QUESTION 1

Nihar's calculator, the DevSach P90X, has a tag that claims the number of hours it lasts on a full charge follows a Normal distribution with a mean of 30 hours and a standard deviation of 3 hours. Let:

A = the probability that the calculator lasts exactly 600 minutes

B = the probability that the calculator lasts less than 16 hours or more than 1 day

C = the probability that the calculator lasts between 13 and 21 hours

D = the hour value for which the probability of the calculator lasting longer than this value is 0.15

Round each answer to 4 decimal places, and find $A + B + C + D$.

QUESTION 2

Gabie does not enjoy probability. Currently, she is attempting to process data she collected on the discrete random variable X, for which the probability distribution is shown below.

x	3	12	23	5	14	10
$P(X = x)$	0.12	0.32	0.21	.07	0.18	0.10

Using Gabie's data, let:

$$\begin{aligned} A &= \text{the value of } E(X) \\ B &= \text{the value of } Var(X) \\ C &= \text{the value of } E(X^2) \\ D &= \text{the value of } \sigma(X) \end{aligned}$$

Round each answer to 2 decimal places, and find $A + B + C + D$.

QUESTION 3

The students at Fairview must choose to work with a pair of tutors made up of 1 female and 1 male. Below is a frequency table showing the number of students who chose each pair.

Person	Deekshita	Sanjita	Tanvi
Rayyan	180	255	200
Rohan	165	250	300
Sina	15	80	20

Assume the event of a Fairview student selecting a particular tutor is denoted by that tutor's name (i.e., $P(Rayyan) =$ the probability of Rayyan being selected as a tutor). If a Fairview student is selected uniformly at random, let:

$$\begin{aligned} A &= P(Sina|Tanvi) \\ B &= P((Rayyan' \cap Sanjita) \cup (Rohan \cap Tanvi)) \\ C &= P((Deekshita \cap Sina) \cup (Deekshita \cap Rohan)) \\ D &= P((Rohan \cap Sanjita) \cup Rayyan) \end{aligned}$$

Find the exact value of $A + B + C + D$.

QUESTION 4

The probability density function of a continuous random variable X is given by $f(x) = \sin x$, for $0 \leq x \leq \frac{\pi}{2}$. Let:

$$A = \text{the difference between the least and greatest values of } X$$
$$B = \text{the mode of the distribution of } X$$

The probability density function of a continuous random variable X is defined by the curve of the graph of $\pi x^2 + 4\pi y^2 - 6\pi x + 9\pi - 4 = 0$ that falls within the first quadrant. Let:

$$C = \text{the upper bound of the distribution of } X$$
$$D = \text{the median of the distribution of } X$$

Find the exact value of $AB + C - D$. Express your answer as an improper fraction with a rational denominator.

QUESTION 5

Karthik is at his fort in the woods, but cannot find his way back home. There are 4 paths he can take. The first path will lead him back to the fort in 4 hours. The second path will lead him home in 3 hours; the third path will lead him to the second path in 1 hour, at which point he can choose to go in one of two directions, with one leading him home in 1 hour and the other leading him back to the fort in 2 hours. The fourth path will magically transport Karthik to his house in a negligible amount of time. Assume that at all times, Karthik is equally likely to choose between the paths/directions available to him. Let:

A = the expected length of time, in hours, it takes Karthik to return home, assuming this time is finite

B = the variance of the length of time, in hours, it takes Karthik to return home, assuming it always takes him a finite amount of time

Find the exact value of $A + B$.

QUESTION 6

Mr. Reedy gives his IB Physics SL students a test on uncertainty. He finds that the scores are normally distributed with a mean of 45 and a standard deviation of 13. He curves the test scores so that they follow a normal distribution with a mean of 85 and a standard deviation of 7. The linear function $f(x)$ takes an original score and outputs a curved score. Let:

$$\begin{aligned} A &= \text{Vishnu's original score, if he earned a 78 after the curve} \\ B &= \text{the slope of } f(x) \\ C &= \text{the } y\text{-intercept of } f(x) \\ D &= \text{the } x\text{-intercept of } f(x) \end{aligned}$$

Round each answer to 2 decimal places, and find $A + B + C + D$.

QUESTION 7

Krunchy Kyle is a world renown peanut manufacturer. Krunchy Kyle claims that the mean of the number of peanuts in each bag it sells is 500 with a standard deviation of 30 peanuts. Puneet decides to test this claim and samples 400 bags of Krunchy Kyle peanuts, calculating a sample mean of 480 peanuts per bag. He conducts a one-tailed Z-test to decide whether or not Krunchy Kyle averages less peanuts per sold bag than it claims. He sets $\alpha = 0.05$ before he begins testing. Let:

$$
\begin{aligned}
A &= \text{1 if Puneet should reject the null hypothesis of his test, 2 otherwise} \\
B &= \text{the true population mean, if the null hypothesis in Puneet's test is false and } \beta = 0.2 \\
C &= \text{the standard deviation of the sampling distribution of the mean} \\
D &= \text{the value of Puneet's test statistic}
\end{aligned}
$$

Round each answer to 2 decimal places, and find $A + B + C + D$.

