For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. What is the derivative of $4x^2 + 3x + 17$ with respect to y at $x = 2$ if x and y are independent of each other?

(A) 19 (B) 0 (C) 11 (D) 17 (E) NOTA

2. Evaluate the following limit: $\lim_{x\to\infty} \frac{\sin^3 x}{x^3}$

(A) 0 (B) 1 (C) 3 (D) DNE (E) NOTA

3. What is the area of the region bounded by the curves $y = 4x - 30$ and $y = x^3 - 6x^2 - 9x + 12$?

(A) $-\frac{625}{2}$ (B) $\frac{625}{4}$ (C) $\frac{625}{2}$ (D) $\frac{125}{2}$ (E) NOTA

4. What is the derivative of the volume of a sphere with respect to the surface area when the radius is 6?

(A) 3 (B) 3π (C) 6 (D) 6π (E) NOTA

5. Evaluate: $\int \frac{d(x^3)}{dx} dx$

(A) $3x^2$ (B) x^3 (C) $\frac{x^4}{4}$ (D) 0 (E) NOTA

6. What is the volume of the shape created when the graph of $4x^2 - 16x + 9y^2 - 90y = -205$ is revolved around the line $y = 2x - 9$?

(A) $24\pi^2\sqrt{5}$ (B) $24\pi\sqrt{5}$ (C) $12\pi^2\sqrt{5}$ (D) $12\pi\sqrt{5}$ (E) NOTA

7. Evaluate: $\sum_{x=1}^{\infty} \left[\frac{d}{dx}\left(\frac{2}{x}\right)\right]$

(A) $\frac{\pi^2}{6}$ (B) $\frac{-\pi^2}{6}$ (C) $\frac{\pi^2}{3}$ (D) $\frac{-\pi^2}{3}$ (E) NOTA

8. Evaluate: $\lim_{x\to 0}(9x^2 + 6x + 1)^{\frac{1}{x}}$

(A) 1 (B) ∞ (C) e^6 (D) e^3 (E) NOTA

9. What is the volume of Gabriel's Horn divided by the surface area of Gabriel's Horn?

(A) 0 (B) 1 (C) 2 (D) DNE (E) NOTA

10. The average value of the function $y = |2\sin(x) + 1|$ over the interval $[0, 2\pi]$ can be expressed in the form $\frac{a\sqrt{b}}{\pi} + \frac{1}{c}$. Find $a + b + c$.

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

11. Evaluate to 2 significant figures: $\int_{-1}^{1} \frac{1}{\sqrt{\pi}} e^{-x^2} dx$

(A) 0.50 (B) 0.68 (C) 0.75 (D) 0.82 (E) NOTA

12. An inverted cone is filled with water. The water is evaporating at a rate proportional to the exposed circle of water. If when the radius is 3 feet, the radius is decreasing at a rate of 6 inches per second, and the volume is decreasing at a rate of 36π cubic feet per second, what is the ratio between the height and the radius?

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

13. A open box is being formed out of a rectangle of cardboard by removing squares from each of the corners and folding the resulting net. If the dimensions of the original rectangle of cardboard is 5 by 8, what is the maximum volume box that can be created?

(A) 18 (B) 20 (C) 24 (D) 30 (E) NOTA

14. What is the first positive integer n such that the region bounded by $y = 2\sin(x)$ and the x-axis from $2n$ to $2n+1$ strictly contains 3 lattice points? (Strict containment means the points are inside the area and not on the boundary)

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

15. Start with 7. For each correct statement, multiply by 6 and add 12. For each incorrect statement, multiply by x^2 and take the derivative with respect to x. What is the final result when x is 3?

I. $\displaystyle\int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \tan(x)dx = 0$
II. If a polynomial has n extrema, the degree of the polynomial is the opposite parity as n.
III. The integral of a function is a function on the domain of the original function.

(A) 2340 (B) 2916 (C) 3456 (D) 4536 (E) NOTA

16. What is the average value of the derivative of $y = x^4 \sinh(\ln(x))$ over the interval [0, 2]?

(A) 4 (B) 6 (C) 8 (D) 12 (E) NOTA

17. Find the sine of the acute angle between the intersection of the tangent lines to $y = 3e^{2x}$ and $y = \dfrac{\sin\left(\frac{2\pi}{3}e^{-x}\right)}{4\pi}$ at $x = \ln(2)$.

