

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- Shreyas is playing Trivia Murder Party with his friends. His question is to calculate $\arcsin(\cos(\frac{7\pi}{6}))$. What should Shreyas put to get the right answer?

(A) $\frac{\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{5\pi}{3}$ (D) $-\frac{\pi}{3}$ (E) NOTA
- Anjali's garden only contains roses that have the same amount of petals as the equation $r = 11 \sin(2\theta) \cos(2\theta)$. How many petals does each rose in Anjali's garden have?

(A) 2 (B) 4 (C) 11 (D) 22 (E) NOTA
- Which of the following equations is equal to the rectangular equation $x^4 - y^4 = 2xy$ in polar form?

(A) $r = \tan \theta$ (B) $r = \tan 2\theta$ (C) $r^2 = \tan \theta$ (D) $r^2 = \tan 2\theta$ (E) NOTA
- Evaluate the limit $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9 - x}$.

(A) $\frac{1}{9}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) Does Not Exist (E) NOTA
- Farzan rips Dylan's NACLO test urging him to do Science Olympiad. The ripped test is the same shape as the conic $x^2 + 6xy + 25y^2 = 1$. What is the area of Dylan's ripped test?

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) NOTA
- Unfortunately Tanvi takes away Farzan's Science Olympiad test until he can evaluate $\sin(67.5)$. What is the answer to Tanvi's question?

(A) $\frac{\sqrt{2 - \sqrt{2}}}{4}$ (B) $\frac{\sqrt{2 - \sqrt{2}}}{2}$ (C) $\frac{\sqrt{2 + \sqrt{2}}}{4}$ (D) $\frac{\sqrt{2 + \sqrt{2}}}{2}$ (E) NOTA
- What is the length of the latus rectum of the conic defined by the equations $x = 4 \sinh \theta + 3$ and $y = 3 \cosh \theta + 5$?

(A) $\frac{32}{3}$ (B) $\frac{9}{2}$ (C) $\frac{18}{5}3$ (D) $\frac{50}{3}$ (E) NOTA
- What is the volume of the tetrahedron defined by the vectors $\langle 1, 4, 6 \rangle$, $\langle 3, 4, 6 \rangle$, $\langle 2, 0, 2 \rangle$?

(A) $\frac{8}{3}$ (B) 8 (C) 16 (D) 96 (E) NOTA
- What is the coefficient of the 3rd term in the binomial expansion of $(3x + 4y)^{1.5}$?

(A) $\frac{2\sqrt{3}}{3}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) $4\sqrt{3}$ (E) NOTA
- A right triangle has integer side lengths, and one side of length 19. Its smallest angle is labelled x and its second-smallest y . If $\sin(2x + y) = \frac{m}{n}$ for m and n relatively prime integers, what is $|m + n|$?

(A) 1 (B) 18 (C) 19 (D) 361 (E) NOTA

11. What is the sum of all values of x that satisfy $\cos^4 x + \sin x = \cos 2x$ in the range $[0, 2\pi]$?
- (A) π (B) $\frac{3\pi}{2}$ (C) $\frac{5\pi}{2}$ (D) $\frac{9\pi}{2}$ (E) NOTA
12. Karthik loves his crazy conic $3x^2 + 2xy + 4y^2 - 8x + 12y = 0$, but to make him sad Rayyan wants to add a number to the left side of the equation such that the conic becomes degenerate! What number should Rayyan add to accomplish this goal and invoke Karthik's fury?
- (A) -5 (B) $\frac{172}{13}$ (C) 20 (D) 134 (E) NOTA
13. Given that x is a cube root of unity and $x \neq 1$, evaluate the limit $\lim_{n \rightarrow \infty} (x^{3n+1} - 1)(2 - x^{3n})(x^{3n+2} + x^{3n+1} + 1)$?
- (A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA
14. Evaluate $e^{\log \pi} - \pi^{\log e}$ to the nearest tenth.
- (A) -0.1 (B) 0 (C) 0.1 (D) 0.2 (E) NOTA
15. Find the difference between the maximum and minimum values of $f(x) = \sin 3x + 4 \sin^3 x + \sqrt{8 + 8 \cos 2x}$.
- (A) 5 (B) 6 (C) 8 (D) 10 (E) NOTA
16. At how many points do at least two of the four graphs $y = \sin^{-1}(x)$, $y = \sin(x^{-1})$, $y = (\sin(x))^{-1}$, and $y = (\sin^{-1}(x^{-1}))^{-1}$ intersect?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA
17. Dylan, Prabhas, and Farzan are designing a DPF encryption system, which uses a public key that is the sum of two consecutive triangular numbers. Which of the following keys could not be a DPF public key?
- (A) 15229352314336644 (B) 61383892279237441 (C) 18255893183048028 (D) 14815496946593476 (E) NOTA
18. Which of the following points in the polar plane lie on the line tangent to the polar graph of $r = 4 \cos \theta$ at the polar point $(2\sqrt{2}, \frac{\pi}{4})$?
- (A) $(2\sqrt{2}, \frac{\pi}{2})$ (B) $(4\sqrt{3}, \frac{2\pi}{3})$ (C) $(-4, \frac{11\pi}{6})$ (D) $(8\sqrt{6}, -\frac{13\pi}{12})$ (E) NOTA
19. Mihir the mean master poses you this mean question: For any two positive numbers, which of the following can be less than the geometric mean of their harmonic mean and arithmetic mean?
- I. Their sum
 II. Their product
 III. Their arithmetic mean
 IV. Their geometric mean
 V. Their harmonic mean
- (A) II, V (B) IV, V (C) III, IV, V (D) I, II, III, IV, V (E) NOTA

