For all questions, answer choice (E) NOTA means that none of the given answers is correct. The $\sqrt{-1} = i$. Good Luck!

1. The probability that Anirudh gets attendance failure in a class is 0.72. The probability that Siddharth gets attendance failure in a class is 0.32. The probability that Siddharth gets attendance failure in a class given that Anirudh does not get attendance failure in that class is 0.42. What is the probability that both Anirudh and Siddharth get attendance failure in a class?
(A) $\frac{478}{625}$ (B) $\frac{253}{1250}$ (C) $\frac{147}{625}$ (D) $\frac{1103}{1250}$ (E) NOTA

2. Alex has been trying to master the art of optimizing and utilizing his brain capacity to its fullest potential to increase his knowledge. To help model his maximization abilities, Alex would like to know what the maximum value of $\frac{x^3y^3z^3}{4}$ is from the equation $6x + 3y + 8z = 36$. Which of the following is the number he should come up with?
(A) 628 (B) 128 (C) 576 (D) 432 (E) NOTA

3. Rohan is tired of living in the shadow of Alex and decides to improve his maximization abilities as well to catch up to Alex's level of knowledge. He would like to know what the maximum possible value of $3x^2 + 5y^2 + 6z^2$ is given that $x^4 + y^4 + z^4 = 3$. Which of the following is the number he should come up with?
(A) $\sqrt{210}$ (B) 14 (C) 21 (D) $2\sqrt{55}$ (E) NOTA

4. Josh likes to throw up gang signs at people and one of them is referred to as the "Korean heart". Recognizing that cardioids share a similarity to hearts, he sought to determine the longest distance between two intersection points of the polar curves $r = 2 + 2\cos(\theta)$ and $r = 2 + 2\sin(\theta)$. What is this distance?
(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) NOTA

5. Tanvi is notorious for having a very loud and problematic voice. Her vocal range starts from 0 dB and has a maximum of the same value as the amplitude of the function $f(x) = 130\tan(x - \pi) + 10$. What is the loudest that Tanvi's voice can reach? (Answers are in dB)
(A) 120 (B) 130 (C) 140 (D) 150 (E) NOTA

6. In regular pentagon $ABCDE$, given that $\overline{AD} = 12$ and $\tan(36^\circ) = \sqrt{5 - 2\sqrt{5}}$, what is $[ABCDE]$?
(A) $90\sqrt{5 - 2\sqrt{5}}$ (B) $108\sqrt{5 - 2\sqrt{5}} + 9\sqrt{10 - 4\sqrt{5}}$ (C) $144\sqrt{5 - 2\sqrt{5}} + 18\sqrt{5 - 4\sqrt{5}}$
(D) $72\sqrt{5 - 2\sqrt{5}} + 18\sqrt{10 - 2\sqrt{5}}$ (E) NOTA

7. What is the sum of the fourth powers of roots for the equation $3x^3 + 9x^2 + 8 = 0$.
(A) $\frac{31}{3}$ (B) 113 (C) -81 (D) -35 (E) NOTA

8. Sanjita feels a personal connection to match paired t-tests. She wants to determine the t-value for a given set of data. This value can be represented by the magnitude of the vector that is constructed from $< 7, 5, 4 > \times < -3, -2, 3 >$. What t-value should she come up with?
(A) -19 (B) $\sqrt{1553}$ (C) $\sqrt{1619}$ (D) 14 (E) NOTA

9. Suppose two square matrices A and B are defined such that $A + B = AB$. How many of the following statements must be true?

I. $AB = BA$

II. $AB^{-1} = B^{-1}A$

III. All elements in A are equal to 0 or all elements in B are equal to 0.

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

10. What is the height of the polar curve $r = 4\sin(11\theta)$? Use the approximation of $\sin\frac{3\pi}{22} \approx 0.415$. Answers are rounded to the nearest tenths place. (Height is defined as the positive difference between the maximum and minimum y-coordinate of a graph, considering the rectangular form of the points on the polar curve.)
(A) 7.6 (B) 7.8 (C) 7.9 (D) 8.0 (E) NOTA

11. Rayyan is training harder than ever in his quest to play in the NFL. If he can throw 2 balls in a seconds, where a is the period of the function $f(x) = 3\cos(\pi x)$, how many balls can he throw in 1 minute?
(A) 60 (B) 120 (C) 180 (D) 240 (E) NOTA

12. The vector $< 1, 2 >$ is projected onto both $< 15, 20 >$ and $< 7, 24 >$. The two vectors resulting from these projections are then added together, yielding the resultant vector $< k, 2k >$ where k is a scalar. What is k?
(A) $\frac{48}{25}$ (B) $\frac{49}{25}$ (C) $\frac{243}{125}$ (D) $\frac{242}{125}$ (E) NOTA

13. Let $\vec{r} =< 5, 8, 7 >$, $\vec{y} =< 1, 2, 3 >$, and $\vec{d}$ be the projection of $\vec{y}$ onto $\vec{r}$. What is the smaller angle formed between $\vec{r}$ and $\vec{d}$?
(A) $\sin^{-1}(\frac{\sqrt{483}}{23})$ (B) $\cos^{-1}(\frac{\sqrt{483}}{23})$ (C) $\tan^{-1}(\frac{\sqrt{42}}{21})$ (D) $\sin^{-1}(\frac{\sqrt{399}}{21})$ (E) NOTA

14. Which of the following is equivalent to $(4\sqrt{3} - 4i)^3$?
(A) $256i$ (B) $256\sqrt{2} + 256i\sqrt{2}$ (C) $-256\sqrt{2} - 256i\sqrt{2}$ (D) $256\sqrt{3} - 256i$ (E) NOTA

