For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Find the exact value of $\sin\left(\frac{\pi}{12}\right)$.

   (A) $\frac{\sqrt{6}-\sqrt{3}}{4}$ (B) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (C) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (D) $\frac{\sqrt{6}-2\sqrt{3}}{4}$ (E) NOTA

2. Find the area enclosed within both of the shapes defined by the polar equations:

$$\begin{aligned} r &= 2\cos(\theta) \\ r &= 3+\cos(\theta) \end{aligned}$$

   (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) π (D) $\frac{3\pi}{2}$ (E) NOTA

3. Nihar Kommu really likes dogs. In fact, his love for dogs can be modeled by the function $f(t)=\left(\frac{2+t}{t}\right)^t$, where t is time measured in seconds. Find:

$$\lim_{t\to\infty} f(t)$$

   (A) ∞ (B) 1 (C) e (D) $\ln(2)$ (E) NOTA

4. Bryan has just turned 417 years old! He has a cake, which is infinitely big and described by the Cartesian plane. He cuts it according to the polar equation:

$$r=\frac{1+6\cos(\theta)}{8+4\sin(\theta)}$$

   How many different pieces of cake does he have?

   (A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

5. Find the sum of the coefficients in the expansion of $(2x+3y+z)^4$.

   (A) 864 (B) 1296 (C) 1728 (D) 65536 (E) NOTA

6. Brandon must fill in for Nihar whenever he is cannot attend a meeting. To help Brandon prove his worth, find an equation of a plane perpendicular to the plane $3x+4y-z=9$ and containing the points $(3,5,2)$ and $(1,4,6)$.

   (A) $3x+2y+z=21$ (B) $22x-16y+7z=0$ (C) $11x-6y+4z=11$ (D) $3x-2y+z=1$ (E) NOTA

7. Find the square of the minimum distance between any two roots of $x^8=256$ in the complex plane.

   (A) 16 (B) $4\sqrt{2}$ (C) $16-4\sqrt{2}$ (D) $8-4\sqrt{2}$ (E) NOTA

8. Evaluate the indefinite integral:

$$\int_{-\infty}^{\infty} \frac{-1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx$$

   (A) 1 (B) -1 (C) e (D) $\frac{1}{\sqrt{2e\pi}}$ (E) NOTA

9. The instantaneous rate of change at a point $(x, f(x))$ can be defined as:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Given this information, find the instantaneous rate of change of $f(x) = \dfrac{x-5}{x-11}$ at $x = 2$.

(A) $\dfrac{1}{3}$ (B) $-\dfrac{1}{27}$ (C) $\dfrac{2}{27}$ (D) $\dfrac{1}{9}$ (E) NOTA

10. Find the element in the second row and third column of the inverse of the following matrix:

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 2 \\ 3 & 7 & 9 \end{bmatrix}$$

(A) $-\dfrac{5}{21}$ (B) 0 (C) $\dfrac{5}{21}$ (D) -5 (E) NOTA

11. Kira is very good at Precalculus, and has given you this problem. Simplify where defined:

$$\frac{\sqrt{2 - 2\cos(2x)} - \tan(\frac{x}{2})\sin(x) + 2\sin^2(\frac{x}{2})}{2}$$

(A) $\sin(x)$ (B) $\cos(x)$ (C) $\tan(x)$ (D) $2\cos(x)$ (E) NOTA

12. Let $f(x) = x^2 - 4x + 4$ and the function $g(x)$ be defined as follows:

$$g(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases}$$

Find:

$$\lim_{x \to 2} g(f(x))$$

(A) Does Not Exist (B) 0 (C) 1 (D) -1 (E) NOTA

13. Find the sum of the squares of the distinct roots of $f(x) = x^4 - 2x^3 - 19x^2 + 68x - 60$.

(A) 46 (B) 42 (C) 38 (D) 34 (E) NOTA

14. Jason is going insane and is traveling in a circle of radius 4 units at a rate of $\frac{\pi}{3}$ radians per second, beginning at time $t = 0$ seconds. Michael is standing 8 units from Jason and 12 units from the center of Jason's circle at $t = 0$. Find the distance, in units, between Michael and Jason at time $t = 20$ seconds.

(A) $2\sqrt{13}$ (B) $2\sqrt{7}$ (C) $4\sqrt{13}$ (D) $4\sqrt{7}$ (E) NOTA

15. Jessica becomes curious of the situation between Jason and Michael described in the previous problem, and wonders how their distance changes. Find the rate of change of the distance between the two at time $t = 20$.

