For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Anirudh and Siddharth are designing a special $2D$ tennis racket. The racket's head is in the shape of two concentric rectangles. The length of the outer rectangle is 8 inches longer than the length of the inner rectangle, and the width of the outer rectangle is 6 inches longer than the width of the inner rectangle. The area of the inner rectangle is 100 square inches. What is the minimum possible area of the region between the two concentric rectangles? All answers are expressed in square inches.
(A) $48\sqrt{6}+48$ (B) $96\sqrt{2}+48$ (C) $80\sqrt{3}+48$ (D) $132\sqrt{2}$ (E) NOTA

2. Let $y=\ln\left(x^2\right)$ be the solution to the equation $3y''-y'''+y'=0$. What is the sum of the real solutions for the value of x?
(A) 3 (B) 6 (C) 9 (D) 12 (E) NOTA

3. Evaluate

$$\lim_{y\to\infty}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sqrt[y]{\frac{1+\tan^y(x)}{\sec^y(x)}}\right)dx.$$

(A) $\sqrt{2}$ (B) $\sqrt{2}-1$ (C) $\frac{1}{2}$ (D) $\sqrt{3}-1$ (E) NOTA

4. Find the coefficient of the x^{38} term in the Maclaurin polynomial expansion associated with the function $f(x)=5x^{10}e^{\frac{-x^4}{3}}$.
(A) $\frac{1}{104976}$ (B) $\frac{-1}{734832}$ (C) $\frac{-1}{314928}$ (D) $\frac{-1}{2204496}$ (E) NOTA

5. Given that $f(x)=10x^2-3x$, $g(x)=5x$, and $h(x)=g\left(f\left(\frac{-x}{2}\right)\right)$, evaluate $h'(-4)$.
(A) 185 (B) $\frac{-185}{4}$ (C) $\frac{-185}{2}$ (D) 187 (E) NOTA

6. Evaluate the area of intersection of the polar curves $r=8$ and $r=8-2\cos\theta$.
(A) $32-\pi$ (B) $32\pi-64$ (C) $32+33\pi$ (D) $65\pi-32$ (E) NOTA

7. The region between the curves $x=2y^2+4$ and $x=7-4y^2$ is the base of a solid. Find the volume of this solid with cross-sections perpendicular to the $x-$axis that are equilateral triangles.
(A) $\frac{3\sqrt{3}}{4}$ (B) 3 (C) $\frac{2\sqrt{3}}{4}$ (D) $\frac{\sqrt{3}}{4}$ (E) NOTA

8. What is the maximum value of $f(x)=\ln(-x^2-3x-2)$ over the domain $x\in(-2,-1)$?
(A) $\ln(3)$ (B) $\ln\left(\frac{1}{2}\right)$ (C) $\ln\left(\frac{1}{4}\right)$ (D) $\ln(2)$ (E) NOTA

9. Given that $f(x)=4\cos^3 x+4\sin^3 x-3\sin x-3\cos x$, evaluate $f^{(6)}\left(\frac{\pi}{3}\right)$ (this means that $f(x)$ is differentiated 6 times and evaluated at $x=\frac{\pi}{3}$).
(A) $-729\left(\frac{\sqrt{3}-1}{2}\right)$ (B) 0 (C) 6 (D) $729\left(\frac{\sqrt{3}-1}{2}\right)$ (E) NOTA

10. Let X be a continuous random variable with the following probability density function

$$f(x)=\begin{cases}e^{-x} & x\geq 0\\ 0 & x<0\end{cases}.$$

Evaluate the mean value of X.
(A) $\frac{1}{e}$ (B) $\frac{e}{4}$ (C) $\frac{e}{2}$ (D) e (E) NOTA

11. Let $f(x)$ be a quintic polynomial with 5 distinct real roots. Four of these roots are at $x = -1, 2, 3$, and 4. Given that $f'(5) = 0$, at what value of x is the fifth real root of $f(x)$?
(A) 5 (B) $\frac{11}{2}$ (C) 6 (D) $\frac{13}{2}$ (E) NOTA

12. Evaluate
$$\frac{d}{dx}\left(\prod_{y=1}^{10}\left(\frac{yx+3}{y}\right)\right) \text{ at } x = 0.$$
(A) $\frac{729}{896}$ (B) $\frac{297}{896}$ (C) $\frac{8019}{8960}$ (D) $\frac{2673}{8960}$ (E) NOTA

