1. The $\boxed{\text{contrapositive}}$ of a true statement is always true.

2. The number of diagonals in a 2020-gon is equal to $\frac{(2020)(2020-3)}{2}$ or $1010(2017) = 2037170$. At a given vertex there are 2017 possible diagonals, as a diagonal cannot connect a vertex to itself or either of its 2 neighbors. Then there are $2(2017) = 4034$ diagonals that should be erased - 2017 for both A_{19} and A_{2019}. However, this method double-counts the diagonal from A_{19} to A_{2019}, so to correct the double-count we should subtract 1 from the erased diagonals. In other words, 4033 diagonals are erased - 1 from A_{19} to A_{2019}, 2016 from A_{19} to other vertices, and 2016 from A_{2019} to other vertices. Thus the answer is $2037170 - 4033 = \boxed{2033137}$.

3. The intersection of the first two lines is at $x-4 = 2x-1$ so $x = -3$ and the point is $(-3,-7)$. The intersection of the first and third is at $x-4 = 3x-2$, so $2x = -2$ and $x = -1$, meaning the point is $(-1,-5)$. The intersection of the final two lines is at $2x-1 = 3x-2$, so $x = 1$ and the point is $(1,1)$. Using the shoelace formula on the four points $(-1,-5)$, $(1,1)$, $(-3,-7)$, and $(-1,-5)$, we get the area of the triangle as $\frac{|((-1)(1)+(1)(-7)+(-3)(-5))-((1)(-5)+(-3)(1)+(-1)(-7))|}{2} = \frac{|7-(-1)|}{2} = \boxed{4}$.

4. Brahmagupta's formula ($\sqrt{(s\text{–}a)(s\text{–}b)(s\text{–}c)(s\text{–}d)}$) is used to find the area of a cyclic quadrilateral whose side lengths are a,b,c, and d (4,5,7,10 for this problem). The semiperimeter or s is equal to $\frac{4+5+7+10}{2}$ or 13. Plugging these numbers in the formula gives us $\sqrt{9*8*6*3}$ or $\sqrt{1296}$ which simplifies to $\boxed{36}$.

5. The remaining angles are equal to 30 degrees because this is an isosceles triangle. We can split up the triangle into two 30-60-90 triangles such that the side facing the 60-degree angle would have a length of 10 units. Using the 1-$\sqrt{3}$-2 ratio, the altitude of the original triangle would be $\frac{10}{\sqrt{3}}$ or $\frac{10\sqrt{3}}{3}$ units and both of the legs would be equal to $\frac{20\sqrt{3}}{3}$ units long. This means that the perimeter is equal to $\frac{40\sqrt{3}}{3} + 20$ units and the area would be equal to $\frac{100\sqrt{3}}{3}$ units^2. Adding the two values up gives us our desired value of $\boxed{\frac{140\sqrt{3}}{3} + 20}$.
Another way to solve this problem would be to use the Law of Sines.

6. One common formula in geometry is area = semiperimeter*inradius or a=s*i. The semiperimeter is equal to $\frac{\frac{40\sqrt{3}}{3}+20}{2}$ or $\frac{20\sqrt{3}}{3} + 10$ units and the area is equal to $\frac{100\sqrt{3}}{3}$ units^2. This means that the value of the inradius should be $\frac{\frac{100}{\sqrt{3}}}{\frac{20}{\sqrt{3}}+10}$ or $20 - 10\sqrt{3}$ units. Squaring that ($(a-b)^2 = a^2 - 2ab + b^2$) and multiplying it by π to find the area of the incircle gives us $\boxed{(700-400\sqrt{3})\pi}$ units^2.

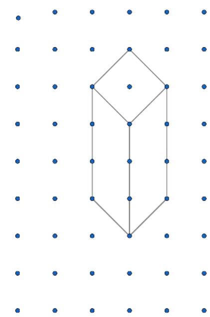
7. We can see that $7^2 + 8^2 < 14^2$ so this triangle must be $\boxed{\text{obtuse}}$.

