For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Given a certain statement is true, which of the following must be true?
   (A) Its converse
   (B) Its inverse
   (C) Its contrapositive
   (D) Depends on the statement
   (E) NOTA

2. The vertices of a 2020-gon are labeled in order as A_1, A_2, ..., A_{2020}. The diagonals of that polygon are drawn. Justin erases all of the diagonals that contain A_{19} or A_{2019}. How many diagonals are left?
   (A) 2035153 (B) 2035155 (C) 2033134 (D) 2033136 (E) NOTA

3. What is the area of the triangle bounded by the lines: $y = x - 4$, $y = 2x - 1$, $y = 3x - 2$?
   (A) 3 (B) 4 (C) 6 (D) 8 (E) NOTA

4. Akhil's favorite formula is the Brahmagupta formula. What is the area of a cyclic quadrilateral whose side lengths are 4,5,7, and 10?
   (A) 72 (B) 36 (C) 78 (D) 1296 (E) NOTA

5. Triangle ABC is an isosceles triangle. If the length of AB is 20 units and angle C is equal to 120 degrees. What is the sum of the numerical values of the area and perimeter of the triangle?
   (A) 180 (B) $\frac{40\sqrt{3}}{3} + 20$ (C) $\frac{140\sqrt{3}}{3} + 20$ (D) Not possible (E) NOTA

6. Sruthi decides to inscribe a circle into the triangle from the previous question. What is the area of the inscribed circle?
   (A) $\frac{10 - 5\sqrt{3}}{4}$ (B) $(700 - 400\sqrt{3})\pi$ (C) $\frac{100\pi}{3}$ (D) $\frac{(175 - 100\sqrt{3})\pi}{16}$ (E) NOTA

7. Which of the following is a valid classification of a triangle whose side lengths have a ratio of 7:8:14?
   (A) Acute (B) Right (C) Obtuse (D) Need more info (E) NOTA

8. Fun fact: "RICE" is a permutation of "ERIC". Eric's rice bowl is a perfect hemisphere large enough to contain exactly 18π cubic inches of rice. The bowl itself is 1 inch thick. What is the surface area of the rice bowl when it has nothing in it?
   (A) 18π (B) 36π (C) 50π (D) 57π (E) NOTA

9. Two spheres have a radius of 10 and 20 units, respectively. If they are concentric, what is the volume within the larger sphere but outside of the smaller sphere?
   (A) $\frac{28000\pi}{3}$ (B) $\frac{20000\pi}{3}$ (C) $\frac{32000\pi}{3}$ (D) $\frac{25000\pi}{3}$ (E) NOTA

10. Akash knows that the value of $\sin\theta$ equals 0.1 and that θ is acute. What is the value of $\cot\theta$?
    (A) $3\sqrt{11}$ (B) $\frac{3\sqrt{11}}{10}$ (C) $\frac{\sqrt{11}}{33}$ (D) 10 (E) NOTA

11. An isometric drawing on a 6-by-8 paper of a rectangular prism has vertices at (2,6), (3,7), (4,6), (3,5), (2,3), (3,2), and (4,3). Knowing that the isometric drawing's dimensions are numerically equal to the dimensions of the real prism, what would the surface area of the rectangular prism be if it was brought into three-dimensional space?
    (A) $6\sqrt{2} + 2$ (B) $12\sqrt{2} + 8$ (C) $12\sqrt{2} + 4$ (D) 12 (E) NOTA

12. Tanmay's 2nd meme machine (if you are curious, his first meme machine was a pentagon that he later transformed into a hexagon) is a regular octagon $ABCDEFGH$ with a side length of 12 units. What is the area of $ACEF$?
(A) $144\sqrt{2}+216$ (B) $144\sqrt{2}+288$ (C) $216\sqrt{2}+144$ (D) $216\sqrt{2}+288$ (E) NOTA

