For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Find the last digit of $1 + 2^2 + 3 + 2^4 + 5 + 2^6 + \cdots + 99 + 2^{100}$.

   (A) 0 (B) 2 (C) 4 (D) 6 (E) NOTA

2. Find the domain of the function (in interval notation):

   $$f(x) = \sqrt{2^x - 4} + \sqrt{3^x - 27}$$

   (A) $(0, 2) \cup (3, \infty)$ (B) $[2, \infty)$ (C) $[2, 3]$ (D) $[3, \infty)$ (E) NOTA

3. Shubham loves swimming, and he successfully convinced his friend Karthik to swim with him. Shubham and Karthik start at the opposite ends of the same lane, which is 56 meters long. Shubham swims at 5 meters/second and Karthik swims at 2 meters/second. At the instant they meet each other or an end of a lane, they instantly turn around and continue swimming. Shubham and Karthik swim like this for 100 seconds. How many times will they meet each other?

   (A) 5 (B) 6 (C) 7 (D) 8 (E) NOTA

4. Find the sum of the REAL solutions of $x^4 + 2x^3 - 3x^2 - 24x - 36 = 0$.

   (A) -2 (B) -1 (C) 0 (D) 1 (E) NOTA

5. A certain parabola has vertex $(1, 2)$ and focus $(3, 2)$. This parabola intersects the line $x = 3$ at two points (x_1, y_1) and (x_2, y_2). Evaluate $x_1 + y_1 + x_2 + y_2$.

   (A) 8 (B) 10 (C) 12 (D) 14 (E) NOTA

6. What is the minimum POSITIVE value of $|60x + 12y - 288z|$ if x, y, and z are integers?

   (A) 4 (B) 6 (C) 12 (D) 24 (E) NOTA

7. Find the sum of the integer solutions of the equation:

   $$|x + 1| + |x + 2| + |x + 3| = x + 4$$

   (A) -3 (B) -1 (C) 0 (D) 1 (E) NOTA

8. Find the sum of the positive integers x at most 100 that satisfy $4 \mid x^x - 1$.

   (A) 1008 (B) 1175 (C) 1200 (D) 1225 (E) NOTA

9. Evaluate

   $$\sqrt{3\sqrt{9\sqrt{27\sqrt{81\sqrt{\cdots}}}}}$$

   (A) 3 (B) 9 (C) 27 (D) 81 (E) NOTA

10. Tanmay trains tirelessly to track trains. To teach the "Train Tracker" the technique to tame the table, transpose the table to tell the truth, $\mathbb{T}^T$, then tell $\mathbb{T}^T_{2,3}$:

    $$\mathbb{T} = \begin{bmatrix} t & r & a & i & n \\ t & r & u & t & h \\ t & r & a & c & e \end{bmatrix}$$

    (A) t (B) r (C) u (D) e (E) NOTA

11. Suppose that x and y are real numbers that satisfy

$$x + y^2 = y + x^2 = 1$$

If $x + y$ is not an integer, then $x + y$ must be $-a \pm \sqrt{b}$, where a and b are positive integers. Find $a + b$.

(A) 4 (B) 6 (C) 7 (D) 8 (E) NOTA

12. Suppose that x is the unique prime number that satisfies

$$x^2 - 1 = \left(\frac{x-1}{10}\right)!$$

where the factorial function is only defined on the integers. How many factors does $x + 1$ have?

(A) 2 (B) 4 (C) 8 (D) 12 (E) NOTA

13. Call a positive integer N *2-powerful* if there exist positive integers x and y such that $|2^x - 10^y| = N$. Which integer below is not *2-powerful*?

(A) 22 (B) 24 (C) 26 (D) 28 (E) Multiple integers fail.

14. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. If $q + \lfloor t \rfloor = 20.19$ and $\lfloor q \rfloor + t = 20.91$, find $q + t$.

(A) 20 (B) 20.1 (C) 21 (D) 21.1 (E) NOTA

15. The number $41! - 40! - 39! - 38!$ has a 4-digit prime factor. What are the last two digits of it?

(A) 31 (B) 43 (C) 59 (D) 97 (E) NOTA

16. Suppose there exists a positive real number x such that

$$\log_x 2^x + \log_x x^3 + \log_x 4^x + \log_x x^5 + \cdots + \log_x x^9 + \log_x 10^x = 24 + 2x$$

Then, $x = a\sqrt{b}$, where a and b are positive integers, and b is not divisible by the square of a prime. Find $a + b$.

