For all questions, answer choice (E) NOTA means that none of the given answers is correct. For all problems, $i = \sqrt{-1}$. Good Luck!

1. The slope of a parallel line is equal to that of the original line but its y-intercept must be different. The only choice that satisfies this is $y = -\frac{123}{456}x + 1$, $\boxed{A}$.

2. $3^2 - 2^2 = 9 - 8 = 1 = 3 - 2 = \pi - e$, $\boxed{A}$.

3. $\sqrt{2^{6^{2^{1^{4^4}}}}} = \sqrt{2^{6^{2^{1^{256}}}}} = \sqrt{2^{6^{2^1}}}$ (previous steps could have been skipped because 1 raised to any power is equal to 1) $= \sqrt{2^{6^2}} = \sqrt{2^{36}} = 2^{18} = 262144$, $\boxed{C}$.

4. If $d(r-1) = 1$, then r must be 2, which clearly fails. If $d(r-1) = 2$, then $d(r+1) = 8$. $d(r-1) = 2$ implies $r - 1$ is a prime, and $r - 1 = 2$ is clearly not going to cut it, meaning it must be an odd prime. This means $r + 1$ must also be odd, with 8 divisors. Any number with 8 divisors must be of the form p^7 (min 2187), p^3q (min 135), or pqr (min 105). Clearly $r + 1 = 105 \implies r = 104$ is the best we can do in this case ($r - 1 = 103$ is prime), but we must also rule out higher cases.

   If $d(r+1) = 16$ and $d(r-1) = 4$, then $r+1$ must be in the form p^{15} (way too large), p^7q (too large), p^3q^3 (min 216), p^3qr (min 120), or $pqrs$ (min 210). All are larger than 105. Clearly no numbers less than 105 have 20+ divisors. $r = 104$ is the best we can do, so $d(r) = 4(2) = 8$, $\boxed{D}$.

5. $\log_2 8^3 = 3\log_2 8 = 3 \cdot 3 = 9$ and $(\log_2 8)^3 = 3^3 = 27 \to 9 - 27 = -18$, $\boxed{C}$.

6. $1 - 4x^2 - 12xy - 9y^2 = 1 - (4x^2 + 12xy + 9y^2) = 1^2 - (2x + 3y)^2 = (1 - 2x - 3y)(1 + 2x + 3y)$, $\boxed{D}$

7. We can rearrange the equation to $x^2 - x = 0$. This is the same as $x(x - 1) = 0$ meaning that $x = 0, 1$, $\boxed{C}$.

8. $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}4 & 3\\1 & 2\end{bmatrix} = \begin{bmatrix}1(4)+2(1) & 1(3)+2(2)\\3(4)+4(1) & 3(3)+4(2)\end{bmatrix} = \begin{bmatrix}6 & 7\\16 & 17\end{bmatrix}$, $\boxed{C}$

9. $\frac{5i+4}{\sqrt{3}+2+i} = \frac{5i+4}{\sqrt{3}+2+i} \cdot \frac{\sqrt{3}+2-i}{\sqrt{3}+2-i} = \frac{5i\sqrt{3}+6i+13+4\sqrt{3}}{8+4\sqrt{3}} = \frac{5i\sqrt{3}+6i+13+4\sqrt{3}}{4(2+\sqrt{3})} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{4i\sqrt{3}-3i-5\sqrt{3}+14}{4}$, $\boxed{A}$.

10. We add the exponents to get i^S where $S = 1 + \frac{1}{2} + \frac{1}{4} + \cdots = 2$ due to infinite geometric series formula. Then $i^2 = -1$, $\boxed{B}$.

11. First, note that $n^2 + 4n + 4 = (n + 2)^2$. We are trying to find the intersection points of $y = f(x) = (x + 2)^2$ and $y = g(x) = 2^x$. The former is a parabola symmetrical about $x = -2$, while the latter is an exponential function. Sketching out the two, we see they must intersect three times - twice for $x < 0$ (near the parabola's vertex) and once for $x > 0$ (because $f(0) > g(0)$ but eventually $g(x)$ grows larger than $f(x)$). First we check all the negative values. $f(-1) > g(-1)$ but $f(-2) < g(-2)$, so there must be a cross between -1 and -2. $f(-3) > g(-3)$, so there must be a cross between -2 and -3. This removes the two negative possibilities for crosses at integer x. Finally we check positive integers with a simple binary search. $f(0) > g(0)$, but $f(10) < g(10)$, so there must be a cross between 0 and 10. $f(5) > g(5)$, $f(7) < g(7)$, and we find that $f(6) = g(6)$. Thus $x = n = 6$ is the only integer value at which they cross, and the answer is 6, $\boxed{C}$.

