1. The expression $\cos(\frac{7\pi}{6})$ evaluates to $-\frac{\sqrt{3}}{2}$. Since the arcsin function is restricted to the range $[\frac{-\pi}{2}, \frac{\pi}{2}]$, $\arcsin(-\frac{\sqrt{3}}{2})$ evaluates to $\boxed{-\frac{\pi}{3}}$.

2. Using double angle identities, we can simplify $r = 11\sin(2\theta)\cos(2\theta)$ into the rose curve $r = \frac{11}{2}\sin(4\theta)$. The number of petals in a rose curve with the equation $r = a\sin(n\theta)$ is 2n given that n is even. This means that there are 2(4) or $\boxed{8}$ petals on each rose in Mihir's garden.

3. To convert rectangular equations into polar form, we can use various identities, mainly $x = r\cos(\theta)$, $y = r\sin(\theta)$, and $x^2 + y^2 = r^2$. By substituting these values into our equation we get $r^4(\cos^4\theta - \sin^4\theta) = r^2(2\sin\theta\cos\theta)$. The left side simplifies to $r^4(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = r^4\cos 2\theta$. The right side simplifies to $r^2\sin 2\theta$, so we divide to get $\boxed{r^2 = \tan 2\theta}$.

4. We can factor the denominator making our expression $\frac{3-\sqrt{x}}{(3+\sqrt{x})(3-\sqrt{x})}$ which simplifies to $\frac{1}{3+\sqrt{x}}$. Now we can simply substitute in 9 for x and we get $\boxed{\frac{1}{6}}$.

5. The formula for the area of a conic in the form $Ax^2 + Bxy + Cy^2 = 1$ is $\frac{2\pi i}{\sqrt{B^2 - 4AC}}$. This gives us $\frac{2\pi i}{\sqrt{36 - 4(1)(25)}}$ or $\frac{2\pi i}{8i}$ and finally $\boxed{\frac{\pi}{4}}$

6. $\sin(67.5°) = \sin(\frac{135°}{2})$. This is a simple use of a half angle identity; $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$. We now get $\sin(\frac{135°}{2}) = \pm\sqrt{\frac{1-(\frac{-\sqrt{2}}{2})}{2}}$. We can now finally simplify our equation into $\sin(\frac{135°}{2}) = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$.

7. By putting our two parametric equations in terms of $\sinh\theta$ and $\cosh\theta$ we get that $\frac{x-3}{4} = \sinh\theta$ and $\frac{y-5}{3} = \cosh\theta$. Since $\cosh^2\theta - \sinh^2\theta = 1$, we can square both sides of our expressions and add them to get $\frac{(y-5)^2}{9} - \frac{(x-3)^2}{16} = 1$. This gives us the equation of a hyperbola. The length of the latus rectum of a hyperbola in form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ is $\frac{2b^2}{a}$ or $\frac{2(16)}{3}$ or $\boxed{\frac{32}{3}}$.

8. To find the volume of a tetrahedron defined by 3 vectors, we can put the 3 vectors into a matrix and take one-sixth the absolute value of the determinant of said matrix. For example, using the 3 vectors given in the problem we get the matrix $\begin{vmatrix} 1 & 4 & 6 \\ 3 & 4 & 6 \\ 2 & 0 & 2 \end{vmatrix}$. Now all we have to do is find the determinant of this matrix which is -16 and after taking one-sixth of the absolute value we get an area of $\boxed{\frac{8}{3}}$.

9. The third term in the binomial expansion of $(ax + by)^n$ is $\binom{n}{2}(ax)^{n-2}(by)^2$. By substituting values from our equation we get $\binom{3/2}{2}(ax)^{-1/2}(by)^2$. To extend the choose function beyond natural numbers, we can define $\binom{n}{2}$ as $\frac{n!}{2!(n-2)!}$ or $\frac{n(n-1)}{2}$. By substituting 3/2 for n, we get $\binom{3/2}{2} = 3/8$. Now we can get our final answer of $(\frac{3}{8})(\frac{1}{\sqrt{3}})(16) = \boxed{2\sqrt{3}}$.