8. Since the bowl has thickness, it can actually be thought of as two concentric hemispheres. The inner hemisphere is the one that actually contains the rice, so it must have a volume of 18π. The volume of this hemisphere is also $\frac{1}{2}(\frac{4}{3}^3) = \frac{2}{3}^3$, so by setting the two volumes equal to each other we see that $r^3 = 27$ and thus $r = 3$ inches. The surface area of the inner hemisphere is thus $\frac{1}{2}(4^2) = 2^2 = 18\pi$. The outer hemisphere, meanwhile, has a radius of $3+1 = 4$ inches. Its surface area is then $2\pi(4)^2 = 32\pi$. Finally, we must consider the top surface of the bowl, an annulus formed by two concentric circles of radii 3 and 4. The area of this annulus is $\pi(4^2 - 3^2) = 7\pi$, so the total surface area is $18\pi + 32\pi + 7\pi = \boxed{57\pi}$.

9. The volume of the bigger sphere is $\frac{4(20^3)\pi}{3}$ and the volume of the smaller sphere is $\frac{4(10^3)\pi}{3}$. The difference would be $\frac{4(8000)\pi}{3} - \frac{4(1000)\pi}{3} = \boxed{\frac{28000\pi}{3}}$.

10. The hypotenuse of a right triangle is 10 units and one of the legs is 1 unit long (this means that the sin value of one of the angles is equal to 0.1). The other leg of this triangle is equal to $3\sqrt{11}$ units long. Recall that cot or cotangent is equal to $\frac{1}{\tan}$ or $\frac{adjacent}{opposite}$, so $\cot\theta = \boxed{3\sqrt{11}}$.

11. The isometric drawing contains a rectangular prism with a length of $\sqrt{2}$ units, a width of $\sqrt{2}$ units, and a height of 3 units.



The surface area of this rectangular prism would be $2((\sqrt{2}*\sqrt{2})+(\sqrt{2}*3)+(3*\sqrt{2}))$ or $\boxed{12\sqrt{2}+4}$.

12. We can extend AF and BC to meet at I, CD and EF meet at J. Both $\triangle BIC$ and $\triangle DJE$ are 45-45-90 triangles so the length of BI, IC, DJ, and EJ are $6\sqrt{2}$. The area of rectangle $AFJI$ would be $(6\sqrt{2}+12+6\sqrt{2})(12+6\sqrt{2}) = (12)(\sqrt{2}+1)(6)(2+\sqrt{2}) = 72(3\sqrt{2}+4) = 216\sqrt{2}+288$. Since $\triangle AIC$ and $\triangle EJC$ aren't a part of the octagon we have to remove their areas $2*\frac{(12+6\sqrt{2})(6\sqrt{2})}{2} = 72\sqrt{2}+72$. $216\sqrt{2}+288-72(\sqrt{2}+1) = \boxed{144\sqrt{2}+216}$.

13. Using various methods (ex. the formula, splitting it up into 6 equilateral triangles, etc.), we find that the area of the hexagon is equal to $150\sqrt{3}$ units^2. The perimeter of the hexagon is equal to 60 units, so our answer would be $\boxed{\frac{5\sqrt{3}}{2}}$ units.

14. Drawing a figure reveals that the problem is asking for the perimeter of a triangle ($\triangle FGH$) whose vertices are the midpoints of a triangle ($\triangle CDE$). The latter shares a vertex with another triangle ($\triangle ABC$) whose base is parallel to the base of the former triangle. $\angle ABC \cong \angle CDE, \angle BAC \cong \angle CED, \angle ACB \cong \angle DCE$. This means that $\frac{AC}{CE} = \frac{12}{9} = \frac{20}{DE}$. Cross-multiplying gives us that $DE = 15$. $\angle HDG \cong \angle CDE$ We know that all of the side lengths of $\triangle FGH$ are half of the side lengths of DCE so $CE = 4*2 = 8$. The perimeter of $\triangle DCE$ is $15+9+8$ so the perimeter for $\triangle FGH$ must be half of that or $\frac{15+9+8}{2}$ or $\boxed{16}$.

