QUESTION 1

$\triangle ABC$ is an isosceles triangle with side $\overline{AC}$ as its base. $\triangle ABC$ has $\overline{BD}$ as the perpendicular bisector of $\overline{AC}$, with D on $\overline{AC}$. $\overline{AE}$ is the angle bisector of $\angle A$ with E on $\overline{BC}$. $\overline{AE}$ and $\overline{BD}$ intersect at F. $\angle AFD + \angle BFE + \angle DFE = 240°$ and $\overline{BF}$ is 8 units.
Let:

$$\begin{aligned} A &= \text{the length of } \overline{AB} \\ B &= \text{the length of } \overline{BE} \\ C &= \text{the length of the inradius of } \triangle ABC \\ D &= \text{the length of the circumradius of } \triangle ABC \end{aligned}$$

Find $AB + C + D$.

QUESTION 2

Vishnav (A), Russell the human (B), Russell the dog (C), and Rufus the dog (D) are trying to meet each other to "hang out". However, they face a series of obstacles and need your help! (Each part should be solved separately.)

A is located at point $(5, 1)$, B is located at point $(7, 5)$, D is located at point $(13, -3)$, and ... Russel the dog forgot where he was! Let W be the sum of the coordinates of Russell the dog's position, if $ABCD$ forms a rectangle.

A, B, C, and D are located on circle O such that $\overline{AC} \perp \overline{BD}$ and $\overline{AC}$ and $\overline{BD}$ intersect at E. $\overline{AE} = 15$, $\overline{BE} = 6$, and $\overline{EC} = 8$. They all agreed that they wanted to meet at a location equidistant to all four of them, as each is too lazy to walk more than they need to. Let X be how far each of them walk.

Russell the dog knows that $ABCD$ forms an orthogonal quadrilateral. He also knows that $\overline{AB} = 10$, $\overline{BC} = 12$, and $\overline{CD} = 9$. Let Y be the length of $\overline{AD}$.

They finally decide to meet at the Park (E). $ABCD$ forms a rectangle with E located in the interior of the rectangle. Rufus knows that $\overline{AE} = 12$, $\overline{BE} = 10$, and $\overline{CE} = 6$. He doesn't know the distance he's supposed to travel, $\overline{DE}$. Let Z be the distance Rufus must travel to reach E.

Find $W + X^2 + Y^2 + Z^2$.

QUESTION 3

Let:

A	$=$	the sum of the acute angles in a regular 10-pointed star
B	$=$	the area of the circumcircle of a right triangle where the length of one median that extends to a leg of the triangle is 8 units, and the length of the median that extends to the other leg is 14 units
C	$=$	the sum of the squares of the diagonals of a parallelogram with side lengths 10 and 12
D	$=$	the volume of a regular tetrahedron with side length 12 units

Find $A + \dfrac{B}{\pi} + C + D$.

QUESTION 4

Let:

A = the geometric mean of 12, 14, 10, and 9

B = the area of a regular octagon with side length 8

C = the area of the annulus formed by two concentric circles, given that the radii of the outer and inner circles are 10 and 5 respectively

D = the ratio between a diagonal of a regular pentagon and one of its sides

Find $A + B + \dfrac{C}{\pi} + 2D$.

QUESTION 5

Let:

A = the length of the angle bisector $\overline{OM}$ in $\triangle LOG$, where $\overline{LO} = 5$, $\overline{OG} = 9$, and $\overline{LG} = 12$ and M is on side $\overline{LG}$

B = the area of the graph that satisfies the following equations: $y \leq 3 - 4|x - 2|$ and $y \geq 4|x - 2| - 5$

C = the lateral area of a cone with a diameter of 6 and a height of 4

D = the maximum number of distinct side lengths in an equiangular hexagon

Find $A^2 + B + \dfrac{C}{\pi} + D$.

QUESTION 6

Let:

A = the volume of a sphere circumscribed about a cube with side length 8

B = the height of the intersection of two ropes strung from the tops of two vertical poles to the bases of the other pole, one of height 30 and the other of height 40

Consider a convex quadrilateral with 3 side lengths 2, 2, and 7. Let C equal the number of distinct integer lengths possible for the fourth side.

Consider a cylinder of height 21 and radius $\frac{5}{2\pi}$. On the cylinder there is a one-dimensional ribbon that starts at one base, wraps around the cylinder 4 times and ends at the same location on the other base as on the first base. Let D be the length of the ribbon.

Find $\dfrac{A}{\sqrt{3}\pi} + B + C + D$.

QUESTION 7

Trapezoid $ABCD$ has $\overline{AB} \parallel \overline{CD}$, $\overline{AD} = 17$, $\overline{AB} = 7$, $\overline{BC} = 18$, and $\overline{CD} = 14$. The median of $ABCD$ is $\overline{XY}$, with X on side $\overline{AD}$, and Y on side $\overline{BC}$. The diagonals of the trapezoid intersect the median at points S and T.
Let:

A = the length of $\overline{ST}$

B = the length of the line segment that passes through the intersection of $\overline{AC}$ and $\overline{BD}$, is parallel to $\overline{CD}$, and has endpoints on $\overline{AD}$ and $\overline{BC}$

C = the length of the line segment that is parallel to $\overline{AB}$ and divides $ABCD$ into two quadrilaterals of equal area

D = the height of trapezoid $ABCD$

Find $6(A + B + CD)$.