10. A simple check shows that no right triangle with integer side lengths has a hypotenuse of 19, so one of the legs must have length 19. If the hypotenuse is a and the other leg is b, then $a^2 - b^2 = (a-b)(a+b) = 361$. Either $a - b = 19$

and $a+b=19$, in which case b must be 0 (impossible), or $a-b=1$ and $a+b=361$, in which case $a=181$ and $b=180$. Since $x+y=90^\circ$ as they are angles in a right triangle, $\sin(2x+y)=\sin(x+90^\circ)=-\cos(x)$ for acute x. x is the smallest angle so it must be opposite the leg of length 19, so $-\cos(x)=-\frac{180}{181}$ and $|m+n|=-180+181=\boxed{1}$.

11. Subtracting from both sides we get $\cos^4 x-\cos 2x+\sin x=0$. Then we can simplify the left side to $\cos^4 x-\cos^2 x+\sin^2 x+\sin x=\cos^2 x(\cos^2 x-1)+\sin^2 x+\sin x=\cos^2 x(-\sin^2 x)+\sin^2 x+\sin x=\sin^2 x(1-\cos^2 x)+\sin x=\sin^2 x(\sin^2 x)+\sin x=\sin^4 x+\sin x=(\sin x)(\sin x+1)(\sin^2 x-\sin x+1)$. The third factor has no real solutions, so the only solutions are $\sin x=0$ and $\sin x=-1$, which are satisfied in the range only with $x=0$, $x=\pi$, $x=2\pi$, and $x=\frac{3\pi}{2}$. The sum is $\boxed{\frac{9\pi}{2}}$.

12. For a conic in the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ to be degenerate, $\begin{vmatrix} A & 0.5B & 0.5D \\ 0.5B & C & 0.5E \\ 0.5D & 0.5E & F \end{vmatrix}=0$. Plugging in the numbers from the problem with an unknown value for F, we get $(12F-108)-(F+24)+(-88)=0$, which simplifies to $11F=220$ so $F=\boxed{20}$.

13. We can simplify our equation into $(x^{3n}x^1-1)(2-x^{3n})(x^{3n}x^2+x^{3n}x^1+1)$. Since x is a cube root of unity we know that $x^3=1$ and that $x^{3n}=(x^3)^n=1^n=1$. We now simplify our expression to $(x-1)(2-1)(x^2+x+1)=(x^3-1)=(1-1)=\boxed{0}$

14. Let $e^{\log\pi}=n$. Then, taking the log of both sides and removing the exponent we get $\log n=(\log\pi)(\log e)$. By using log rules, we can put the alternate term as the exponent: $\log n=\log\pi^{\log e}$. Thus, $n=\pi^{\log e}$ so the difference is $n-n=\boxed{0}$.

15. $\sin 3x+4\sin^3 x+\sqrt{8+8\cos 2x}=3\sin x-4\sin^3 x+4\sin^3 x+\sqrt{8+8(1-2\sin^2 x)}=3\sin x+\sqrt{16(1-\sin^2 x)}=3\sin x+4\cos x$. The amplitude of this function is $\sqrt{3^2+4^2}=5$, and the period is the least common multiple of the two individual periods which are both $\frac{2\pi}{1}=2\pi$. Thus the product is $5(2\pi)=\boxed{10\pi}$.

16. The key part of this question is to imagine the graph of $\sin(\frac{1}{x})$. As x nears 0, $\frac{1}{x}$ increases to infinity very quickly, so $\sin(\frac{1}{x})$ varies an infinite amount of times between -1 and 1. Meanwhile, the graph of $\arcsin(x)$ is a continuous curve that passes through the origin. Therefore, the rapidly alternating former graph must intersect the continuous latter graph an infinite amount of times as x nears 0, so none of the answer choices are correct and the answer must be E ($\boxed{\infty}$).