13. Evaluate
$$\lim_{y\to\infty}\int_1^3\left(\frac{y+x+4}{y+4}\right)^{y+3} dx.$$
(A) 0 (B) e (C) e^2 (D) Does Not Exist (E) NOTA

14. Evaluate
$$\int_0^{\frac{\pi}{2}}\left(\frac{3}{4+2\cos(x)}\right) dx.$$
(A) $\frac{\pi\sqrt{2}}{3}$ (B) $\frac{\pi\sqrt{2}}{6}$ (C) $\frac{\pi\sqrt{3}}{6}$ (D) $\frac{\pi\sqrt{3}}{12}$ (E) NOTA

15. Evaluate
$$\sum_{n=0}^{\infty}\frac{n^2+7n}{7^{n+1}}.$$
(A) $\frac{7}{27}$ (B) $\frac{35}{36}$ (C) $\frac{25}{108}$ (D) $\frac{7}{36}$ (E) NOTA

16. Find the volume of the solid formed by revolving the part of the cycloid parameterized as $x = 3t - 3\sin(t)$ and $y = 3 - 3\cos(t)$ from $0 \le t \le 2\pi$ around its horizontal base.
(A) $81\pi^2$ (B) $135\pi^2$ (C) $175\pi^2$ (D) $243\pi^2$ (E) NOTA

17. The curve $y = x^2 - 9$ intersects the positive x-axis and the y-axis at points A and B respectively. Let point C be the center of mass of the region bounded by $y = x^2 - 9$ that is in the fourth quadrant. What are the coordinates of the centroid of ΔABC?
(A) $\left(\frac{11}{8}, \frac{-21}{5}\right)$ (B) $\left(\frac{9}{8}, \frac{-18}{5}\right)$ (C) $\left(\frac{5}{4}, -4\right)$ (D) $\left(\frac{4}{3}, \frac{-22}{5}\right)$ (E) NOTA

18. There exists a triangle in the first quadrant with one vertex at the origin, one vertex that lies on the curve $y = 5x$, and one vertex that lies on the curve $y = 3x$. Given that the rate of change of x with respect to time is 2 units per second, how fast is the area of this triangle changing when $x = 10$? All answers are expressed in square units per second.
(A) 30 (B) 40 (C) 50 (D) 60 (E) NOTA

19. Rayyan recently built a new yacht for himself that can be represented as the solid formed by rotating the parametric curve defined by $x = 2t + 4$ and $y = -4t + 3$ within $1 \le t \le 3$ about the $y-$axis. He hired Tanvi and Sanjita to paint the entire exterior surface of his yacht. Tanvi paints at a rate of $2\pi\sqrt{5}\frac{\text{square units}}{\text{minute}}$ and Sanjita paints at a rate of $6\pi\sqrt{5}\frac{\text{square units}}{\text{minute}}$. Given that Sanjita arrived late and started painting the house x minutes after Tanvi began painting, and they completed painting the house after 5 additional minutes to the time that Tanvi painted alone, what is the value of x? Assume that the curve representing Rayyan's yacht traces only once for the given range of t's.
(A) 12 (B) 9 (C) 15 (D) 9π (E) NOTA

20. Evaluate

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{\sqrt{5}}{\sqrt{20n^2-5(i+n)^2}}\right).$$

(A) 0 (B) $\frac{\pi}{6}$ (C) 1 (D) $\frac{\pi}{3}$ (E) NOTA

21. Which of the following three series converge absolutely:

$$I:\sum_{n=1}^{\infty}\frac{-2\cos(n)}{7n^2}\qquad II:\sum_{n=10}^{\infty}\frac{(-1)^{n+1}}{\ln(n)}\qquad III:\sum_{n=2}^{\infty}\frac{(-1)^n\left(3n^6+14\right)}{7n^{10}-5n^2}.$$

(A) I and II only (B) I and III only (C) II and III only (D) I, II, and III (E) NOTA