(A) 24 (B) 30 (C) 32 (D) 36 (E) NOTA

17. Let N denote the largest integer x such that $\frac{x^2+3x+2020}{x+7}$ is an integer. Which positive three-digit integer divides N?

(A) 107 (B) 149 (C) 157 (D) 167 (E) NOTA

18. Suppose there exists a positive real value q that satisfies

$$\frac{3q}{2^1} + \frac{6q}{2^2} + \frac{9q}{2^3} + \frac{12q}{2^4} + \cdots = \frac{2}{q^1} + \frac{4}{q^2} + \frac{6}{q^3} + \frac{8}{q^4} + \cdots$$

If $2q - q^2 = \frac{m}{n}$, where m and n are relatively prime positive integers, find $m + n$.

(A) 7 (B) 9 (C) 11 (D) 13 (E) NOTA

19. Consider a positive three-digit integer N, which has digits a, b, and c, in that order. It is known that $N = abc + a + b + c + 100$. Find the largest prime factor of N.

(A) 7 (B) 19 (C) 31 (D) 43 (E) NOTA

20. An ellipse has foci F_1 and F_2. There exists a point P on the ellipse such that $PF_1 = 5$, $PF_2 = 12$, and $F_1F_2 = 13$. The line through P and F_1 intersects the ellipse at another point $Q \neq P$. Find F_1Q.

(A) $\frac{27}{10}$ (B) $\frac{26}{11}$ (C) $\frac{30}{13}$ (D) $\frac{30}{11}$ (E) NOTA

21. Consider the function

$$F(x) = x + \frac{x}{x + \frac{x}{x+\cdots}}$$

If $F(F(F(\alpha))) = 2$, then evaluate 777α.

(A) 210 (B) 240 (C) 256 (D) 280 (E) NOTA

22. Rohan, Tanusri, Akash, and Vishnav are distributing $A > 3$ apples amongst themselves. No apple can be broken, and every person wants a positive number of apples. Rohan wants the number of apples he receives to be a perfect square. Tanusri wants the number of apples she receives to be a perfect cube. Akash wants a composite number of apples. Vishnav wants a prime number of apples. However, Vishnav immediately realizes that if they distributed the apples to satisfy everyone, Vishnav would only get 2 apples, always. Who gets the most apples?

(A) Akash (B) Rohan (C) Tanusri (D) It could be anyone. (E) NOTA

23. Using the exact setup and observation in the previous question (Vishnav always get 2 apples), what is the sum of the possible values of A?

(A) 17 (B) 18 (C) 19 (D) ∞ (E) NOTA

24. There exists a positive real number r that satisfies $0 < r < 1$ and

$$r^2 + 1 = \sqrt{5r^3 + 50r^2 + 5r}$$

If $r^{1.5} = \sqrt{a+1} - \sqrt{a}$, for some positive integer a, find the sum of the digits of a.

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

25. Suppose that there exists non-real numbers x and y which satisfy

$$x + y = x^2 + y^2 = x^4 + y^4$$

Find the value of $|x^7 + y^7|$.

(A) 0 (B) 0.5 (C) 1 (D) 1.5 (E) NOTA

26. Find the sum of all integers x which satisfy

$$2^x \leq (x+1)(x+3)(x+5)$$

(A) 60 (B) 62 (C) 63 (D) 66 (E) NOTA

27. The three positive, distinct real numbers a, b, and c add to 6. Let N be the greatest real number that is always less than $\dfrac{1}{a(a-b)(a-c)} + \dfrac{1}{b(b-a)(b-c)} + \dfrac{1}{c(c-a)(c-b)}$. Find the last digit of $\lfloor 100N \rfloor$.

(A) 0 (B) 2 (C) 3 (D) 6 (E) NOTA

28. Dylan and Farzan are exploring the caves where extremely smart cavemen lived. Farzan noticed the following written on the wall:

$$16 = 66 \log 2 + \log \log 2$$

Farzan sees this and thinks that the cavemen are stupid, as the equation is false. However, Dylan realizes that if the base of the logarithm was not 10 but actually b, then the equation is true. Find the sum of the digits of b^2.

(A) 10 (B) 13 (C) 16 (D) 19 (E) NOTA

29. Suppose that $f(x) = x^2 - 2x$. Find the sum of the REAL solutions of the equation $f(f(f(x))) = f(f(x)) + f(x)$.

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

30. Suppose that x and y are positive reals such that $y > x$ and $(2x)^y = y^x$. If $x + y = \sqrt{6}$, then $x^2 + y^2 = \frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

(A) 12 (B) 15 (C) 17 (D) 19 (E) NOTA