17. All triangular numbers can be expressed in the form $\frac{n(n+1)}{2}$ for a natural number n. If we substitute x and x+1 into our equation to find consecutive triangular numbers we get that the triangular numbers are in the form $\frac{x(x+1)}{2}$ and $\frac{(x+1)(x+2)}{2}$. By adding these two expressions together, we find that the sum of our consecutive triangular numbers are in the form $\frac{(2x+2)(x+1)}{2}$ and finally $(x+1)^2$. This means that the key is a square number. All square numbers can either end with a 1,4,9,6 or 0. Since no square number can have 8 as their last digit, we know that $\boxed{18255893183048028}$ is not the public key.

18. $r=4\cos\theta\to r^2=4r\cos\theta\to x^2+y^2=4x\to x^2-4x+y^2=0\to(x-2)^2+y^2=4$. Thus the graph is a circle centered at $(2,0)$ with a radius of 2. The polar point $(2\sqrt{2},\frac{\pi}{4})$ is equivalent to the rectangular point $(2,2)$, which is the highest point on the circle with a horizontal tangent line. Thus the tangent line is $y=2$ in rectangular form or $r\sin\theta=2\to r=2\csc\theta$ in polar form. Plugging each of the answer choices into this new equation, we find that the only choice which satisfies the equation is $\boxed{(-4,\frac{11\pi}{6})}$ because $2\csc(\frac{11\pi}{6})=2(-2)=-4$.

19. $G(A(a,b),H(a,b))=\sqrt{(\frac{a+b}{2})(\frac{2ab}{a+b})}=\sqrt{ab}=G(a,b)$. By AM-GM inequality the arithmetic mean (and by extension the sum, which is twice the arithmetic mean) can never be less than the geometric mean. Also, the

geometric mean can never be less than itself. So the only options which can be less than the geometric mean are the product (if both numbers are less than 1, for instance) and the harmonic mean (which is always less than the geometric mean except in the equality case). Thus the answer is $\boxed{II, V}$.

20. Let the side length of the square base be s. There are two possibilities - the space diagonal is always the same, but the face diagonal could either be on a square face or a rectangular face. Given the face diagonal is on the square face, then the length of the diagonal is $s\sqrt{2}$ and the opposite side is the height of 6, so $\tan(30°) = \frac{1}{\sqrt{3}} = \frac{6}{s\sqrt{2}} \rightarrow s = 3\sqrt{6} \rightarrow V = 6(3\sqrt{6})^2 = \boxed{324}$.

21. First we must find the equation of the plane, which we can either do with a system of 3 equations or cross-products. The latter method, which is quicker, requires finding 2 of the vectors between the 3 points, such as $< 4-0, 3-6, 1-(-5) > = < 4, -3, 6 >$ and $< -8-0, 5-6, 4-(-5) > = < -8, -1, 9 >$. Using the formula for cross-product, we find that the cross-product of these 2 vectors is $< -21, -84, -28 >$ which simplifies to $< 3, 12, 4 >$. This vector is perpendicular to both of the previous 2 vectors, and is thus perpendicular to the Moho plane, so we can use it to define the plane as $3x + 12y + 4z = d$. Then, we can substitute any of the 3 points to find $d = 52$. Lastly, using point-to-plane formula we find that the distance is $\frac{3(7) + 12(2) + 4(5) - 52}{\sqrt{3^2 + 12^2 + 4^2}} = \frac{13}{13} = \boxed{1}$.

