

1. D. The slope-intercept form of a line ($y = mx + b$) is used in this question. It is important to note that m is equivalent to the slope of the line, and b is equivalent to the y -intercept of the line. This means that the constant term in the equation, $\frac{22}{3}$ is the y -intercept.
2. D. $\frac{133.2}{360} = \frac{1332}{3600} = \frac{666}{1800} = \frac{111}{300} = \frac{37}{100} = 0.37$. Thus the answer is 37%
3. D. In order to rationalize the denominator, we must multiply the numerator and denominator by the conjugate of the denominator. In this case, the conjugate is $3\sqrt{3} + 8\sqrt{6}$ (note the sign change). From here, we reach $\frac{(3\sqrt{3} + 8\sqrt{6})(5\sqrt{3} + 2\sqrt{6})}{27 - 384}$. Further simplification yields $\frac{-47 - 46\sqrt{2}}{119}$.
4. B. By adding together the three equations we get $3x - 2y = 5$. From here we can guess and check x and y until we arrive at $x, y, z = 1, -1, 1$. This yields $\frac{1(-1)}{1} = -1$.
5. C. $f^{-1}(x)$ indicates the inverse of a function. Because of this, we are trying to find x such that $-9x + 5 = 11$. Solving for x gives $-\frac{2}{3}$.
6. B. We note that the ratio between Akhil's height and shadow is $\frac{3}{8}$. This means that to find the length of the shadow of the pole, we can compute $\frac{32 \cdot 8}{3} = \frac{256}{3}$.
7. E. The product of the roots of a quadratic is $\frac{c}{a}$, and the sum is $-\frac{b}{a}$. The product divided by the sum is $-\frac{c}{b} = \frac{7}{8}$.
8. A. By looking at the equation, we know that $a = 4$ and $b = -2$. Thus, Tanmay's location is $(12, 5)$. The distance from the origin is $\sqrt{12^2 + 5^2} = 13$.
9. A. Let's break this problem up into its two terms. The third root of 512 is 8, which is the first term. The second term is $12^4 = 20736$. Subtracting gives us $8 - 20736 = -20728$.
10. E. First, we need to decide how each variable is related. The amount of artists is directly proportional to the amount of paintings, because the more artists you have, the more paintings you should have. Artists and hours are inversely proportional because the more artists you have, the less time you need. This lets us know that $\frac{AH}{P}$ is equal to some constant k . With the numbers we are given, we can compute $\frac{12 \cdot 15}{4} = 45$ for k . Now we know that $\frac{23H}{13} = 45$, so $H = 25.43$.
11. A. $\frac{(72.5 - 4.2) \cdot 100}{4.2} = \frac{6830}{4.2} \approx 1626$
12. C. In order to efficiently calculate a and b , we can sort the set first. After sorting the set, we have 12, 14, 14, 17, 19, 20, 21. The median is the fourth element in the set, 17. The mean is equal to $\frac{117}{7}$. Then the answer is $\frac{1989}{7} \approx 284$
13. D. The prime factorization of 32412 is $(2^2)(3)(2701)$. $2 \cdot 1 \cdot 1 = 2$.
14. B. $f(f(x)) = f(x^2 - 2020) = (x^2 - 2020)^2 - 2020$. By difference of squares using $2020 = (\sqrt{2020})^2$, this factors to $(x^2 - 2020 - \sqrt{2020})(x^2 - 2020 + \sqrt{2020})$. So, for the expression to be equal to zero, either $x^2 = 2020 + \sqrt{2020}$ or $x^2 = 2020 - \sqrt{2020}$. This means the roots are $-\sqrt{2020 + \sqrt{2020}}$, $\sqrt{2020 + \sqrt{2020}}$, $-\sqrt{2020 - \sqrt{2020}}$, $\sqrt{2020 - \sqrt{2020}}$, so the sum of the squares is $2020 + \sqrt{2020} + 2020 + \sqrt{2020} + 2020 - \sqrt{2020} + 2020 - \sqrt{2020} = 4(2020) = 8080$.
15. D. The distance \overline{AB} is $\sqrt{1^2 + 2^2} = \sqrt{5}$, the distance \overline{BC} is $\sqrt{3^2 + 1^2} = \sqrt{10}$, the distance \overline{AC} is $\sqrt{2^2 + 3^2} = \sqrt{13}$. Adding the squares, we get $13 + 5 + 10 = 28$.