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1 (E) NOTA

18. Evaluate $\displaystyle\lim_{i\to\infty}\sum_{k=1}^{\infty}\left(\frac{1}{i}+\frac{k}{2i^2}+\frac{k^2}{4i^3}+\frac{k^3}{8i^4}+\ldots+\frac{k^n}{2^n i^{n+1}}+\ldots\right)$

(A) $\ln 2$ (B) $\ln 3$ (C) $2\ln 2$ (D) $2\ln 3$ (E) NOTA

19. Which of these functions grows the fastest?

(A) $y = x!$ (B) $y = \left(\frac{x}{3}\right)^n$ (C) $y = (e^x)^x$ (D) $y = x^x$ (E) NOTA

20. What is the eccentricity of the conic formed by the solution of the differential equation $\dfrac{dy}{dx} = \dfrac{2x+1}{3y-2}$ if when $x = 1.3$, $y = 12$?

(A) $\frac{\sqrt{15}}{3}$ (B) $\frac{\sqrt{10}}{2}$ (C) $\frac{\sqrt{15}}{5}$ (D) $\frac{\sqrt{10}}{3}$ (E) NOTA

The following information will be used for questions 21 and 22.

In computer science, complexity is a method of determining the speed of an algorithm with relation to the number of inputs. It is calculated for a function $f(n)$ by creating a function $O(n)$ such that there exists a constant c, $\lim_{n\to\infty} cO(n) = \lim_{n\to\infty} f(n)$. For example, $f(n) = 3n+2$ is $O(n)$, and $f(n) = 7n^2 + 17n - 4$ is $O(n^2)$.

21. Which of the following is the complexity of $\int (\ln x)^2 dx$?

(A) $O(x^2)$ (B) $O(x^2 \ln^2 x)$ (C) $O(x \ln x)$ (D) $O(x \ln^2 x)$ (E) NOTA

22. Which of these scenarios is $O(n^2)$?

(A) The number of handshakes at a party if everyone shakes hands with everyone else once
(B) The number of bacteria in a Petri dish, if they split in a specified amount of time
(C) A binary search of a set of sorted data
(D) The number of sandwiches needed if each person eats one sandwich
(E) NOTA

23. Evaluate the convergence of the following series: $\displaystyle\sum_{i=1}^{\infty} \sin\left(\frac{i\pi}{180}\right)$

(A) Converges conditionally
(B) Converges absolutely
(C) Diverges
(D) Need more information
(E) NOTA

24. Find the interval of convergence of the following series: $\displaystyle\sum_{i=1}^{\infty} \frac{(4-x)^i}{3^i}$

(A) (1,7) (B) [1,7) (C) (1,7] (D) [1,7] (E) NOTA

25. An ant is at one end of a 1-meter-long rope. Every second, the ant moves one centimeter towards the other end of the rope. The rope is then stretched by one meter, so that the proportions of rope on either side of the ant stay the same. For example, after one second, the ant moved 1 centimeter. The rope is stretched, and the ant is now 2 centimeters along a 2 meter rope. Does the ant reach the end of the rope?

(A) Yes, in finite time (B) Yes, in infinite time (C) No, never (D) Need more information (E) NOTA

26. A pie is taken out of the oven at 300° Fahrenheit. If after 20 minutes, the pie is at 250° Fahrenheit, and the ambient temperature is 50°, how hot in degrees Fahrenheit is the pie 1 hour after the pie is taken out of the oven?

(A) 210 (B) 178 (C) 152 (D) 130 (E) NOTA

27. What is the volume of the figure obtained by revolving the region bounded by $y = x^2 - 4x$ and the x-axis around the x-axis?

(A) $\dfrac{256\pi}{45}$ (B) $\dfrac{512\pi}{45}$ (C) $\dfrac{256\pi}{15}$ (D) $\dfrac{512\pi}{15}$ (E) NOTA

28. What is the reciprocal of the coefficient of the x^5 term in the Maclaurin series expansion of $4\sin(5x)$?

(A) $\dfrac{625}{96}$ (B) $\dfrac{96}{625}$ (C) $\dfrac{625}{6}$ (D) $\dfrac{6}{625}$ (E) NOTA

29. Alice and Bob are playing a game. They both choose a random number between 0 and 8. Bob wins if his number is greater than the square of Alice's number. What is the probability that Bob wins?

    (A) $\frac{\sqrt{2}}{6}$ (B) $\frac{\sqrt{2}}{8}$ (C) $\frac{\sqrt{3}}{6}$ (D) $\frac{\sqrt{3}}{8}$ (E) NOTA

30. Evaluate the derivative of $y = |x+1|$ at $x = 1$.

    (A) 1 (B) -1 (C) 0 (D) DNE (E) NOTA