QUESTION 8

Let:

A = the length of side $\overline{DO}$ where $\triangle DOG$ has median $\overline{DE}$ with E on side $\overline{OG}$, median $\overline{OF}$ with F on side $\overline{DG}$, $\overline{DG} = 16$ units, $\overline{OG} = 12$ units, and $\overline{OF}$ and $\overline{DE}$ perpendicular to each other

B = the difference in areas of two circles that have a common external tangent of length 75 units, a common internal tangent of length 13 units, and 85 units between their centers (Hint: There exists the Pythagorean Triple 13, 84, 85)

Find $A + B$.

QUESTION 9

Begin with a total of 2018. If the statement is true, add the value in the parentheses, and if the statement is false, subtract the value in the parentheses.

- (47) A conditional statement and its converse will not necessarily have the same truth value.
- (-23) A conditional statement and its contrapositive will always have the same truth value.
- (13) $[p \Leftrightarrow q]$ is the logical equivalent of [If $p \rightarrow q$ and if $q \rightarrow p$].
- (-10) A bicentric quadrilateral is a quadrilateral with two incircles.
- (109) All quadrilaterals are capable of tessellating a plane.
- (-37) Euler is considered the Father of Geometry.
- (11) 2017 is a prime number.

What is the final total?

QUESTION 10

Akash is secretly a professional Pokemon master. One day, when he was practicing throwing pokeballs (spheres), he was challenged by his nemesis, Vishnav, and to win the duel Akash has to answer the following questions. Help Akash win! Let:

A = the surface area of a pokeball if the diameter of the cross section parallel to the great circle of the pokeball 5 units above its great circle is 24 units

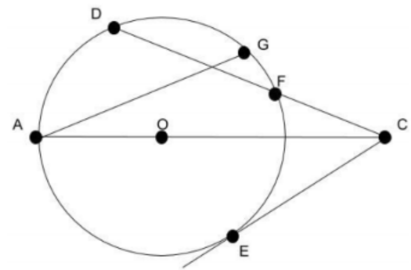
B = the radius of a pokeball if it was inscribed inside of a cone of radius 6 units and height 8 units

C = the area of the largest triangle that could fit inside of the positive trajectory of a pokeball, if the trajectory of the pokeball flying through the air could be modeled by $y = -2x^2 + 11x - 12$

D = the sum of twice the x-coordinate and thrice the y-coordinate of a pokeball's landing position, given that pokeball was thrown with starting position $(-4, 3)$, and it landed at the point equivalent to rotating its starting point 270° counterclockwise about the origin

Find $\sqrt{\dfrac{A}{\pi}} + B + 32C + D$.

QUESTION 11



Let:

A = the measure of major arc DAF in degrees, if $\angle DCA$ on circle O measures 33°, and $\angle DOA$ is equal to 4 times $\angle FOC$

B = the radius of circle O if $\overline{DC}$ intersects $\overline{AG}$ at H, I is on $\overline{AC}$, $\overline{GI} \perp \overline{AC}$, $\overline{DH}$ is 5 units, $\overline{HF}$ is 8 units, $\overline{AH}$ is 6 units longer than $\overline{GH}$, and minor arc ADG is 120°

C = the length of $\overline{FC}$ if $\overline{DC}$ intersects $\overline{AG}$ at H, $\overline{AH}$ is 15 units, $\overline{HG}$ is 2 units, $\overline{DH}$ is 6 units, and $\overline{CE}$ is $4\sqrt{5}$ units

D = the area of circle O if $\angle AGO$ is 30°, $\overline{AG} \parallel \overline{EC}$, and $\overline{EC} = 10\sqrt{3}$ units, where $\overline{CE}$ is tangent to circle O

Find $\sqrt{3}\left(A + B + C + \dfrac{D}{\pi}\right)$

QUESTION 12

Let:

$$\begin{aligned} A &= \text{the sum of the coordinates of the centroid of a triangle with points at (2,3), (12,12), and (7,6)} \\ B &= \frac{2}{3} + \cfrac{4}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \dots}}} \\ C &= \text{the length of the space diagonal of a rectangular prism with a length of 18, a width of 18, and a height of 9} \end{aligned}$$

Chanda is mad at Vishnav for stealing her journal, so she uses her super strength to take Vishnav and throw him at the ground. If Vishnav is 72 inches tall (starting height) and dimensionless, and Vishnav bounces back up to $\frac{2}{3}$ of the previous height he had and he continues bouncing for an infinite amount of time, let D be the total vertical distance Vishnav will travel, in inches.

Find $3(A + B + C + D)$.

QUESTION 13

Let:

$$
\begin{aligned}
A &= \text{the number of diagonals in a convex dodecagon} \\
B &= \text{the distance between the orthocenter and circumcenter of a triangle with side lengths of 13, 84, and 85} \\
C &= \text{the number of squares in a 5 by 5 grid} \\
D &= \text{the maximum number of pieces into which a circular pumpkin pie can be cut with only 7 cuts}
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 14

Let:

$$\begin{aligned} A &= \text{the inradius of a triangle with side lengths of 13, 14, and 15} \\ B &= \text{the area of a cyclic quadrilateral with side lengths of 2, 4, 8, and 6} \\ C &= \text{the number of prime factors of 2018} \\ D &= \text{the number of edges in a polyhedron with 2018 vertices and 50 faces} \end{aligned}$$

Find $A + B + C + D$.