16. C. Because $i = \sqrt{-1}$, i has cycles that repeat. We can use the modulo of the exponent to know the answer without directly computing it. In the explanation I will use n to refer to the exponent that i is being raised to. If $n \bmod 4$ is 0, then the expression will be equivalent to 0. If $n \bmod 4$ is 1, then the expression will be equivalent to i . If $n \bmod 4$ is 2, then the expression will be equivalent to -1 . If $n \bmod 4$ is 3, then the expression will be equivalent to $-i$. 2020 is divisible by 4, so the expression is equivalent to $\boxed{1}$.
17. C. First we simplify the denominator of the large fraction: $\sqrt{x} = \frac{2\sqrt{x} + x\sqrt{x}}{2+x}$, so the denominator simplifies to $\frac{2\sqrt{x} + x\sqrt{x} + 1}{2+x}$. This means the whole fraction is just the reciprocal of that, or $\frac{2+x}{2\sqrt{x} + x\sqrt{x} + 1}$. Multiplying both sides by the denominator we get $2x\sqrt{x} + x^2\sqrt{x} + x = 2+x$, which simplifies to $x^2\sqrt{x} + 2x\sqrt{x} - 2 = 0$. To make this problem solvable, we make a variable $n = \sqrt{x}$ and substitute it. Since $x = n^2$, the equation becomes $n^5 + 2n^3 - 2 = 0$. If n is negative, then n^5 and $2n^3$ will be negative, so the left-hand side will definitely be negative - thus, n cannot be negative, which is good because it means every solution for n corresponds to a solution for \sqrt{x} . This means that the product of the roots to the n equation, which is $-\frac{-2}{1} = 2$ by Vieta's, is the square root of the product of the roots to the x equation. Thus, the product of the roots to the original equation is $2^2 = \boxed{4}$.
18. A. The domain of $\boxed{(-\infty, \infty)}$ is valid here, because 3 can be raised to any real power without making the denominator undefined.
19. A. As x becomes smaller and smaller to negative infinity, 3^x becomes roughly $\frac{1}{3^\infty} \approx 0$. This means that the denominator becomes about 2, and the fraction becomes $\frac{5}{2}$. Meanwhile, as x grows to positive infinity, 3^x becomes infinite so the denominator of the fraction is infinite and the fraction becomes 0. Thus the answer would be $\boxed{(0, \frac{5}{2})}$ with the parentheses signifying that the bounds are not attainable.
20. A. If we make an arbitrary variable x equivalent to the expression, we can manipulate the expression to find x .

$$\begin{aligned} x &= \sqrt{90 + \sqrt{90 + \sqrt{90 + \dots}}} \\ x &= \sqrt{90 + x} \\ x^2 &= 90 + x \\ x^2 - x - 90 &= 0 \\ (x - 10)(x + 9) &= 0 \end{aligned}$$

Note that since the square root of a number cannot be negative, the only possible answer is $\boxed{10}$.

21. D. Though the numbers in this question may seem too large to compute efficiently, we can use rounding to make it easier. For instance, $5^6 = (5^4)(5)(5) \approx (600)(5)(5) = (3000)(5) = 15000$. Obviously $10^4 = 10000$. $2^{14} = (2^{10})(2^4) \approx (1000)(16) = 16000$. $3^9 = (3^6)(3)(9) \approx (700)(3)(9) = (2000)(9) = 18000$. Since 18000 is the largest by a considerable margin, we can say that the largest expression was $\boxed{3^9}$. (The actual values are 15625, 10000, 16384, and 19683)
22. B. We can combine the equations $a + b = c$ and $c + d = a$ to get $a + b + d = c + d = a$. By subtracting both sides of the equation by a , we get $b + d = 0$. Now it is known that $d = -b$, $b + c = d$ turns into $b + c = -b$, $c = -2b$. At this point, what we can do is find every variable in terms of b (we already have c and d , just need to find a). $a + b = c$ reduces into $a + b = -2b$ leading to $a = -3b$. As a final calculation, the sum $a + b + c + d = -3b + b - 2b + -b = -5b$. Because b is a negative integer, the largest possible value of b is -1 and thus the smallest possible value of $-5b$ is $\boxed{5}$.
23. D. Since Mrs. Pickett has the right quadratic and linear terms, the sum of her roots will be correct. The sum of her roots is 3, meaning that the sum of the actual quadratic's roots is also 3. Mrs. Cross has the right quadratic and constant terms, so the product of her roots is the product of the actual roots, -40 . Since we have the sum and product of the roots, we can make the actual quadratic $x^2 - 3x - 40 = 0$. When you factor this out, you get roots $\boxed{-5, 8}$.
24. C. In order to tackle this problem properly, we'll start from the inside. Each of these can be broken up by writing the root as $\sqrt{a + b\sqrt{c}} = k + \sqrt{n}$. To find k and n , you must square both sides. For instance, if $\sqrt{3 - 2\sqrt{2}} = \sqrt{a} - \sqrt{b}$,