15. The distance between (20,20) and (20,19) is 1. The distance between (20,20) and (21,20) is also equal to 1. So, we must add one to the x-coordinate and subtract 1 from the y-coordinate to get an answer of $\boxed{(21,19)}$.

16. This problem is more difficult than it appears because the bottles can be stacked efficiently either parallel or perpendicular to the box's base. Instead of considering cases in which the bottles are rotated, we can consider cases in which the box is rotated (and the bottles always remain with circular faces upon the ground) because these are equivalent. Since the bottles are each 4 units tall, we must consider 2 possible scenarios - one where the 3-by-4 face is on the ground, and one where the 3-by-20 face is on the ground. We do not need to consider the scenario in which the 4-by-20 face is on the ground, because this would leave the height of the box as 3 units but the bottles would be too tall to fit in this case.

    If the 3-by-4 face is on the ground, then the box is 20 units tall so exactly 5 levels of bottles can be stacked vertically. Thus the maximum number of bottles depends entirely on how many circles can be fit in a 3-by-4 rectangle. The answer seems to be 2 because the total diameter of two circles lengthwise would be $4 \leq 4$, and the total diameter widthwise would be $2 \leq 3$. Still, we must consider placing a third circle in the space between the two circles. For them to be packed as efficiently as possible, each of the three circles would be tangent to the other two. If this was the case, then the three circles would form an equilateral triangle of side length 2 with their radii. The height of the triangle would be $\sqrt{3}$, and there would be 2 additional radii outside the triangle, resulting in a total width of

$2+\sqrt{3}$ while maintaining the original length of 4. Since $2+\sqrt{3} > 3$, this most efficient arrangement still would not fit into a 4-by-3 rectangle, so at most 2 bottles can fit in each level. The maximum number of bottles for this case is $(2)(5) = 10$.

If the 3-by-20 face is on the ground, then the box is 4 units tall so only 1 level of bottles can be placed horizontally. Thus the maximum number of bottles depends entirely on how many circles can fit in a 3-by-20 rectangle. Naively considering the case in which the circle's centers all lie in a straight line, we know that 10 circles will fit because each contributes a diameter of 2 for a total length of 20. No circle can be placed adjacent to these because the width of the rectangle is 3 and the width of the circle arrangement is 2. But, if we use our triangular packing strategy from before, we can close up a little more space. To figure out how much, we can consider the packing function $P(x)$, which returns the minimum n such that x circles of radius 1 can fit in a 3-by-n rectangle. Obviously $P(1) = 2$, because the circle's diameter will be the length. Then, adding another circle and fitting it to be tangent to the opposite side of the rectangle as the first circle creates a right triangle with their radii. The hypotenuse is $2r = 2$ and the base is $r = 1$, so the triangle contributes $\sqrt{3}$ to the rectangle's minimum length. The length necessary for two circles is thus the first circle's radius of 1, plus the "connection length" of $\sqrt{3}$, plus the second circle's radius of 1, yielding $P(2) = 2+\sqrt{3}$. This pattern continues as more circles are added, meaning $P(x) = 2+(x-1)\sqrt{3}$. The original question (how many circles can fit in a 3-by-20 rectangle) can be rephrased as "for what number of circles is the minimum length of the bounding rectangle closest to 20". In other words, for what value of x does $P(x) = 20$. Setting $20 = 2+(x-1)\sqrt{3}$, we find that $x = \frac{18}{\sqrt{3}}+1 \approx \frac{18}{1.7}+1 \approx 10.1+1 \approx 11.1$. The exact value does not matter, because we must have a whole number of circles - the floor function of $P^{-1}(20)$, which must be 11. This proves that 11 circles can fit in a 3-by-20 rectangle, so 11 bottles can fit into the box in this case.