12. Multiply both sides by x to get $x^3 + 2x^2 + 3x + 6 = 0$. The Rational Root Theorem tells us that the possible roots of this equation are ±1, ±2, ±3, ±6. Through trial and error, you will find that -2 is a solution and that $(x + 2)$ is a factor of $x^3 + 2x^2 + 3x + 6$. $\frac{x^3+2x^2+3x+6}{x+2} = x^2 + 3$. The other values of x are $\pm\sqrt{3}i$ because $x = \sqrt{-3}$, $\boxed{D}$.

13. $\begin{vmatrix}n+1 & n+3 & n+5\\n+7 & 0 & n+9\\n+11 & n+13 & n+15\end{vmatrix} = (n+1)((0)(n+15) - (n+9)(n+13)) - (n+3)((n+7)(n+15) - (n+9)(n+11)) +$ $(n+5)((n+7)(n+13) - (0)(n+11)) = 40n + 320$, so this implies $\sum_{n=1}^{10}(40n + 320) = 360 + 400 + ... + 720 = \frac{1}{2} \cdot (360 + 720)(10) = 5400$, $\boxed{A}$.

14. The probability that Dylan will win in the first round is $\frac{3}{10}$ (this is the probability that Eric will laugh in the first round). If Dylan were to win in the second round, we would need to account for both him and Eric not laughing in the first round and Eric laughing in the second round so he probability Dylan wins in the second round is $\frac{7}{10} \cdot \frac{6}{10} \cdot \frac{3}{10}$. Similarly, the probability that he wins in the third round is $\frac{7}{10} \cdot \frac{6}{10} \cdot \frac{7}{10} \cdot \frac{6}{10} \cdot \frac{3}{10}$. Every round, the probability of

Dylan winning is the probability of him winning in the last round multiplied by $\frac{6\cdot 7}{10\cdot 10}$. This is a geometric series so the probability that Dylan will win in any round is $\frac{\frac{3}{20}}{1-\frac{42}{100}} = \frac{15}{29}$, $\boxed{B}$.

15. Suppose that q was a decomposable number divisible by no perfect square other than 1. This means $\sqrt{a}+\sqrt{b} = \sqrt{q} \implies a = b+q-2\sqrt{bq}$. This means that bq is a perfect square. However, since q is the product of distinct primes, then in order to make the exponents of these primes even, b must be divisible by these primes too, making $b \geq q$. However, this is problematic, since $\sqrt{a} = \sqrt{q}-\sqrt{b} \leq 0$, contradiction. This rules out $2021 = 43\cdot 47$, $2022 = 2\cdot 3\cdot 337$, and $2026 = 2\cdot 1013$. Since $\sqrt{2023} = 17\sqrt{7} = 10\sqrt{7}+7\sqrt{7} = \sqrt{700}+\sqrt{343}$, 2023 is decomposable, and our answer is $\boxed{C}$.

16. The table has all of the probabilities filled out using the information from the problem. $20\% + 40\% = 60\%$, $\boxed{B}$.

	A (Y)	A (N)	Total
G (Y)	10	40	50
G (N)	20	30	50
Total	30	70	100

17. $x^3 = 27 \to (a+bi)^3 = 27 \to a^3+3a^2bi+3ab^2i^2+b^3i^3 = 27$. Simplifying gives us $a^3+3a^2bi-3ab^2-b^3i = 27$. Since there is no imaginary term in 27, $3a^2bi - b^3i = 0$. This means that $3a^2 = b^2$. We also know that $a^3-3ab^2 = 27$. Substituting in $3a^2$ for b^2 gives us $a^3-9a^3 = -8a^3 = 27 \to a = \frac{-3}{2}$. From this, we find that the value of b is $\pm\frac{3\sqrt{3}}{2}$. This means that the two non-integer solutions to $x^3 = 27$ are $\frac{-3}{2} \pm \frac{3i\sqrt{3}}{2}$, $\boxed{D}$.