22. Multiply both sides by the denominator to get $(x^2 + y^2)^3 = (x + y)^2 - (x^2 + y^2) = x^2 + y^2 + 2xy - (x^2 + y^2) = 2xy$. Then, convert to polar form to get $(r^2)^3 = r^6 = 2(r\sin\theta)(r\cos\theta) = r^2 \sin 2\theta$ so $r^4 = \sin 2\theta$. The distance from the origin r is of course maximized when $\sin 2\theta = 1$, meaning $\theta = \frac{\pi}{4}$. Then $x = r\cos\theta = \frac{\sqrt{2}}{2} = y$, so $xy = (\frac{\sqrt{2}}{2})^2 = \boxed{\frac{1}{2}}$.

23. Let $AB = c$. Then $\frac{a}{\sin A} = \frac{b}{\sin 2A} = \frac{c}{\sin(180° - A - 2A)} = \frac{c}{\sin(180° - 3A)} = \frac{c}{\sin 3A}$. Then, simplifying the a and b fractions we get $b = a(\frac{\sin 2A}{\sin A}) = a(2\cos A)$. Meanwhile, simplifying the a and c fractions we get $c = a(\frac{\sin 3A}{\sin A}) = a(3 - 4\sin^2 A) = a(-1 + 4 - 4\sin^2 A) = a(-1 + 4\cos^2 A)$. So $ac = a^2(4\cos^2 A) - a^2 = (2a\cos A)^2 - a^2 = b^2 - a^2$, meaning $c = \boxed{\frac{b^2 - a^2}{a}}$. Alternatively, you can also use Law of Cosines or make an educated guess using 30-60-90 triangles.

24. $\frac{\sqrt{6} + \sqrt{2}}{2} = 2\cos\frac{\pi}{12}$. By multiplying our equation by x and rearranging it we get $x^2 - (2\cos\frac{\pi}{12})(x) + 1 = 0$. The discriminant of this quadratic is $4\cos^2\frac{\pi}{12} - 4 = -4\sin^2\frac{\pi}{12}$ so the roots are $\cos\frac{\pi}{12} \pm i\sin\frac{\pi}{12} = (\pm\frac{\pi}{12})$. Regardless of which root is chosen, $x^{588} + x^{-588} = (49\pi) + (-49\pi) = -1 + 0\pi - 1 + 0\pi = \boxed{-2}$.

25. To play the game optimally, you must leave your opponent with 10 coins on your 2nd to last turn to guarantee that you can win on your next turn. This also means that throughout the game you must leave your opponent with a multiple of 10 amount of coins to guarantee a win. So, the first player must take 2^{2020} mod 10 coins in order to win the game. This is simply the last digit of 2^{2020}, which can be easily calculated by noting that the last digits of powers of two cycle in a pattern of 2,4,8,6. Since 2020 is divisible by 4, we conclude that the last digit and our final answer is $\boxed{6}$.

26. $r^2 + \theta^2 = 1$, so we can quickly see that neither r nor θ can be greater than 1, and as each nears 0 the other nears 1. So as the radius of a bounding circle nears 0 (as can be seen with a radius of 0.001), the curve approaches the line $\theta = 1$. For such a small radius we can simply treat the curve as 2 crossing lines $\theta = 1$ and $\theta = -1$. Between the 2 lines on the positive side is a subtended angle of 2 radians, and the same angle is on the negative side, so overall 4 radians of the circle are covered by the glasses. Thus the ratio of areas is $\frac{1}{(\frac{4}{2\pi})} = \frac{\pi}{2} \approx \boxed{1.6}$.

27. Let $a = 2x$, $b = 3y$, and $c = z$. Then the fraction becomes $\frac{2ab + 2bc + 2ac}{a^2 + b^2 + c^2} = \frac{(a + b + c)^2 - (a^2 + b^2 + c^2)}{a^2 + b^2 + c^2} = \frac{(a + b + c)^2}{a^2 + b^2 + c^2} - 1$. By Cauchy-Schwarz Inequality, $(a + b + c)^2 \leq (1^2 + 1^2 + 1^2)(a^2 + b^2 + c^2)$ so $\frac{(a + b + c)^2}{a^2 + b^2 + c^2} \leq 3$. Therefore $\frac{2ab + 2bc + 2ac}{a^2 + b^2 + c^2} + 1 \leq 3$ so the maximum value is $\boxed{2}$.