then $3 - 2\sqrt{2} = a + b - 2\sqrt{ab}$. Then $a + b = 3$ and $ab = 2$, so the only options are $a = 1$ and $b = 2$ or the reverse. However, it must be $\sqrt{2} - 1$ rather than $1 - \sqrt{2}$ because the latter is negative, so the simplification is $\sqrt{2} - 1$. The same can be done again and again until the original expression is simplified:

$$\begin{aligned} & \sqrt{9 - 2\sqrt{23 - 6\sqrt{10 + 4\sqrt{3 - 2\sqrt{2}}}}} \\ &= \sqrt{9 - 2\sqrt{23 - 6\sqrt{10 + 4(\sqrt{2} - 1)}}} \\ &= \sqrt{9 - 2\sqrt{23 - 6\sqrt{6 + 4\sqrt{2}}}} \\ &= \sqrt{9 - 2\sqrt{23 - 6(2 + \sqrt{2})}} \\ &= \sqrt{9 - 2\sqrt{11 - 6\sqrt{2}}} \\ &= \sqrt{9 - 2(3 - \sqrt{2})} \\ &= \sqrt{3 + 2\sqrt{2}} = \boxed{\sqrt{2} + 1} \end{aligned}$$

25. C. To make it from one vertex to the opposite in six moves, exactly 2 must be in each dimension (x , y , and z). Thus, we can characterize the sequence of moves as the arrangement of six characters $xyyzzz$. The total number of different ways to permute this sequence, taking into account the repeated letters (indistinguishable moves), is $\frac{(2+2+2)!}{2!2!} = \boxed{90}$.
26. B. The chances are the same that Nitish will sit on the left or the right of Shrung. So the amount of total permutations is $5! = 120$, which we then divide by 2 to get a final answer of $\boxed{60}$.
27. D. $145 \equiv 2 \pmod{13}$. So $145^{89} \equiv 2^{89} \pmod{13}$. $2^6 = 64 \equiv -1 \pmod{13}$, so $2^{84} = (2^6)^{14} = (-1)^{14} = 1 \pmod{13}$. Then we multiply this by $2^5 = 32$ to get $145^{89} \equiv 2^{89} \pmod{13} \equiv 32 \pmod{13} \equiv 6 \pmod{13}$. For the second term, we must note that $3^3 = 27 \equiv 1 \pmod{13}$. $3^{2001} = (3^3)^{667} \equiv 1^{667} \pmod{13} \equiv 1 \pmod{13}$. Then we multiply by 3 to get $3^{2002} \equiv 3 \pmod{13}$. Thus the final sum is $6 + 3 = \boxed{9}$.
28. D. There are a total of $\binom{10}{6} = 210$ different sets possible. If the second smallest is 3, then one number is picked from the set $[1, 2]$, and 4 are picked from the interval $[4, 10]$. There are $\binom{2}{1}\binom{7}{4} = 70$ ways to do this. $\frac{70}{210} = \boxed{\frac{1}{3}}$.
29. C. First, we subtract the second equation from the first, giving $x - y = y^2 - x^2$. By using difference of squares on the right side, we get $-(y - x) = (y - x)(x + y)$. Bringing the left side over, this becomes $(y - x)(x + y) + (y - x) = 0$, and then we can factor out $y - x$ to get $(y - x)(x + y + 1) = 0$. In other words, there are two possibilities: either $y - x = 0$ or $x + y + 1 = 0$. In the first case, $y = x$, so the equations both simplify to $y + 16 = y^2$ or $y^2 - y - 16 = 0$. Using quadratic formula on this, the positive solution is $\frac{1 + \sqrt{65}}{2}$. In the second case, $x + y + 1 = 0$ and $x = y^2 - 16$, so $(y^2 - 16) + y + 1 = 0$ and $y^2 + y - 15 = 0$. Solving with quadratic formula again, we get $y = \frac{-1 + \sqrt{61}}{2}$. These are the only two positive roots, and their sum is $\frac{\sqrt{61} + \sqrt{65}}{2}$. Thus, the answer is $61 + 65 = \boxed{126}$.
30. D. Let the second largest root be a , and the common ratio of the geometric series be r . By the definition of a geometric series, the smallest root is then $\frac{a}{r}$ and the largest is ar . So if we use Vieta's rules on each coefficient we get the following set of equations:

$$\begin{aligned} \text{Sum: } & \frac{a}{r} + a + ar = 63 \\ \text{Two-at-a-Time: } & \frac{a}{r}(a) + \frac{a}{r}(ar) + a(ar) = \frac{a^2}{r} + a^2 + a^2r = c \\ \text{Product: } & \left(\frac{a}{r}\right)(a)(ar) = a^3 = 1728 \end{aligned}$$

Since $a^3 = 1728$, then $a = 12$. Then, we see that the second equation is simply a times the first, so $c = 63a = 63(12) = \boxed{756}$.