18. We can find the differences of each term in the sequence and work backwards from there. $[1,2,4,9] \to [1,2,5] \to [1,3] \to [2]$. Since the degree of the polynomial is 3, we only take the difference of the numbers in the sequence 3 times. The last list of differences should contain the same number (in this case, 2). $2+3 = 5 \to 5+5 = 10 \to 9+10 = 19$, $\boxed{A}$.

Another method would be to plug the x-values in a cubic (like $ax^3+bx^2+cx+d = y$) to create a system of equations and then solve for a, b, c, and d. Then, you would plug in 4 in the cubic to find $f(4)$.

19. We can multiply both sides by $(x-9)^3$ to get that $7x-4 = A(x-9)^2+B(x-9)+C$. Plugging in $x = 10$, since $10-9 = 1$, gives us $A+B+C = 66$, $\boxed{C}$.

20. Of the two inner summations, the left evaluates to $\frac{1/a^2}{1-1/a} = \frac{1}{a^2-a} = \frac{1}{(a-1)(a)}$ and the right evaluates to $\frac{1/a^2}{1+1/a} = \frac{1}{a^2+a} = \frac{1}{(a)(a+1)}$ by geometric series formula. By partial fraction decomposition, the sum of the summations is $\frac{1}{a-1}-\frac{1}{a}+\frac{1}{a}-\frac{1}{a+1} = \frac{1}{a-1}-\frac{1}{a+1}$. Then we sum this over all a to get $\frac{1}{1}-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+\ldots$. We can see that every term from $\frac{1}{3}$ onward cancels out with its additive inverse. Thus the total sum is $1+\frac{1}{2} = 1.5$, $\boxed{D}$.

21. Let $S = \sum_{x=1}^{\infty} \frac{x^2}{4^x}$. We can multiply both sides by 4 to get $4S = \frac{1^1}{4^0}+\frac{2^2}{4}+\frac{3^2}{4^2}+\frac{4^2}{4^3}\cdots$. This means that $3S = 1+\frac{2^2-1^2}{4}+\frac{3^2-2^2}{4^2}+\frac{4^2-3^2}{4^3}\ldots$. We can express every numerator as a difference of of squares $((a+1)^2-(a)^2 = (a+1-a)(a+1+a) = (2a+1))$. This means that $3S = 1+\frac{3}{4}+\frac{5}{4^2}+\frac{7}{4^3}\ldots$. Multiplying both sides by 4 gives us $12S = 4+3+\frac{5}{4}+\frac{7}{4^2}\ldots$. By subtracting $3S$ from $12S$ we get that $9S = 6+\frac{2}{4}+\frac{2}{4^2}+\frac{2}{4^3}\ldots$. We can find the sum of $9S-6$ but using the formula for the sum of a geometric sequence: $9S-6 = \frac{\frac{2}{4}}{1-\frac{1}{4}} = \frac{2}{3}$. This means that $S = \frac{\frac{2}{3}+6}{9} = \frac{20}{27}$, $\boxed{D}$.

22. Notice that $g(x)$ is equal to the sum of the x^{th} row of Pascal's triangle. This means that the value of $g(x)$ is 2^x (this is equal to the sum of the coefficients when $(a+b)^{64}$ is expanded). Since $f(1) = 1$, this means that $f(64) = 2+2^2+...+2^{64} = 2^{65}-2$, $\boxed{C}$ (this can be found by either using the formula for the sum of a finite geometric sequence or the fact that $2^0+2^1+...+2^n = 2^{n+1}-1$).

23. The equation implies that the sum of the distances from z and $1+i$ and $-2-3i$ is 13. This means the equation encloses an ellipse, by the geometrical definition of an ellipse. This ellipse has foci at $1+i$ and $-2-3i$, meaning

the distance between these foci is 5. The major axis of this ellipse has length 13, meaning the minor axis has length 12. The area of this ellipse is thus $6(6.5)\pi = 39\pi$, $\boxed{A}$.

24. The slope of the line that intersects $(3,4)$ and (a,b) is $\frac{b-4}{a-3}$. The slope of the line perpendicular would be $\frac{3-a}{b-4}$, $\boxed{A}$.