28. To figure this out we can check the number of AP's that end with each natural number. There are 0 that end with 1 or 2, because 3 numbers are necessary to form an AP. There is 1 that ends with 3 ([1, 2, 3]) and 1 that ends with 4 ([2, 3, 4]). Then there are 2 that end with 5 ([3, 4, 5], [1, 3, 5]) and 2 that end with 6 ([4, 5, 6], [2, 4, 6]). There are 3 that end with 7 and 8, 4 that end with 9 and 10, and so on, because each odd number adds a new group of AP's with a larger common difference. So the total number of AP's less than 2 is $2(0)$, less than 4 is $2(1+0)$, less than 6 is $2(2+1+0)$, and so on (no need to worry about odd numbers because the summation only asks for evens). Thus for even n, $a_n = 2(\frac{n}{2}-1)(\frac{n}{2})(\frac{1}{2}) = \frac{n^2}{4} - \frac{n}{2}$. Then $\sum_{i=1}^{50} a_{2i} = \sum_{i=1}^{50} i^2 - i = \sum_{i=1}^{50} i^2 - \sum_{i=1}^{50} i = \frac{(50)(51)(101)}{6} - \frac{(50)(51)}{2} = \frac{(50)(51)(98)}{6} = (50)(17)(49) = (2)(5^2)(7^2)(17)$, so the sum of the prime factors is $2+5+7+17 = \boxed{31}$.

29. $\text{Re}(z^{2^m}) = \text{Re}((2^m(\frac{180}{127})) = \cos(2^m(\frac{180}{127})$. Let $n = \frac{180}{127}^\circ$. Then we have $\cos(n)\cos(2n)\cos(4n)...\cos(64n)$. Multiply by $\sin(n)$ to get $\sin(n)\cos(n)\cos(2n)... = \frac{1}{2}\sin(2n)\cos(2n)\cos(4n)... = \frac{1}{4}\sin(4n)\cos(4n)\cos(8n)...$ and so on, eventually resulting in $\frac{1}{128}\sin(128n)$. Remember to divide back by the $\sin(n)$ we multiplied in to get $\frac{\sin(128n)}{128\sin(n)}$. $\sin(128\cdot\frac{180}{127}) = \sin(180\cdot\frac{128}{127}) = \sin(180(1+\frac{1}{127})) = \sin(180+\frac{180}{127}) = -\sin(\frac{180}{127})$. So we have $\frac{-\sin(\frac{180}{127}^\circ)}{128\sin(\frac{180}{127}^\circ)} = \boxed{-\frac{1}{128}}$.

30. By using angle difference identities on our hint equation we get $\sin(x+1^\circ)\cos(x) - \sin(x)\cos(x+1^\circ) = \sin(1^\circ)$. Now we divide by $\sin(x)\sin(x+1^\circ)$ to get $\cot(x) - \cot(x+1^\circ) = \frac{\sin(1^\circ)}{\sin(x)\sin(x+1^\circ)}$. Our summation is now equivalent to $\frac{1}{\sin(1^\circ)}\sum_{x=1^\circ}^{89^\circ} \cot(x) - \cot(x+1^\circ)$. This simplifies to $(\frac{1}{\sin(1^\circ)})(\cot(1^\circ) - \cot(2^\circ) + \cot(2^\circ) - \cot(3^\circ) + \cot(3^\circ) - \cot(4^\circ) + ...\cot(87^\circ) - \cot(88^\circ) + \cot(88^\circ) - \cot(89^\circ) + \cot(89^\circ) - \cot(90^\circ))$ and finally $\boxed{\cot(1^\circ)\csc(1^\circ)}$.