Comparing the two cases, we see that $11 > 10$, so the maximum number of bottles that can fit into the box is at least 11, but it is clearly at most 19 due to volume reasons, as the volumes of the bottles cannot exceed the volume of the box. This means that the answer must be $\boxed{E}$ (the answer is most likely 11, however there is no rigorous proof of this known to the problem authors). When solving difficult math problems like this, it's best to think *outside the box!*

17. Let y be the value of the angle in degrees. This tells us that $x = 2\pi * x * \frac{y}{360}$. Solving that gives us the value of y as $\frac{180}{\pi}$ degrees. There are 360 degrees or 2π radians in a circle; using proportions, the value of y would be equal to $\boxed{1}$ radian (fun fact: this is the definition of a radian).

18. The rule for this is that (x,y) becomes (-y,x) ((6,7) becomes $\boxed{(-7,6)}$) after being rotated 90 degrees counterclockwise around the origin. This isn't something you need to memorize but you should be able to see a relationship by experimenting (choose coordinates where the difference between the x-values and y-values is very high so you can visualize it better).

19. Let P_n denote the perimeter of S_n, and A_n denote the area of S_n. A triangle formed by connecting the midpoints of a larger triangle will always be similar to the larger triangle. Specifically, each side length will be half the length of the corresponding side of the larger triangle, meaning $P_{n+1} = \frac{1}{2}P_n$. Additionally, since the scale factor is one-half, $A_{n+1} = (\frac{1}{2})^2 A_n = \frac{1}{4}A_n$. The sum of all the perimeters is a geometric series with first term P_1 and common ratio $\frac{1}{2}$. The sum of the series is $\frac{a}{1-r} = \frac{P_1}{1-1/2} = \frac{P_1}{1/2} = 2P_1 = 2$, meaning $P_1 = 1$. Similarly, the sum of the areas is $\frac{a}{1-r} = \frac{A_1}{1-1/4} = \frac{A_1}{3/4} = \frac{4}{3}A_1 = 4$, meaning $A_1 = 3$. The inradius of S_1 is the area over the semiperimeter: $\frac{A_1}{P_1/2} = \frac{3}{1/2} = \boxed{6}$.

20. Let M be the midpoint of side AB. Since all of the vertices cannot be reached, we have $s < MD = \sqrt{(8\sqrt{3})^2+4^2} = \sqrt{208}$. Since all of the sides can be reached, $s \geq d(M, DE) = 8\sqrt{3}$. The only integer in this range is clearly $s = 14$. Note that DE is opposite to side AB. This means that the points that can be reached on side DE is a chord of the circle with center M and radius $s = 14$. Note that the distance from this chord to the center of the circle is $8\sqrt{3}$,

so the length of this chord is $2\sqrt{14^2-(8\sqrt{3})^2}=4$. Our answer is $\frac{4}{8}=\boxed{0.5}$.

21. The central angle of the polygon is $\dfrac{360^\circ}{15}=24^\circ$. Line segment AB is a side from the 15-gon. The incircle would intersect AB at its midpoint or point C. The triangle formed when connecting A, C, and the center of the inradius (or point D) is a right triangle. $\angle ADC$ is half the central angle, or 12 degrees. $\tan 12^\circ=\dfrac{\sin 12^\circ}{\cos 12^\circ}$. $\sin 12^\circ=\sqrt{\dfrac{1-0.9}{2}}$ and $\cos 12^\circ=\sqrt{\dfrac{1+0.9}{2}}$. This means that $\tan 12^\circ=\sqrt{\dfrac{1-0.9}{1+0.9}}=\sqrt{\dfrac{0.1}{1.9}}=\sqrt{\dfrac{1}{19}}=\dfrac{1}{\sqrt{19}}=\dfrac{\sqrt{19}}{19}$. $\tan\angle ADC=\dfrac{AC}{CD}$ so $AC=(CD)\tan(\angle ADC)=19(\dfrac{\sqrt{19}}{19})=\sqrt{19}$. Finally, $AB=2AC=\boxed{2\sqrt{19}}$.