13. Dylan drew a regular hexagon with a side length of 10 units. What is the numerical ratio between the area of that hexagon and its perimeter?
(A) $5\sqrt{3}$ (B) 60 (C) $\frac{5\sqrt{3}}{2}$ (D) $\frac{15\sqrt{3}}{2}$ (E) NOTA

14. Line segments AB and DE are parallel. Line segments AE and BD intersect at point C. The length of AB is 20 units, the length of BC is 12 units, and the length of DE is 15 units. The midpoint of DC is H, the midpoint of CE is F, and the midpoint of DE is G. What is the perimeter of triangle FGH if HG is 4 units long?
(A) $\frac{47}{4}$ (B) 16 (C) 32 (D) 47 (E) NOTA

15. What are the coordinates of (20,20) after it is reflected across $y = x - 1$?
(A) (20, 20) (B) (19, 21) (C) (19, 19) (D) (21, 19) (E) NOTA

16. Shubham has to carry a box full of rigid bottles and put it in a car. Every bottle is a cylinder with a radius of 1 unit and a height of 4 units. The box is a rectangular prism whose base is 3 units in length, 4 units in width. Given that the height of the box is 20 units, how many bottles can he fit into this box such that the top of the box can be placed without interference?
(A) 8 (B) 9 (C) 10 (D) 20 (E) NOTA

17. A circle with a radius of x units exists such that the length of an arc is also x units. What is the value of the angle that the arc intercepts, in radians?
(A) 180 (B) $\frac{1}{\pi}$ (C) 1 (D) $\frac{x}{\pi}$ (E) NOTA

18. Segment AB has endpoints at (5,6) and (7,8). What are the coordinates of the segment's midpoint after it is rotated 90 degrees counterclockwise around the origin?
(A) (6, 7) (B) (-6, 7) (C) (-7, 6) (D) (-7, -6) (E) NOTA

19. Sierpinski draws a triangle S_1. He connects the midpoints of S_1 to form a second triangle S_2. Then he connects the midpoints of S_2 to form a third triangle S_3. He repeats this process infinitely many times. If the sum of the areas of all the numbered triangles he draws (S_1, S_2, S_3, ...) is 4, and the sum of the perimeters of these same triangles is 2, what is the inradius of S_1?
(A) $\frac{3}{2}$ (B) 2 (C) 3 (D) 6 (E) NOTA

20. Nitish decided to punish his pet circle Rico by putting it in his barn, regular hexagon $ABCDEF$ with side length of 8 meters. He ties it to a rope of length s meters long, where s is some positive integer, and ties the other end of the rope to the midpoint of AB. Rico's area is negligible, and it is restricted by the barn's boundary and the rope. It is known that Rico can reach a point on all of the sides of the hexagon, but he cannot reach all of the vertices of the hexagon. What fraction of the points on side DE can be reached by Rico? Round your answer to the nearest tenth.
(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.5 (E) NOTA

Please use the formula $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ and $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$) for questions 21 – 24.

21. Karthik circumscribes a regular pentadecagon (15 sides) about a circle of radius 19. Using the approximation of $\cos 24^\circ \approx 0.9$, what is the side length of the pentadecagon?
(A) $\sqrt{19}$ (B) $2\sqrt{19}$ (C) $\sqrt{38}$ (D) $2\sqrt{38}$ (E) NOTA

22. Using the approximation from the previous question, what is the area of a regular triacontagon (30 sides) with a side length of 2 units?
(A) $30\sqrt{5}+15\sqrt{19}$ (B) $15\sqrt{5}+30\sqrt{19}$ (C) $60\sqrt{5}+30\sqrt{19}$ (D) $30\sqrt{5}+60\sqrt{19}$ (E) NOTA

23. What is the area of a regular icositetragon (24 sides) with the circumradius being 2 units long?
(A) $12(\sqrt{6}-\sqrt{2})$ (B) $12\sqrt{6}$ (C) $12(\sqrt{3}+\sqrt{6})$ (D) $12(\sqrt{6}+\sqrt{2})$ (E) NOTA