15. Express $0.\overline{2018}_9$ in base 10.
(A) $\frac{295}{1312}$ (B) $\frac{2018}{9999}$ (C) $\frac{2}{9}$ (D) $\frac{89}{441}$ (E) NOTA

16. Sanjita's favorite shape is a triangle. She decides to make a bracelet in the shape of one and "names it" $\triangle RYD$ (vertices labeled respectively). She knows that $\angle RYD = 60°$ and $\angle YRD = 40°$. Thinking in a pretentious manner, she contructs a point S on $\overline{RD}$ such that $\angle DYS = 40°$ and a point C on $\overline{RY}$ such that $\angle YDC = 70°$. Given that $\overline{YS}$ and $\overline{DC}$ intersect at point X, what is the measure of the smaller angle formed by the intersection of $\overrightarrow{RX}$ and $\overline{YD}$ that lies within the triangle?
(A) 50° (B) 60° (C) 70° (D) 80° (E) NOTA

17. Find the number in row 2 and column 3 of the inverse of the following 4x4 matrix:

$$\begin{bmatrix} 3 & -1 & 1 & -2 \\ -2 & 2 & -3 & 2 \\ 0 & 2 & 1 & -2 \\ 3 & -3 & 2 & 0 \end{bmatrix}$$

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$ (E) NOTA

18. Rohan famously has possession of a case for his banana, and now wants to have a case that will accommodate for any exotic fruit. He wants the case to be a 3-D prism with two bases, each base having an area represented by the function $f(x) = 5 + 5x + 5x^2 + 5x^3 + 5x^4 + 5x^5$ graphed in the Argand plane with the corresponding roots connected. Given that the height of the figure is 6, what is its volume?
(A) $\frac{375\sqrt{3}}{2}$ (B) $9\sqrt{3}$ (C) $225\sqrt{3}$ (D) $\frac{15\sqrt{3}}{2}$ (E) NOTA

19. What is the remainder when 123^{131} is divided by 17?
(A) 4 (B) 10 (C) 13 (D) 11 (E) NOTA

20. What is the sum of the elements of X^{-1} where X=

$$\begin{bmatrix} 3 & -2 & 2 \\ -3 & 1 & 1 \\ 2 & 2 & -3 \end{bmatrix}$$

(A) $\frac{5}{17}$ (B) $\frac{29}{17}$ (C) $\frac{61}{17}$ (D) $\frac{-5}{17}$ (E) NOTA

21. Solve for x:

$$\log_4 x = \log_{16}\left(\sin(163^\circ) \times \tan(28^\circ) \div \cos(17^\circ) + \sin(28^\circ) \div \cos(332^\circ) + \tan(197^\circ) + \log_{16}(2e^{\frac{5i\pi}{2}})^4\right)$$

Hint: If $\alpha + \beta = \frac{\pi}{4}$, then $(\tan\alpha + 1)(\tan\beta + 1) = 2$.

(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) NOTA

22. What is the product of the solutions to the equation $\sin(2x) - 4\cos(x) - \sin(x) + 2 = 0$ where $-\pi \leq x \leq \pi$?

(A) $\frac{-\pi^2}{12}$ (B) $\frac{-\pi^2}{9}$ (C) $\frac{-\pi^2}{18}$ (D) $\frac{-\pi^2}{3}$ (E) NOTA

23. Which of the following is an eigenvector of $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$?

(A) $\begin{bmatrix} 5 \\ \frac{5}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{3}{2} \\ -\frac{3}{4} \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (E) NOTA

24. For the conic of the equation $16x^2 - 9y^2 - 96x + 36y - 36 = 0$, the asymptotes are represented by the equation $y = \pm\frac{A}{B}(x - C) + D$ What is the value of $A + B + C + D$?

(A) 12 (B) 18 (C) 24 (D) 15 (E) NOTA

25. If $\vec{v} =< 3, -2, -5 >$ and $\vec{w} =< -4, -1, 7 >$, what is $\vec{v} \cdot \vec{w}$?

(A) 58 (B) -45 (C) -49 (D) -8 (E) NOTA

26. What is the cylindrical coordinate representation of the Cartesian point $(-6\sqrt{3}, -6, 6)$?

(A) $(6\sqrt{5}, 210^\circ, 6)$ (B) $(6\sqrt{5}, 240^\circ, 12)$ (C) $(12, 240^\circ, 6\sqrt{5})$ (D) $(12, 210^\circ, 6)$ (E) NOTA

27. How many solutions are there to the equation $\sin(-2x) + \sin(-4x) = 0$ where $-2\pi \leq x < 0$?

(A) 4 (B) 6 (C) 8 (D) 9 (E) NOTA

28. Evaluate:

$$\sum_{n=1}^{\infty} \frac{4n^2}{3^{n+1}}$$

(A) $\frac{7}{4}$ (B) $\frac{3}{2}$ (C) $\frac{9}{4}$ (D) 2 (E) NOTA

29. In which quadrant would the graph of $-4(-100 + 155i)^{20}$ be in the complex plane?

(A) I (B) II (C) III (D) IV (E) NOTA

30. Rayyan, the energetic child, can't stop moving. His friends document his movement on a Cartesian plane over the course of 10 seconds. The resulting pathway he makes is in the shape of a polynomial with an odd degree, all real and non-repeating roots, and one that crosses the x-axis exactly 5 times. The following points lie on the graph: $(0, 6)$, $(1, -3)$, $(2, -5)$, $(3, 10)$, $(4, 13)$, and $(5, 4)$. What is the y-coordinate for the point at $x = 6$?
(A) 31 (B) 17 (C) 80 (D) 122 (E) NOTA