22. The central angle of this polygon is $\dfrac{360^\circ}{30}=12^\circ$, so we can use a similar strategy to the previous question. AB is a side of the polygon, C is the midpoint of AB, and D is the center of the polygon. ACD is once again a right triangle because DC will always be perpendicular to the side. $AC=\dfrac{AB}{2}=\dfrac{2}{2}=1$. $\tan\angle ADC=\dfrac{AC}{CD}=\dfrac{1}{CD}$, so $CD=\cot\angle ADC=\cot\dfrac{12^\circ}{2}=\cot 6^\circ$. From last problem's solution we know that $\cos 12^\circ=\sqrt{\dfrac{1.9}{2}}=\dfrac{\sqrt{3.8}}{2}$. By the half-angle formulas, $\cot 6^\circ=\dfrac{\cos 6^\circ}{\sin 6^\circ}=\sqrt{\dfrac{2+\sqrt{3.8}}{2-\sqrt{3.8}}}$. Multiplying the numerator and denominator by $2+\sqrt{3.8}$ we get $\dfrac{2+\sqrt{3.8}}{\sqrt{0.2}}=\sqrt{5}(2+\sqrt{3.8})=2\sqrt{5}+\sqrt{19}$, the length of CD. Then the area of the right triangle is $\sqrt{5}+0.5\sqrt{19}$. There are two of these triangles for each of the polygon's sides, and 30 sides, so the total area is $60(\sqrt{5}+0.5\sqrt{19})=\boxed{60\sqrt{5}+30\sqrt{19}}$.

23. A 24-gon can be split into 24 isosceles triangles with the angle measures being, $\dfrac{165}{2}$, $\dfrac{165}{2}$, and 15. The length of the legs for this problem would be 2 units. We can use the formula $\dfrac{1}{2}(a)(b)(\sin c)$ or $\dfrac{1}{2}(2)(2)(\sin 15)$ to find the area of one of these triangles. Using the half-angle formula for sin gives us $\sin 15=\dfrac{\sqrt{6}-\sqrt{2}}{4}$. $24*\dfrac{1}{2}(2)(2)(\dfrac{\sqrt{6}-\sqrt{2}}{4})=\boxed{12(\sqrt{6}-\sqrt{2})}$.

24. Point A is the center of the circle whose radius is $2\sqrt{3}-2$ and point B is the center of the other circle. The hint tells us that $\triangle ABC$ is a right triangle (point C is one of the two intersections between circles A and B). If we knew the angle measures of this triangle, we can find the area of the two sectors (note: the two sectors aren't congruent) and subtract the area of the triangle to find the area of the intersection. To find the value of $\angle CBA$ we can work back from the half-angle formula for sin. $\dfrac{2\sqrt{3}-2}{4\sqrt{2}}=\sqrt{\dfrac{1-\cos\theta}{2}}$. Solving this equation gives us that $\cos\theta=\dfrac{\sqrt{3}}{2}$, which is also the value of $\cos 30$, so the measure of angle CBA is $\dfrac{30}{2}$ or 15 degrees. This means that $\triangle ABC$ is a 15-75-90 triangle. The sum of the area of the sectors is $((2\sqrt{3}-2)^2*(\dfrac{75}{360})*\pi)+((2\sqrt{3}+2)^2*(\dfrac{15}{360})*\pi)=\dfrac{15\pi}{360}(5(16-8\sqrt{3})+(16+8\sqrt{3})=\dfrac{\pi}{24}(96-32\sqrt{3})=4\pi-\dfrac{4\pi\sqrt{3}}{3}$ or $\dfrac{12\pi-4\pi\sqrt{3}}{3}$. The area of the triangle is $\dfrac{(2\sqrt{3}-2)(2\sqrt{3}+2)}{2}$ or 4. The difference between the total area of the sectors and the triangle is $\dfrac{12\pi-4\sqrt{3}\pi-12}{3}$, the area of intersection of one half of the intersection (there are two triangles that cover the intersection). Then the answer must be double that value, or $\boxed{\dfrac{24\pi-8\sqrt{3}\pi-24}{3}}$.