20. A space diagonal and face diagonal of a rectangular prism intersect at a common point, with a 30° angle between them. The height of the prism is 6, and it has a square base. Given the face diagonal lies on one of the square bases, what is the volume of the prism?
- (A) 108 (B) 432 (C) 324 (D) 216 (E) NOTA
21. Mihir and Prabhas are drilling through the earth when they realize they are close to the Moho Discontinuity, a plane containing the points $(4, 3, 1)$, $(0, 6, -5)$, and $(-8, 5, 4)$. If the drill is currently at $(7, 2, 5)$, what is the shortest distance from the drill to the Moho Discontinuity?
- (A) $\frac{\sqrt{78}}{6}$ (B) 1 (C) $\sqrt{13}$ (D) $\sqrt{78}$ (E) NOTA
22. A point P is on the graph of $(x^2 + y^2)^2 = \frac{((x + y)^2 - (x^2 + y^2))}{x^2 + y^2}$ such that it maximizes the distance between P and the origin. If P can be represented as (x, y) , what is xy ?
- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) NOTA
23. Angle B of triangle ABC is twice angle A. If $BC = a$ and $AC = b$, which of the following is equal to AB?
- (A) $\sqrt{a^2 + b^2}$ (B) $\frac{\sqrt{a^2 + b^2}}{a}$ (C) $b^2 - a^2$ (D) $\frac{b^2 - a^2}{a}$ (E) NOTA
24. $x + \frac{1}{x} = \frac{\sqrt{6} + \sqrt{2}}{2}$, what is $x^{588} + \frac{1}{x^{588}}$?
- (A) -2 (B) 0 (C) $\frac{\sqrt{3}}{2}$ (D) 2 (E) NOTA
25. Shrung and Shubham are playing a game with a pot of 2^{2020} coins. Each turn, one of them can draw 1 to 9 coins from the pot (inclusive). The player who draws the final coin will win the game. If both Shrung and Shubham play optimally, how many coins should the first player draw to guarantee they win the game?
- (A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA
26. Rohan's distinguished glasses are modeled by the parametric polar equations $r = \cos(t)$ and $\theta = \sin(t)$, where t ranges from -2π to 2π . Rohan's hidden cyclops eye is represented by a circle with radius 0.001 centered at the origin. To the nearest tenth, what is the ratio of the area of the eye to the area of the eye covered by the glasses?
- (A) 1.6 (B) 2.4 (C) 3.1 (D) 6.3 (E) NOTA
27. For three positive real numbers x, y, z , find the greatest possible value of $\frac{12xy + 6yz + 4xz}{4x^2 + 9y^2 + z^2}$.
- (A) 1 (B) $\frac{11}{7}$ (C) 2 (D) 3 (E) NOTA
28. An AP-3 is an increasing arithmetic progression of length 3 - for instance, $(1, 3, 5)$ or $(2, 5, 8)$. Let a_n denote the number of AP-3's in the set of the first n natural numbers. What is the sum of the prime factors of $\sum_{i=1}^{50} a_{2i}$?
- (A) 14 (B) 31 (C) 24 (D) 45 (E) NOTA

29. If $z = \text{cis}\left(\frac{180^\circ}{127}\right)$ then compute $\prod_{m=0}^6 \text{Re}(z^{2^m})$.

(A) $-\frac{1}{128}$

(B) $-\frac{1}{64}$

(C) $\frac{1}{64}$

(D) $\frac{1}{128}$

(E) NOTA

30. Solve the summation $\sum_{x=1^\circ}^{89^\circ} \frac{1}{\sin(x)\sin(x+1^\circ)}$. (Hint: $\sin((x+1^\circ)-x) = \sin(1^\circ)$)

(A) $\sin(1^\circ)\cos(1^\circ)$

(B) $\cot(1^\circ)$

(C) $\tan(1^\circ)\sin(1^\circ)$

(D) $\cot(1^\circ)\csc(1^\circ)$

(E) NOTA