The x matrix is the coefficient matrix with all x coefficients replaced by the constants on the right side of each equation: $\begin{vmatrix} 11 & -1 & -8 \\ -18 & 6 & 13 \\ -5 & 7 & 11 \end{vmatrix} = 11((6)(11) - (7)(13)) - (-1)((-18)(11) - (13)(-5)) + (-8)((-18)(7) - (6)(-5)) =$ $11(-25) + 1(-133) - 8(-96) = -275 - 133 + 768 = \boxed{360}$

The y matrix is the coefficient matrix with all y coefficients replaced by the constants on the right side of each equation: $\begin{vmatrix} 2 & 11 & -8 \\ 4 & -18 & 13 \\ 10 & -5 & 11 \end{vmatrix} = 2((-18)(11) - (-5)(13)) - 11((4)(11) - (13)(10)) + (-8)((4)(-5) - (-18)(10)) =$ $2(-133) - 11(-86) - 8(160) = -266 + 946 - 1280 = \boxed{-600}$

The z matrix is the coefficient matrix with all z coefficients replaced by the constants on the right side of each equation: $\begin{vmatrix} 2 & -1 & 11 \\ 4 & 6 & -18 \\ 10 & 7 & -5 \end{vmatrix} = 2((6)(-5) - (-18)(7)) - (-1)((4)(-5) - (-18)(10)) + 11((4)(7) - (6)(10)) = 2(96) +$ $1(160) + 11(-32) = 192 + 160 - 352 = \boxed{0}$

Final Answer: $360 - 600 + 0 = \boxed{-240}$

14. A: $\frac{4}{\frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{6}} = \frac{4}{(\frac{30+15+6+5}{30})} = \frac{4}{(\frac{56}{30})} = \frac{30}{14} = \boxed{\frac{15}{7}}$.

B: The harmonic mean for 2 numbers $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ simplifies to $\frac{2ab}{a+b}$. For 3 and 9, it is $\frac{2(3)(9)}{3+9} = \frac{54}{12} = \boxed{\frac{9}{2}}$

C: $\frac{2(4)(6)}{4+6} = \frac{48}{10} = \boxed{\frac{24}{5}}$

D: $\frac{2(5)(7)}{5+7} = \frac{70}{12} = \boxed{\frac{35}{6}}$

Final Answer: $ABCD = (\frac{15}{7})(\frac{9}{2})(\frac{24}{5})(\frac{35}{6}) = \boxed{270}$

15. Thaw rate is surface area divided by volume: $\frac{lw}{lwh} = \frac{1}{h}$. Therefore the thaw rate is inversely proportional to the height, so to order from greatest to least in thaw rate we must order from least to greatest in height. $4 < 6 < 8 < 20$, so the desired order is $\boxed{D, A, B, C}$