25. Let us consider a function $f(x,y)$ that returns the number of possible values of z such that $x<y<z$; x, y, and z are in the J-list; and the three numbers can be the side lengths of a non-degenerate triangle. By triangle inequality, $x+y>z$, meaning $y<z<x+y$. If we consider $f(x,y)$ with a constant x as y increases from

$x+1$ to 99, we see that the function can be split into two distinct segments. When $y \le 100-x$, $x+y \le 100$, so we don't have to worry about the bounds of the J-list because z will always be within it. This means that $f(x,y) = (x+y-1)-(y+1)+1 = x-1$ when $y \le 100-x$. But when $y > 100-x$, $x+y > 100$, so some viable values of z will be left out because they are greater than 99. In this range, $f(x,y) = (99)-(y+1)+1 = 99-y$.

Now we have to eliminate the y and simply find the number of triplets with least side x in each range. In the first range, there are $(100-x)-(x+1)+1 = 100-2x$ possible values of y, and for each value of y, $f(x,y) = x-1$, so the total number of triplets is $(100-2x)(x-1) = -2x^2+102x-100$. The second range is trickier: $f(x,y)$ "counts down" from $99-y = 99-(100-x+1) = x-2$ to 1, so the total number of triplets can be found using the formula for the sum of the first $x-2$ natural numbers. This is $\frac{(x-2)(x-1)}{2} = \frac{x^2-3x+2}{2}$, so adding the two quadratics together gives us $J_1(x) = \frac{-3x^2+201x-198}{2}$.

$J_2(x) = 2J_1(x)-21 = -3x^2+180x-198$, a quadratic which is maximized at $x = \frac{-b}{2a} = 30$, which is fortunately a member of the J-List. The maximum value is $J_2(30) = -3(30)^2+180(30)-198 = \boxed{2502}$.

26. We can rearrange the inequality $\frac{a_1+a_2+\ldots+a_n}{2} > a_j$ to $a_1+a_2+\ldots+a_n > 2a_j$. The LHS contains all the side lengths of the polygon so it must contain a_j; this gives us $(a_1+a_2+\ldots+a_n)-a_j > 2a_j - a_j = a_j$. This is basically an extended version of the triangle inequality, claiming that the sum of all sides but one in a non-degenerate polygon will always be greater than the remaining side. Thus, the maximum possible value of H can be found by looking for scenarios where the inequality is not satisfied - one side is greater than the sum of all the others. Obviously, the worst-case scenario occurs when one side is 99 and others work their way up from 10. By inspection, we find that $10+11+12+13+14+15+16 = 91 < 99$, but adding 17 to the right-hand side results in a sum of 108 which is greater than 99. This means that no polygon of side lengths [10, 11, 12, 13, 14, 15, 16, 99] exists, but adding any additional stick in the given range will allow Harshil to create a non-degenerate polygon. Therefore, the maximum value of H is 8. Meanwhile, removing any of the small side lengths from that list will also prevent the inequality from being satisfied, so any value of H less than 8 is possible too. The sum of all possible values of H must be $8+7+6+...+1 = \frac{(8)(9)}{2} = 36$, so the answer is $\boxed{6}$.

27. It may seem like this is a formula you were expected to memorize but that's not true! When we connect the center of the circle to a side of the polygon, we would get an isosceles triangle where the vertex angle is equal to $\frac{360}{a}$ degrees. Dropping an altitude from the center gives us two right triangles whose angle measures are 90 degrees, $\frac{180}{a}$ degrees, and $90-\frac{180}{a}$ degrees. $\tan\frac{180}{a} = \frac{\frac{b}{2}}{altitude}$, rearranging this gives us $altitude = \frac{b}{(2*\tan\frac{180}{a})}$. That is also equal to the apothem. Using the formula $area = semiperimeter * apothem$ gives us $area = \frac{a*b}{2} * \frac{b}{(2*\tan\frac{180}{a})}$.

