1. We can split this into $(1+3+5+7+\cdots 99)+(2^2+2^4+\cdots 2^{100})$. The first part is equal to 50^2, so it ends in 0. Taking the last digits of each of the exponentials in the second part gives $4+6+4+6+\cdots 4+6$, which ends in a 0. Therefore, the entire sum ends in a $0 \implies \boxed{A}$

2. Note that the function is defined if and only if $2^x \geq 4$ and $3^x \geq 27$. This happens if and only if $x \geq 2$ and $x \geq 3$. Therefore, the domain is $[3, \infty) \implies \boxed{D}$.

3. After $\frac{56}{2+5} = 8$ seconds, they will meet each other for the first time, and after another 8 seconds, they will be back at their starting positions. Therefore they will meet at the $8, 24, 40, 56, 72, 88, 104 \cdots$ second marks. Therefore, in 100 seconds, they will meet 6 times $\implies \boxed{B}$.

4. Testing integer roots with the Factor Theorem gives $x = 3$ is a solution. Factoring it out, we see $(x-3)(x^3+5x^2+12x+12) = 0$. Checking negative integers (as all coefficients are positive), we see that $x = -2$ also works, so $(x-3)(x+2)(x^2+3x+6) = 0$. However, since the quadratic has nonreal roots, our answer is $3 - 2 = 1 \implies \boxed{D}$.

5. Notice that the line is perpendicular to the axis. Therefore, the line is symmetrical to the axis, meaning the midpoint of the intersections with the parabola is on the axis. In this scenario, the midpoint of the two intersection points is $(3, 2)$, the point on the axis and the line $x = 3$. Therefore $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (3, 2) \implies x_1 + x_2 = 6, y_1 + y_2 = 4 \implies x_1 + x_2 + y_1 + y_2 = 10 \implies \boxed{B}$.

6. Clearly $|60x + 12y - 288z| = 12|5x + y - 24z|$ is a positive multiple of 12, so the least value it can be is $12 \implies \boxed{C}$. It is attained when $(x, y, z) = (5, 0, 1)$.

7. Suppose that $x < -3$. This means that $x+1, x+2, x+3$ are all negative, so the equation turns into $-x-1-x-2-x-3 = x+4 \implies 4x+10 = 0 \implies x = -2.5$, which is not an integer. Now, suppose that $x > -1$. This means that $x+1, x+2, x+3$ are all positive, so the equation turns into $x+1+x+2+x+3 = x+4 \implies 2x+2 = 0 \implies x = -1$. However, we said that $x > -1$, so this is a contradiction. Therefore, the only solutions lie in the range $-3 \leq x \leq -1$. Checking the three integers gives that -2 and -1 work, so our answer is $-3 \implies \boxed{A}$.

8. Clearly x cannot be even, otherwise $x^x - 1$ is odd. Therefore, by Fermat's Little Theorem, $x^4 \equiv 1 \pmod 4$, so $x^x - 1 \equiv (x \pmod 4)^x - 1 \equiv (x \pmod 4)^{x \pmod 4} - 1$. Clearly, we must have $x \equiv 1, 3 \pmod 4$. If $x \equiv 3 \pmod 4$, then $3^3 - 1 = 26$ is a multiple of 4, contradiction. Therefore, $x \equiv 1 \pmod 4$, which works because $1^1 - 1 = 0$ is a multiple of 4. Adding all of these numbers gives $1+5+9+13+\cdots+97 = 0.5(1+97)25 = 49 \cdot 25 = 1225 \implies \boxed{D}$.

9. Rewriting this expression with exponents gives $3^{1/2}9^{1/4}27^{1/8}\cdots = 3^{1/2+2/4+3/8+4/16+\cdots}$. To find the value of the exponent, note that $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{8}\cdots = \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)+\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)+\cdots = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\cdots = 2$. Therefore, our answer is $3^2 = 9 \implies \boxed{B}$.

10. Note that by the definition of transpose $\mathbb{T}^T_{2,3} = \mathbb{T}_{3,2}$. Clearly, the element in the third row and second column is $r \implies \boxed{B}$.