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{2\pi\sqrt{13}}{13}$ (C) $\dfrac{2\pi\sqrt{39}}{13}$ (D) $\dfrac{2\pi\sqrt{39}}{39}$ (E) NOTA

16. Evaluate:

$$\sum_{i=1}^{\infty} \frac{1}{i}$$

(A) Diverges (B) 2 (C) 4 (D) $\ln(2)$ (E) NOTA

17. Evaluate:

$$\sum_{i=1}^{\infty} \frac{i^2}{2^i}$$

(A) Diverges (B) 4 (C) 6 (D) 16 (E) NOTA

18. Evaluate:

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

(A) Diverges (B) 1 (C) 2 (D) e (E) NOTA

19. Joshua dislikes summations like the ones just given and would much rather be doing Geometry. He thinks of a number a, the number of faces in a dodecahedron. Find the volume of a regular octahedron with side length equal to a.

(A) $1728\sqrt{2}$ (B) $1296\sqrt{2}$ (C) $864\sqrt{2}$ (D) $432\sqrt{2}$ (E) NOTA

20. Let triangle ABC be an equilateral triangle with a point D in the interior of the triangle such that $\angle ADB = 110^\circ$ and $\angle BDC = 120^\circ$. If the line segments $\overline{AD}$, $\overline{BD}$, and $\overline{CD}$ are taken to form a new triangle, find the smallest angle in this new triangle.

(A) 60° (B) 50° (C) 45° (D) 40° (E) NOTA

21. Alex and Lindsay are big fans of Rickards. Find the number of distinct ways to arrange "RICKROCKS" if an "R" must come first.

(A) 40320 (B) 20160 (C) 10080 (D) 5040 (E) NOTA

22. Let:

$$\begin{aligned} x(t) &= 4\csc(t) + 3 \\ y(t) &= 3\cot(t) + 5 \end{aligned}$$

Find the eccentricity of the conic defined by these parametric equations in the xy-plane.

(A) $\frac{5}{3}$ (B) $\frac{5}{4}$ (C) $\frac{\sqrt{7}}{3}$ (D) $\frac{\sqrt{7}}{4}$ (E) NOTA

23. Cindy is marking off her territory with 12 miles of fencing. She wants to create a triangular enclosure with one side against the completely linear Crystal River (eliminating the need for fencing on this side). The last vertex of the triangle must be 4 miles from the river. Find the maximum possible area of the enclosure that suits Cindy's needs.

(A) $8\sqrt{5}$ (B) $8\sqrt{3}$ (C) 16 (D) $12\sqrt{3} - 12$ (E) NOTA

24. Anirudh has a great supporter in Kev. Kev is at the point $(6, 2, 9)$ and wants to meet up with Anirudh, who is at the point on the line $\frac{x-3}{2} = \frac{y+5}{3} = \frac{z-1}{5}$ that is closest to Kev. What is the shortest possible distance from Kev to Anirudh?

(A) $\sqrt{122}$ (B) $\frac{7\sqrt{114}}{3}$ (C) $\frac{7\sqrt{38}}{38}$ (D) $\frac{7\sqrt{114}}{38}$ (E) NOTA

25. Andrew is very good at probability, and his favorite number is 41. Find the probability that a 5-digit number whose digits sum to 41 is divisible by 11.

(A) $\frac{2}{15}$ (B) $\frac{11}{36}$ (C) $\frac{3}{35}$ (D) $\frac{6}{35}$ (E) NOTA

26. Evaluate:

$$\int_1^2 \frac{x-1}{2x-1} dx$$

(A) $\frac{1-\ln(3)}{4}$ (B) $\frac{2-\ln(3)}{4}$ (C) $\frac{4-\ln(3)}{4}$ (D) 1 (E) NOTA

27. A cone of radius 10 centimeters and height $\frac{12}{\pi}$ centimeters is lowered point first at a rate of 1 centimeter per second into a cylinder of radius $10\sqrt{2}$ centimeters and height $\frac{80}{\pi}$. The cylinder is partially filled with water, up to a height of $\frac{40}{\pi}$. How fast, in centimeters per second, is the water level rising at the instant the cone is completely submerged?

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{\pi}{12}$ (D) $\frac{72\pi}{5}$ (E) NOTA

28. In memory of Aman Raj, cipher:

$$\lim_{x\to\infty} \left(\frac{3^x + 4^x}{4}\right)^{\frac{1}{x}}$$

(A) 4 (B) 1 (C) 0 (D) ∞ (E) NOTA

29. Evaluate the following indefinite integral, given the assumption that the functions are defined:

$$\int_0^{e^{-\pi}} \cos(\ln(x)) + \sin(\ln(x)) dx$$

(A) e (B) 1 (C) $\sin(1)$ (D) 0 (E) NOTA

30. Congratulations on reaching the end! Here's another question as a reward. Find the volume when the region bounded by:

$$f(x) = \frac{4\sqrt{-x^2 - 10x - 16}}{3}$$
$$g(x) = 0$$

is rotated around the y-axis.

(A) $60\pi^2$ (B) $45\pi^2$ (C) $30\pi^2$ (D) 60π (E) NOTA