25. Both the original point and its reflection lie on a line perpendicular to $y = ax$, so it has a slope of $-\frac{1}{a}$. Using point-slope form, the equation of the line is $y-1 = -\frac{1}{a}(x)$. Now we can find the intersection point between this line and the reflecting line: $-\frac{1}{a}(x)+1 = ax \rightarrow (a+\frac{1}{a})x = 1 \rightarrow x = \frac{a}{a^2+1}$. Thus $y = \frac{a^2}{a^2+1}$. This point is the midpoint of $(0,1)$ and the reflected point. Let the reflected point be (q,r). Then $\frac{0+q}{2} = \frac{a}{a^2+1}$, so $q = \frac{2a}{a^2+1}$. Meanwhile $\frac{1+r}{2} = \frac{a^2}{a^2+1}$, so $r = \frac{a^2-1}{a^2+1}$. For the reflected point to be on $y = x$ we must have $q = r$, so $a^2-1 = 2a$, $a^2-2a-1 = 0$, and thus by quadratic formula $a = 1+\sqrt{2} \approx 2.41$. The floor of 10 times this is 24, meaning the answer is $\boxed{D}$.

26. First we complete the square to find the equation of the ellipse: $(x-3)^2-9+4(y-1)^2-4+9 = 0 \rightarrow (x-3)^2+4(y-1)^2 = 4 \rightarrow \frac{(x-3)^2}{4}+\frac{(y-1)^2}{1} = 1$. Thus the center of the ellipse is $(3,1)$, and the ellipse is horizontal with $a = 2$ and $b = 1$. Shubham is at $(2,1)$, so we can translate the ellipse over to center $(0,0)$ to make calculations easier: now Shubham is at $(-1,0)$, which is equivalent to $(1,0)$, and the ellipse is centered at $(0,0)$. Now the equation of the ellipse is $\frac{x^2}{4}+\frac{y^2}{1} = 1$, so we rearrange to get $y^2 = 1-\frac{x^2}{4}$. Consider a point (x,y) on the ellipse. The distance from this point to $(1,0)$ is $\sqrt{(x-1)^2+y^2}$, but plugging in the value of y^2 we get $\sqrt{(x-1)^2+1-x^2/4}$. To minimize the distance, we just have to minimize the part within the square root: $x^2-2x+1+1-x^2/4 = \frac{3}{4}x^2-2x+2$. Using vertex formula to find the minimum, the x value that minimizes is $\frac{4}{3}$. Then we plug that back into the distance formula $\sqrt{\frac{3}{4}x^2-2x+2}$ to get $\sqrt{2/3} = \frac{\sqrt{6}}{3}$, for an answer of $\boxed{B}$.

27. Since the hyperbola is vertical, we can deduce that the hyperbola is in the form of $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2} = 1$. In this format, the center of the hyperbola is (h,k). The midpoint of the vertices, the center, is $(3,2)$; as a result, $h = 3$ and $k = 2$. The transverse axis, distance between the vertices, is equal to $2a$ or 8. This means that $a = 4$. The slope of the asymptotes are -2 and 2. The slopes of an hyperbola are in the form of $\pm\frac{a}{b}$, meaning that $b^2 = 4$. With all of that, we can deduce that the equation of the hyperbola is $\frac{(y-1)^2}{16}-\frac{(x-3)^2}{4} = 1$, $\boxed{D}$.

28. Throughout this solution, let $f(k) = x-k$. Out of all $\binom{8}{4} = 70$ choices of B, all values of $f(B)$ clearly do not have an x^4 coefficient, since the coefficient of x^4 in both products in 1, cancelling them out. Most choices of B make $f(B)$ have an x^3 term (and thus degree 3), but not all of them. For example, if $B = \{f(1), f(3), f(6), f(8)\}$, then both products will have $-18x^3$ terms. In general, if $B = \{f(a), f(b), f(c), f(d)\}$, then the first product has a $-(a+b+c+d)x^3$ term, and the second product has a $-(36-a-b-c-d)x^3$ term. In order for these to cancel, we must have $a+b+c+d = 18$. This happens precisely when