Simplifying this gives us $\boxed{\frac{a*b^2}{4*\tan\frac{180}{a}}}$.

28. Let points A and B be the centers of the circle. Let points C and D be the intersections of the sides of the square with AB, where C is closer to A and D is closer to B. Let E be a vertex of the square on the circumference of circle A. Call the side length of the square s.
AB is equal to 6 units because the circles touch each other's centers. $AC = DB$ because the figure is symmetrical. $AC+CD+DB = AB$, so $2AC+s = 6$ and $AC = \frac{6-s}{2} = 3-\frac{s}{2}$. Since triangle ADE is right, $AD^2+DE^2 = AE^2$. $AD = AC+CD = 3-\frac{s}{2}+s = 3+\frac{s}{2}$, $DE = \frac{s}{2}$, and $AE = 6$ because it is a radius of circle A. Thus, if we let $n = \frac{s}{2}$, we have $(3+n)^2+n^2 = 6^2$ so $2n^2+6n-27 = 0$. With the quadratic formula we find $n = \frac{-3+3\sqrt{7}}{2}$ so

$s = 2n = 3\sqrt{7} - 3$. The area of the square is $s^2 = (3\sqrt{7} - 3)^2 = \boxed{72 - 18\sqrt{7}}$.

29. A characteristic that belongs to more than one club can either belong to two of them, or all three. Thus, we are looking for the area common to at least two circles - the total intersection of all three circles. Let x be the area of the intersection of 2 circles and y be the area of the part that all 3 circles touch. The answer to this question would be $3x - 2y$. To find the value of x, we can put 2 equilateral triangles in the intersection and find the sum of the area of the 4 sectors and subtract the area of the triangle 2 times. The area of the sector would be $\frac{60}{360}(36)^2\pi = 216\pi$ and the area of the triangle would be $\frac{36^2\sqrt{3}}{4} = 324\sqrt{3}$. The value of x would be $864\pi - 648\sqrt{3}$. To find the area of y, we find the area of the sector 3 times, and subtract the area of the triangle 2 times to get a value of $648\pi - 648\sqrt{3}$. $3(864\pi - 648\sqrt{3}) - 2(648\pi - 648\sqrt{3}) = \boxed{1296\pi - 648\sqrt{3}}$.

30. By sketching out a few examples we see that if Shrung turns x° clockwise while walking, the polygon will have an exterior angle of x° at that vertex and thus an interior angle of $180 - x^\circ$. This means the interior angles of the pentagon are $60^\circ, 60^\circ, 120^\circ$, and 30° in order. Since the interior angles of a pentagon must add to $(5-2)(180) = 540^\circ$, the final interior angle at A must be 270°. The pentagon is concave, so we must split it into a few triangles to find the area.
Noting first that $\angle ABC = 60^\circ$, if we draw an altitude from C to AB it must form a 30-60-90 right triangle with a hypotenuse of 2. Then the length of the leg that is a portion of AB must be 1, but the length of AB is exactly 1, so the altitude from C must intersect AB at A. In other words, $\angle ACB = 90^\circ$, so $\triangle ACB$ is a 30-60-90 right triangle. Its area is $\frac{(1)(\sqrt{3})}{2} = \frac{\sqrt{3}}{2}$

Due to the right angle, C is directly to the right of A and E is directly to the left of A, so CE is horizontal and thus allows $\triangle CDE$ to be cut off completely from $\triangle ABC$. Since $\angle BCD = 60^\circ$ and $\angle BCA = 30^\circ$, $\angle ACD = 30^\circ$. But $\angle DEA = 30^\circ$, so $\triangle CDE$ is isosceles and $CD = DE = 4$. $\angle CDE = 120^\circ$, so the area of the triangle is $\frac{1}{2}(4^2)\sin 120^\circ = 4\sqrt{3}$.

The combined area of the two triangles, and thus the area of the pentagon, is $4\sqrt{3} + \frac{\sqrt{3}}{2} = \boxed{\frac{9\sqrt{3}}{2}}$.