QUESTION 8

Josh loves eclipses. In recent years, scientists have found that the probability that a new moon coincides with a total solar eclipse is 0.042. When this coincidence actually occurs, Josh correctly predicts it 22% of the time. When this coincidence does not actually occur, Josh incorrectly predicts it will 65% of the time. Let $A =$ the probability that a total solar eclipse will coincide with the next new moon, given that Josh has predicted that it will not.

There are two urns, one with 10 balls numbered 1 to 10 and the other with 100 balls numbered 1 to 100. You randomly select a urn and randomly pull a ball from within it. Let $B =$ the probability that you took a ball from the urn with 10 balls, given you drew a ball with the number 3 on it.

Consider a tetrahedral die that is rolled twice. What is the probability that the number on the first roll is strictly higher than the number on the second roll? Note: a tetrahedral die has only four sides (1, 2, 3, and 4).

Keep each answer as an exact value, and find $A + B + C$ to 2 decimal places.

QUESTION 9

Ninju and Bryan have been addicted to the show *Brothers* recently. They believe that the proportion of each of their friend groups (they have no mutual friends) that likes *Brothers* is equal. To test this claim, they sample 100 of Ninju's 10,000 friends and 250 of Bryan's 7,500 friends. They find that 27% of Ninju's friends like *Brothers* and 36% of Bryan's friends like *Brothers*. A 5% level of significance is set prior to testing. Let:

$$A = \text{the absolute value of the test statistic}$$
$$B = \text{the P-value of the test}$$
$$C = \text{the standard error of the sampling distribution for the difference in the proportions}$$
$$D = \text{1 if Ninju and Bryan reject their null hypothesis, 2 otherwise}$$

Round each answer to 3 decimal places, and find $A + B + C + D$.

QUESTION 10

The following statements have point values indicated by the numbers in the parentheses to the left of each statement. Starting with 0, add the point value of each true statement, and subtract the point value of each false statement.

(13) The area under the Gaussian distribution is 1.

(-14) The Empirical Rule states that the area under the Normal distribution between -2 and 2 is approximately 0.95.

(-8) The Law of Large Numbers states that as the sample size tends to infinity, the distribution of the sample means approaches the Normal distribution.

(15) Simpson's Paradox is a phenomenon in which a trend across data sets is seemingly reversed when the data sets are combined.

(11) To increase the power of a significance test, α and σ can be increased.

(18) Two events A and B are independent if and only if $P(A \cap B) = 0$.

(5) If an observation is at the n^{th} percentile of data, then n percent of observations are less than or equal to that observation.

(-16) For any sets A_1, A_2,...,A_n, $(A_1 \cup A_2 \cup \ldots \cup A_n)^c = {A_1}^c \cap {A_2}^c \cap \ldots \cap {A_n}^c$.

What is the final number of points?

QUESTION 11

Consider two independent random variables X and Y:

$$\mu_X = 23,\ \sigma_X = 3,\ \mu_Y = 12,\ \sigma_Y = 4$$

Let:

$$\begin{aligned} A &= \mu_{3X+4Y} \\ B &= \sigma_{X \pm Y} \\ C &= \sigma^2_{X+5} \end{aligned}$$

Consider two positively correlated random variables W and Z:

$$r^2 = 0.0576,\ \mu_W = 120,\ \sigma_W = 10,\ \mu_Z = 12,\ \sigma_Z = 5$$

Let:

$$D = \sigma_{W+Z}$$

Find the exact value of $A + B + C + D$.

QUESTION 12

The members of Rickards Mu Alpha Theta have just received the 807 "Rickards Invitational" pencils they purchased. Of these pencils, exactly 1 will leave a glittery residue on the hand of the person who uses it. The residue is neither visible nor felt until 2 hours after a person has held it. Unfortunately, the Rickards MAO members only have 2 hours and 1 minute before the pencils must be passed out! 10 Rickards students will attempt to find the "residue" pencil. In how many distinct ways can the 10 students test the pencils to find the "residue" pencil and remove it from the rest of the pencils in time to pass the pencils out (express your answer in terms of factorials)? Note: each person will use at least 1 pencil, it takes a negligible amount of time to use a pencil, the pencils are distinguishable, and once the "residue" pencil has been discerned, it takes a negligible amount of time to remove it from the bunch.

QUESTION 13

A = the variance of a geometric distribution with probability of success $\frac{1}{3}$

B = the mean of a binomial distribution with probability of failure $\frac{3}{4}$ and 92 trials

C = the probability that it takes John more than 2 shots to score a goal in soccer, given he has a $\frac{1}{5}$ probability of scoring on each shot he takes

D = the standard deviation of a binomial distribution with probability of success $\frac{2}{3}$ and 9 trials

Find the exact value of $A + B + C + D$. Express your answer as an improper fraction.

QUESTION 14

At a particular school, the senior class is polled about the academic clubs they are active in. It is found that 80 seniors are in Debate Club, 90 are in Math Club, and 60 are in Chess Club. Furthermore, 17 seniors are in both Chess and Math Club, 13 are in both Debate and Math Club, and 14 are in both Chess and Debate Club. 3 seniors are in all three aforementioned clubs. There are 500 seniors in total at this school. Let:

$$A = \text{the number of seniors in Math Club only}$$
$$B = \text{the number of seniors in both Chess and Debate Club, but not Math Club}$$
$$C = \text{the number of seniors in two clubs, but not three}$$
$$D = \text{the number of seniors that participate in no clubs}$$

Find $A + B + C + D$.