$$\{a,b,c,d\} = \{1,2,7,8\}, \{1,3,6,8\}, \{1,4,5,8\}, \{1,4,6,7\}, \{2,3,5,8\}, \{2,3,6,7\}, \{2,4,5,7\}, \{3,4,5,6\}$$

by simple casework. But, what if something even more heinous happens: the x^2 terms cancel? Note that we only have 4 cases to check, since B and its complement are the same case (e.g. $\{1,2,7,8\}$ and $\{3,4,5,6\}$). Incredibly, in one of these 4 cases, this happens:

$$(x-1)(x-4)(x-6)(x-7)-(x-2)(x-3)(x-5)(x-8) = 16x-72$$

and its mirror (switching B and C). This means in all 70 choices of B, 62 of them give degree 3, 6 of them give degree 2, and the last 2 give degree 1. This means the expected value of the degree is

$$\frac{62\cdot 3+6\cdot 2+2\cdot 1}{70} = \frac{200}{70} = \frac{20}{7}, \boxed{A}$$

29. If any of the terms in $B_{a,d}$ is 0, then clearly the sum would not be defined. Assume that none of the terms is 0:

- $d = 0$: ad is always 0, so this case is not necessary to look at.
- $d > 0$: After some finite amount of terms, the sequence becomes all greater than 1. Let f be the first term greater than 1 in this sequence. This means

$$f(B_{a,d}) = \text{const.}+\frac{1}{f^r}+\frac{1}{(f+d)^{r+1}}+\frac{1}{(f+2d)^{r+2}}\cdots < \text{const.}+\frac{1}{f^r}+\frac{1}{f^{r+1}}+\frac{1}{f^{r+2}}\cdots$$

However, the infinite sum on the right converges by the infinite geometric series formula (and clearly $f(B_{a,d})$ is bounded below by this const. term). This means this case will not produce lethal pairs.

- $d < 0$: After some finite amount of terms, the sequence becomes all less than -1. Let g be the first term less than -1 in this sequence. This means

$$f(B_{a,d}) = \text{const.} + \frac{1}{f^r} + \frac{1}{(f+d)^{r+1}} \cdots < \text{const.} + \frac{1}{|f|^r} + \frac{1}{|f+d|^{r+1}} \cdots$$

$$< \text{const.} + \frac{1}{|f|^r} + \frac{1}{|f|^{r+1}} + \frac{1}{|f|^{r+2}} \cdots$$

  Again, the last infinite series clearly converges by the infinite geometric series, and $f(B_{a,d})$ is bounded below by the negative of the upper bound by the same absolute value logic. This means that no lethal pairs are produced in this case either.

Therefore, lethal pairs can only occur when some term of $B_{a,d}$ is 0, or $d = 0$. If $a = 0$, then ad is again 0. If some other term of $B_{a,d}$ is 0, then there exists a positive integer k such that

$$a + kd = 0 \implies ad = (-kd)d = -kd^2 \leq 0$$

In any case, ad is always at most 0, and since equality can be achieved (e.g. $a = d = 0$), then the maximum is 0, $\boxed{B}$.

30. It is important to visualize and sketch the graphs of the functions. $y = |x|$ looks like a "V" which is symmetric across the y-axis. $y = -x^2 + 8$ is the graph of x^2 flipped across the x-axis and shifted up 8 units vertically; in other words, like a "n" or an upside down "U". Since they intersect at $\frac{-1-\sqrt{33}}{2}$ (between -3 and -2) and $\frac{1+\sqrt{33}}{2}$ (between 2 and 3), we only need to find the number of lattice points with a x-coordinate of -2, -1, 0, 1, 2. The only lattice point which falls within the area bounded by the two graphs with an abscissa of 2 is $(2, 3)$. The coordinates $(1, 2)$, $(1, 3)$, $(1, 4)$, $(1, 5)$, and $(1, 6)$ are within the wanted area whose x-coordinate is 1. This means that there are a total of 6 lattice points if the x-coordinate is positive. Since both the graphs are symmetric across the y-axis, we can deduce that the number of lattice points with a negative x-coordinate is also 6. For the x-coordinate of 0, any integer from 1 to 7, inclusive, satisfy the conditions. Adding the values up gives us $6 + 7 + 6 = 19$, $\boxed{B}$.