22. Find the length of the polar curve $r=\frac{\theta}{2}$ for $\frac{\sqrt{3}}{3}\le\theta\le 1$. Assume that the curve is traced exactly once along this range of θ.
(A) $\frac{1}{2}\left(\frac{3\sqrt{2}-2}{3}+\ln\left(\sqrt{6}+\sqrt{3}\right)\right)$
(B) $\frac{1}{8}\left(\frac{3\sqrt{2}-2}{3}+\ln\left(\frac{3\sqrt{3}}{3}\right)\right)$
(C) $\frac{1}{4}\left(\frac{\sqrt{6}-3}{3}+\ln\left(\sqrt{6}+\sqrt{3}\right)\right)$
(D) $\frac{1}{4}\left(\frac{3\sqrt{2}-2}{3}+\ln\left(\frac{\sqrt{6}+\sqrt{3}}{3}\right)\right)$
(E) NOTA

23. The horizontal tangent line of the curve $x^2+xy+y^2=27$ that passes through the first quadrant, the tangent line to the curve $y=x^3-10x^2+31x-30$ at $x=5$, and the line between the absolute extrema of $y=-x^4+x^3+x^2+4x-3$ where $-2\le x\le 1$ form a triangular region of intersection. What is the area of this triangle?
(A) $\frac{289}{9}$ (B) $\frac{1486}{33}$ (C) $\frac{7803}{55}$ (D) $\frac{2601}{11}$ (E) NOTA

24. Rohan the multitasker is yodeling his thoughts out loud while writing his college application essays, all while dancing along the curve $y=-\frac{x^4}{4}-\frac{3x^3}{2}+15x^2+18x+21$ on a Cartesian plane. He starts at point A where $x=-2$ and runs in the positive $x-$direction until he reaches point B, which is the first point at which the rate of change of y with respect to x is no longer following an increasing path. Given that the origin is point C, evaluate $\sin(\angle BAC)$.
(A) $\frac{77}{29\sqrt{485}}$ (B) $\frac{3}{\sqrt{485}}$ (C) $\frac{97}{29\sqrt{485}}$ (D) $\frac{107}{29\sqrt{485}}$ (E) NOTA

25. If $f(x)=x^{x^x}$, evaluate $\frac{d}{dx}[f^{-1}(e^{e^e})]$.
(A) e^{e^e} (B) $\frac{e^{1-e-e^e}}{1+2e}$ (C) $\frac{e^{1-2e^e}+e^{e-e^e}}{1+2e^e}$ (D) $\frac{e^{e^e-1}-e^e}{2e^e-1}$ (E) NOTA

26. Tanvi still hasn't gotten over her obsession with the function $\tan(v)$. Evaluate

$$\int_0^{\frac{\pi}{2}}\left(\frac{4}{1+\tan^{\pi^2e^{30}}(v)}\right)dv.$$

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) NOTA

27. Evaluate

$$\lim_{x\to-1}\left(\frac{\sin(x+1)}{x+1}\right)^{\frac{1}{2(\cos(x+1)-1)}}.$$

(A) $e^{\frac{1}{6}}$ (B) $e^{\frac{1}{3}}$ (C) $e^{\frac{1}{2}}$ (D) 0 (E) NOTA

28. Tanusri has a deflated balloon that when fully inflated has a volume equal to that of the solid formed by the rotation of a regular hexagon with side length 6 feet about one of its sides. Tanusri begins inflating her balloon at an initial rate of $6\pi\frac{\text{cubic feet}}{\text{second}}$, and that rate increases at a constant rate of $6\pi\frac{\text{cubic feet}}{\text{second squared}}$. How much time does it take for Tanusri to fill the balloon? All answers are expressed in seconds.
(A) 81 (B) $12\sqrt{2}-2$ (C) $5\sqrt{13}-1$ (D) $6\sqrt{17}-3$ (E) NOTA

29. Let $f(x)$ be a differentiable function such that $\int_2^4 f'(2x)f(2x)\,dx = 4$ and $\int_8^0 f'(x)f(x)\,dx = 0$. Evaluate $\int_{\frac{1}{2}}^0 f'(8x)f(8x)\,dx$.
(A) $\frac{1}{8}$ (B) -1 (C) 8 (D) -4 (E) NOTA

30. For $0 < n < 1$, compute

$$\sum_{i=0}^{\infty} n^{i-3} i(-3i-6)(i-1).$$

(A) $\frac{4n-8}{n(1-n)^4}$ (B) $\frac{6n-24}{n(1-n)^4}$ (C) $\frac{3-6n}{n(1-n)^4}$ (D) $\frac{12n-20}{n(1-n)^4}$ (E) NOTA