11. Subtracting the equations gives $x - y + y^2 - x^2 = 0 \implies (x-y)(1-x-y) = 0$. Since $x + y \neq 1$ ($x + y$ is a non-integer), then $x = y$. Therefore $x + y^2 = y + y^2 = 1 \implies y^2 + y - 1 \implies y = \frac{-1\pm\sqrt{5}}{2} = x \implies x + y = -1 \pm \sqrt{5}$. Therefore, $a = 1$ and $b = 5$, so our answer is $6 \implies \boxed{B}$.

12. There are not many factorials that are 1 less than a perfect square. Checking the first few reveals that the smallest possibilities are $4! + 1 = 5^2, 5! + 1 = 11^2$, and $7! + 1 = 71^2$. Clearly the third one molds our equation when $x = 71$, so $x + 1 = 72 = 2^3 \cdot 3^2$ has $(3+1)(2+1) = 12$ factors $\implies \boxed{D}$.

13. Note that $|2^5 - 10^1| = 22$, $|2^{10} - 10^3| = 24$, and $|2^7 - 10^2| = 28$, so these integers are 2-powerful. To show that 26 is not 2-powerful, notice that $|2^x - 10^y|$ is a multiple of 4 when x and y are at least 2. Therefore, if $|2^x - 10^y| = 26$, then at least one of x or y is 1. If $x = 1$, then $|2 - 10^y| = 26 \implies 10^y = -24, 28$, which cannot happen. If $y = 1$, then $|2^x - 10| = 26 \implies 2^x = 36, -16$, which cannot happen. Therefore, 26 is not 2-powerful $\implies \boxed{C}$.

14. Let $\{x\} = x - \lfloor x \rfloor$ denote the fractional part of x, and it is clear that $0 \leq \{x\} < 1$. The equations are equal to $\lfloor q \rfloor + \lfloor t \rfloor + \{q\} = 20.19$ and $\lfloor q \rfloor + \lfloor t \rfloor + \{t\} = 20.91$. Since $\lfloor q \rfloor + \lfloor t \rfloor$ is an integer and $\{q\}$ and $\{t\}$ are in the range $[0, 1)$, then we must have $\{q\} = 0.19$ and $\{t\} = 0.91$. Therefore, $\lfloor q \rfloor + \lfloor t \rfloor = 20$, and thus $q + t = \lfloor q \rfloor + \lfloor t \rfloor + \{q\} + \{t\} = 20 + 0.19 + 0.91 = 21.1 \implies \boxed{D}$.

15. Note that $41! - 40! - 39! - 38! = 38!(41 \cdot 40 \cdot 39 - 40 \cdot 39 - 39 - 1) = 38!((40^3 - 40) - (40^2 - 40) - 40) = 38!(40^3 - 40^2 - 40)$. Factoring the 40 out gives $38! \cdot 40(40^2 - 40 - 1)$. Since 38! and 40 do not have a 4-digit prime factor, it must be a factor of $40^2 - 40 - 1 = 1559$. This is a prime, so this must be the desired factor $\implies \boxed{C}$.

16. Note that $\log_x x^3 = 3$, and using this on the other logarithms of this form gives $\log_x x^3 + \log_x x^5 + \log_x x^7 + \log_x x^9 = 3 + 5 + 7 + 9 = 24$. The other logarithms can be simplified using $\log_x 2^x = x \log_x 2$. Simplifying gives $\log_x 2^x + \log_x 4^x + \log_x 6^x + \log_x 8^x + \log_x 10^x = x(\log_x 2 + \log_x 4 + \log_x 6 + \log_x 8 + \log_x 10) = x \log_x 3840$. Therefore, the equation turns into $x \log_x 3840 + 24 = 24 + 2x \implies \log_x 3840 = 2 \implies x = \sqrt{3840} = 16\sqrt{15} \implies 16 + 15 = 31 \implies \boxed{E}$.

17. Using polynomial long division, we get $\frac{x^2+3x+2020}{x+7} = x - 4 + \frac{2048}{x+7}$. Since $x - 4$ is an integer, then $\frac{2048}{x+7}$ must also be an integer. Therefore, the largest possible value of x is $2048 - 7 = 2041 = 13 \cdot 157$. The only 3-digit integer that divides this is $157 \implies \boxed{C}$.