24. Two circles with radius of $2\sqrt{3}-2$ and $2\sqrt{3}+2$ intersect each other. If the distance between the centers of the circles is $4\sqrt{2}$, what is the area of the intersection? (Hint: $(2\sqrt{3}-2)^2+(2\sqrt{3}+2)^2=(4\sqrt{2})^2$)
(A) $\frac{12\pi-4\sqrt{3}\pi-12}{3}$ (B) $\frac{12\pi+4\sqrt{3}\pi+12}{3}$ (C) $\frac{24\pi-8\sqrt{3}\pi-24}{3}$ (D) $\frac{24\pi+8\sqrt{3}\pi-24}{3}$ (E) NOTA

The inequality $\frac{a_1+a_2+\ldots+a_n}{2} > a_j$ (n is the number of sides in a polygon and a_j is the length of any side) is true for all non-degenerate polygons. Use this fact for questions 25-26.

25. Consider the "J-List": the set of natural numbers between 10 and 99, inclusive. Jenna designs a function $J_1(x)$ such that for any number x in the J-List, $J_1(x)$ returns the number of possible pairs of numbers a and b such that a and b are also in the J-List; $x < a < b$; and x, a, and b are the side lengths of a non-degenerate triangle. To make this question easier to solve, she kindly designs a second function $J_2(x) = 2J_1(x) - 21x$. Find the maximum possible value of $J_2(x)$ across all numbers x in the J-list.
(A) 756 (B) 2400 (C) 2502 (D) 2876 (E) NOTA

26. Harshil has a basket of sticks, each with a distinct integer length greater than 9 but less than 100. He chooses H sticks from that basket and tries to fit them together to make a non-degenerate polygon, with one stick as each side, but the task turns out to be impossible! What is the last digit of the sum of all possible values of H?
(A) 1 (B) 5 (C) 6 (D) 8 (E) NOTA

27. What is the area of a regular polygon with a sides and a side length of b units?
(A) $\frac{a*b}{4*\sin\frac{180}{a}}$ (B) $\frac{a^2*b^2}{2*\tan(180-a)}$ (C) $\frac{a*b^2*\tan\frac{180}{a}}{4}$ (D) $\frac{a*b^2}{4*\tan\frac{180}{a}}$ (E) NOTA

28. Danny made two circles with a radius of 6 units intersect at each other's center. He then perfectly inscribed a square between the intersection of the two circles. What is the area of the square?
(A) $72+18\sqrt{7}$ (B) $\frac{72}{\pi}$ (C) $\frac{72}{\pi}+2\sqrt{7}$ (D) $72-18\sqrt{7}$ (E) NOTA

29. Rohan is making a Venn diagram to compare his 3 favorite clubs: Math Club, Programming Club, and Language Club. The Venn diagram consists of 3 circles, each with a radius of 36 inches, such that the center of each circle is on the circumference of the other two. If each circle represents one club, what is the area of the entire region in which Rohan could place a characteristic that belongs to more than one club?
(A) $1296\pi+648\sqrt{3}$ (B) $1296\pi-648\sqrt{3}$ (C) $648\pi-648\sqrt{3}$ (D) $648\pi+648\sqrt{3}$ (E) NOTA

30. After writing this test, Shrung wanted to go outside and walk. He starts off from point A and goes to point B by walking 1 mile north. He then turns 120° clockwise and travels 2 miles to go to point C. He makes a 120° turn clockwise and travels 4 miles to go to point D. He makes a 60° turn clockwise and travels an unknown distance to go to point E. Finally, he turns clockwise another 150° and travels an unknown distance, returning to point A. What is the area enclosed by Shrung's route?

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{9\sqrt{3}}{2}$ (C) $\frac{15\sqrt{3}}{2}$ (D) Shape cannot exist (E) NOTA