18. We simplify the LHS as follows:

$$S_1 = \frac{3q}{2^1} + \frac{6q}{2^2} + \frac{9q}{2^3} + \frac{12q}{2^4} \cdots$$
$$0.5S_1 = \frac{3q}{2^2} + \frac{6q}{2^3} + \frac{9q}{2^4} + \frac{12q}{2^5} \cdots$$
$$\implies 0.5S_1 = \frac{3q}{2^1} + \frac{3q}{2^2} + \frac{3q}{2^3} + \frac{3q}{2^4} \cdots = 3q \implies S_1 = 6q$$

Similarly, we can simplify the RHS as follows:

$$S_2 = \frac{2}{q^1} + \frac{4}{q^2} + \frac{6}{q^3} \cdots$$
$$\frac{1}{q} S_2 = \frac{2}{q^2} + \frac{4}{q^3} + \frac{6}{q^4} \cdots$$
$$\implies \left(1 - \frac{1}{q}\right) S_2 = \frac{2}{q^1} + \frac{2}{q^2} + \frac{2}{q^3} \cdots = \frac{2/q}{1 - 1/q} = \frac{2}{q-1} \implies S_2 = \frac{2q}{(q-1)^2}$$

Since these two are equal, we have $6q = \frac{2q}{(q-1)^2} \implies (q-1)^2 = \frac{1}{3} \implies q^2 - 2q + 1 = \frac{1}{3} \implies 2q - q^2 = \frac{2}{3} \implies 2 + 3 = 5 \implies \boxed{E}$.

19. By the definition of digits, we know that $N = 100a + 10b + c = abc + a + b + c + 100 \implies 99a + 9b = abc + 100 \geq 99a \implies 99a - abc \leq 100 \implies a(99 - bc) \leq 100$. Note that the minimum value of $99 - bc$ is $99 - 9 \cdot 9 = 18$, so $18a \leq 100 \implies a \leq 5$. Now, we will use casework:

   (a) $a = 5$: This means that $99 - bc \leq 20 \implies bc \geq 79$. This only happens when $b = c = 9$, but 599 does not satisfy the original equation.

   (b) $a = 4$: This means that $99 - bc \leq 25 \implies bc \geq 74$. This only happens when $b = c = 9$, but 499 does not satisfy the original equation.

   (c) $a = 3$: Rearranging $99a + 9b = abc + 100$ gives $99a + 9b - abc = 100$. Since $a = 3$, the LHS is a multiple of 3, while the RHS is not, so contradiction.

   (d) $a = 2$: This means that $198 + 9b = 2bc + 100 \implies 98 = 2bc - 9b = b(2c - 9)$. However, the maximum value of b and $2c - 9$ is 9, so the maximum value of the product is $81 < 98$, contradiction.

   This means that $a = 1$. This implies $99 + 9b = bc + 100 \implies 9b - bc = 1 \implies b(9 - c) = 1 \implies b = 9 - c = 1 \implies b = 1, c = 8$. This means the number is $118 = 2 \cdot 59$, so our answer is $\boxed{E}$.

20. Note that by the definition of an ellipse, $F_1P + F_2P = 17 = F_1Q + F_2Q$. Let $F_1Q = k$ and $F_2Q = 17 - k$. Since $\angle QPF_2 = \angle F_1PF_2 = 90°$, using the Pythagorean Theorem gives $(5 + k)^2 + 12^2 = (17 - k)^2 \implies k^2 + 10k + 169 = k^2 - 34k + 289 \implies 44k = 120 \implies k = \frac{30}{11} \implies \boxed{D}$.

21. First, we will find the inverse of this function, $G(x)$:

$$x = G(x) + \frac{G(x)}{G(x) + \frac{G(x)}{G(x)+\cdots}} = G(x) + \frac{G(x)}{x} = G(x)\left(1 + \frac{1}{x}\right) \implies G(x) = \frac{x^2}{x+1}$$

   Therefore, we know that $\alpha = G(G(G(2))) = G(G(4/3)) = G(16/21) = 256/777 \implies 777\alpha = 256 \implies \boxed{C}$.

22. I claim that there exists a distribution of apples, when $A \geq 11$, such that Vishnav does not get 2 apples. For every odd number that is at least 11, then Rohan and Tanusri can have 1 apples each, Vishnav can have 3 apples and Akash can have $A-5$ apples. This satisfies everyone because $A-5$ is even and at least 4, so it must be composite. Similarly, if A is an even number that is at least 12, then Rohan can have 4 apples, Tanusri can have 1 apple, Akash can have $A-8$ apples, and Vishnav can have 3 apples. This satisfies everyone's restriction because $A-8$ is an even number that is at least 4, so it is composite. Since we have found satisfactory distributions where Vishnav does not get 2 apples when $A \geq 11$, this means that $A \leq 10$.

    Notice that Rohan, Tanusri, Akash, and Vishav will get at least 1, 1, 4, 2 apples, respectively, so $A \geq 8$. Checking $A = 8$ and $A = 10$ forces the distribution to be $(1,1,4,2)$ and $(1,1,6,2)$, respectively, so Vishnav's claim is correct. However, when $A = 9$, the distribution $(1,1,4,3)$ invalidates Vishnav's claim. Therefore, the only distributions are $(1,1,4,2)$ and $(1,1,6,2)$, and in either case Akash gets the most apples $\implies \boxed{A}$.

23. By our previous work, the only values of A are 8 and 10, so our answer is $8 + 10 = 18 \implies \boxed{B}$.

24. Squaring both sides gives $r^4 + 2r^2 + 1 = 5r^3 + 50r^2 + 5r \implies r^4 - 5r^3 - 48r^2 - 5r + 1 = 0$. Dividing by r^2 and letting $m = r + \frac{1}{r}$, we have $r^2 + \frac{1}{r^2} - 5r - \frac{5}{r} - 48 = 0 \implies (m^2 - 2) - 5m - 48 \implies m^2 - 5m - 50 = 0 \implies m = 10, -5$. Since $m = r + \frac{1}{r} > 0$, we must have $r + \frac{1}{r} = 10 \implies r^2 - 10r + 1 = 0 \implies r = 5 \pm 2\sqrt{6}$. Since $r < 1$, we must have $r = 5 - 2\sqrt{6}$. Note that $r^{1.5} = r^{3/2} = (5-2\sqrt{6})^{3/2} = (\sqrt{3}-\sqrt{2})^3 = (5-2\sqrt{6})(\sqrt{3}-\sqrt{2}) = 9\sqrt{3} - 11\sqrt{2} = \sqrt{243} - \sqrt{242}$. This means that $a = 242$, and our answer is $\boxed{A}$.

25. Letting $x + y = s$ and $xy = p$, then our equations turn into $s = s^2 - 2p = (s^2 - 2p)^2 - 2p^2$. Since $s^2 - 2p = s$, we have $s^2 - 2p = (s^2 - 2p)^2 - 2p^2 = s^2 - 2p^2 \implies 2p = 2p^2 \implies p = 0, 1$. If $p = 0$, then the equations turn into $s = s^2 = s^4$, so $s = 0, 1$. If $s = 0$, then $(x, y) = (0, 0)$, which are not nonreal. If $s = 1$, then $(x, y) = (1, 0), (0, 1)$, which are not nonreal. This means that $p = 1$. This turns the equation into $s = s^2 - 2 = (s^2 - 2)^2 - 2$. This means that $s = -1, 2$. If $s = 2$, then $(x, y) = (1, 1)$, which is not nonreal. Therefore, $x + y = -1$ and $xy = 1$, so x and y are roots of the quadratic $m^2 + m + 1$.

    However, since $m^2 + m + 1$ is a factor of $m^3 - 1$, we have $x^3 = y^3 = 1$. This means that $|x^7 + y^7| = |x + y| = |-1| = 1 \implies \boxed{C}$.

26. Note that if $x \leq -5$, then the RHS is positive, but the LHS is nonpositive, contradiction. Eventually, 2^x will outgrow $(x+1)(x+3)(x+5)$, as exponentials grow faster than polynomials, and since $2^{12} = 4096$ is greater than $(12+1)(12+3)(12+5) = 3315$, the inequality is false for $x \geq 12$. This means we must check integers in the range $[-4, 11]$. Checking these gives that all integers in this range work, except for $-3, -2, -1$. This means our desired answer is $-4 + 0 + 1 + 2 + 3 + \cdots 11 = 11 \cdot 12 \cdot 0.5 - 4 = 66 - 4 = 62 \implies \boxed{B}$

27. Simplifying the sum to a single denominator, we get $\frac{bc(b-c) - ac(a-c) + ab(a-b)}{abc(a-b)(a-c)(b-c)}$. If $b = c$, the numerator is $0 - ab(a - b) + ab(a-b) = 0$. Therefore, by Factor Theorem, $b - c$ is a factor of the numerator, and by symmetry $a - b$ and $a - c$ are factors as well. The numerator is therefore in the form $n(b-c)(a-b)(a-c)$, and by expanding we see that $n = 1$. Thus the $(b-c)(a-b)(a-c)$ in the numerator and denominator cancel out, leaving only $\frac{1}{abc}$. To find the minimum of this fraction we must maximize abc. By AM-GM inequality, the maximum value of the product of three positive real numbers with a fixed sum occurs when the three numbers are equal, so the maximum here occurs when $a = b = c = \frac{6}{3} = 2$. Then $\frac{1}{abc} = \frac{1}{2^3} = \frac{1}{8}$. Even though $a = b = c = 2$ is technically impossible due to the limitations of the problem (the numbers must be distinct), the fraction must grow infinitely close to $\frac{1}{8}$ as a, b, and c get infinitely close to 2. This means $\frac{1}{8}$ is indeed the greatest lower bound even though it is not actually an attainable value. $\frac{1}{8} = 0.125$, so $\lfloor 100(0.125) \rfloor = 12$ and the answer is $\boxed{2}$.

28. Using logarithm rules, we get

    $$16 = \log_b 2^{66} + \log_b(\log_b 2) = \log_b(2^{66} \log_b 2) \implies b^{16} = 2^{66} \log_b 2 \implies b^{b^{16}} = 2^{2^{66}}$$

    Now, let $b = 2^a$ (note that a may not be an integer)

    $$2^{a \cdot b^{16}} = 2^{2^{66}} \implies a \cdot b^{16} = 2^{66} = a \cdot 2^{16a} \implies 2^{66-16a} = a$$

    Since the LHS is a decreasing function and the RHS is an increasing function, they can only meet at most one point, and they clearly meet at $a = 4$. This means that $b = 2^4 = 16$, so $16^2 = 256$, and our answer is $13 \implies \boxed{B}$.

29. Note that the equation implies that $f(x)$ is a solution of $f(f(y)) = f(y) + y$. Simplifying this equation gives

$$(y^2 - 2y)^2 - 2(y^2 - 2y) = y^2 - y \implies y^4 - 4y^3 + y^2 + 5y = 0 \implies y(y^3 - 4y^2 + y + 5) = 0$$

Clearly $y = 0$ is a root, and for the cubic, substituting -1, 0, 1, 2, 3, 4 gives -1, 5, 3, -1, -1, 9, respectively. This means by the Intermediate Value Theorem (or basic intuition), there is a root in the intervals $(-1, 0)$, $(1, 2)$, and $(3, 4)$. This means that $f(x)$ can attain 4 distinct values, all greater than -1. Graphing $f(x)$ and the roots as $y = r$, we are looking for the number of solutions. However, all of the horizontal lines intersect the parabola twice, and at different points, so the original equation has 8 real roots. We do not have to worry about complex roots. Now, notice that the roots are the x-coordinates of the intersections of the horizontal lines and the parabola. However, by symmetry, the sum of the x-coordinates of a horizontal line's intersections with the parabola is twice the x-coordinate of the vertex of the parabola, which is 1. Applying this philosophy to the four horizontal lines, our final answer is $4 \cdot 2 \cdot 1 = 8 \implies \boxed{D}$.

30. Let $y = xt$, where t is a real number greater than 1. The given equation turns into $(2x)^{xt} = (xt)^x \implies (2x)^t = xt \implies 2^t x^t = xt \implies x^{t-1} = \frac{t}{2^t} \implies x = \frac{1}{2}\sqrt[t-1]{\frac{t}{2}}$. Since $y = xt$, we have $y = \frac{t}{2}\sqrt[t-1]{\frac{t}{2}}$. Using this parameterization on the second equation gives

$$\frac{t+1}{2}\sqrt[t-1]{\frac{t}{2}} = \sqrt{6}$$

Guessing and checking small values of t gives $t = 3$ as a solution. This means that $x = \frac{1}{2}\sqrt[t-1]{\frac{t}{2}} = \frac{1}{2}\sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{4}$ and $y = \frac{3\sqrt{6}}{4}$. This means that $x^2 + y^2 = \frac{6}{16} + \frac{54}{16} = \frac{60}{16} = \frac{15}{4} \implies 19 \implies \boxed{D}$.