QUESTION 1

Let:

$$\begin{aligned} A &= \text{the smallest root of } x^2 + 9x + 18 \\ B &= \text{the largest root of } x^2 + x - 20 \\ C &= \text{the largest root of } x^2 - 4x - 12 \\ D &= \text{the smallest root of } x^2 - 3x - 28 \end{aligned}$$

Find a quartic trinomial with roots A, B, C, and D and leading coefficient of 1.

QUESTION 2

Let:

$$\begin{aligned} A &= 4(\sqrt{-1})^{260} \\ B &= \text{the distance between } (3,4) \text{ and the origin in the Cartesian plane.} \\ C &= \text{the rationalized form of } \frac{3}{7+\sqrt{29}} \\ D &= \text{the rationalized form of } \frac{8}{4-\sqrt{6}} + \frac{4}{5+\sqrt{5}} \end{aligned}$$

Find $AB(C+D)$.

QUESTION 3

Consider the solutions to the following systems of equations:

A.
$$\begin{aligned} 2x + 4y - 3z &= -13 \\ -5x - 3y - 3z &= -27 \\ 3x + 6y - z &= -9 \end{aligned}$$

B.
$$\begin{aligned} 5x - 8y + 3z &= 12 \\ 2x - 4y + 7z &= 28 \\ -3x + 4y + 2z &= 8 \end{aligned}$$

C.
$$\begin{aligned} 6x - 6y - 4z &= 8 \\ -4x - y - 7z &= 4 \\ 2x + 3y + 5z &= 0 \end{aligned}$$

D.
$$\begin{aligned} -5x + 8y - 2z &= 1 \\ 4x - y - 6z &= 21 \\ 3x - 9y + 3z &= 3 \end{aligned}$$

Take the sum of x, y, and z in each system A., B., C., and D, and find the sum of these sums. If necessary, express your answer as a fraction in simplest form.

QUESTION 4

Let A be the largest possible integer solution to the following inequality and B be the smallest possible integer solution to the following inequality:

$$-25 < 7 - 3x \leq 19$$

Let C be the largest possible integer solution to the following inequality and D be the smallest possible integer solution to the following inequality:

$$| x - 3 | = | 5 - 3x |$$

Find $A - B + C - D$.

QUESTION 5

Let:

A = the distance between the points $(-1, -2)$ and $(2, -6)$

B = the shortest distance between the line $y = x + 6$ and the point $(4, 2)$

C = the ordinate of the intersection between a parabola with leading coefficient of 1 and roots of 1 and -4 and the quadratic equation $y = x^2 - 9x + 20$

D = the shortest distance between the vertex of $y = x^2 + 10x + 29$ and the line $y = x - 3$

Find $\frac{ABD}{C}$.

QUESTION 6

Let:

$$\begin{aligned} A &= \text{the sum of the roots of } 28x^2 + 33x - 28 \\ B &= \text{the product of the roots of } 6x^2 - 29x + 28 \\ C &= \text{the sum of the reciprocals of the roots of } 12x^2 - 38x + 30 \end{aligned}$$

Find ABC. Express your answer as a fraction in simplest form.

QUESTION 7

Let:

$$\begin{aligned} A &= \text{the units digit of } 2^{235} \\ B &= \text{the sum of the first 100 natural numbers} \\ C &= \text{the sum of the first 25 odd numbers} \\ D &= \text{the number of positive factors of 496} \end{aligned}$$

Find $\dfrac{B}{ACD}$. Express your answer as a decimal.

QUESTION 8

Given:

$$\begin{aligned} f(x) &= \frac{3}{\sqrt{3x-3}} + \frac{5}{\sqrt{6x+1}} \\ g(x) &= 4x^2 + 6x + 9 \\ z(x) &= 5x - 6 \end{aligned}$$

Let:

$$\begin{aligned} A &= \text{the smallest integral value in the domain } f(x) \\ B &= g(z(1)) \\ C &= z(9) - 36 \\ D &= g(f(4)) \end{aligned}$$

Find $\dfrac{D-B}{A+C}$.

QUESTION 9

Given the set:

$$\{4, 6, 9, 11, 12, 15, 16, 23\}$$

Let:

A = the arithmetic mean of the set
B = the cube root of the product of the three smallest elements of the set

The first term in an arithmetic sequence is 4. The 20^{th} term in this same sequence is 61. Let C = The 31^{st} term in this sequence.

The 4^{th} term in a geometric sequence is 2. The 8^{th} term in the same sequence is $\frac{1}{128}$. Let D = the 1^{st} term in this geometric sequence.

Find $\frac{4A}{B} - \frac{C}{2} + D$.

QUESTION 10

Let:

$$\begin{aligned} A &= \frac{x^3y^4}{x^2y} \cdot \frac{x^6y^7}{x^5y^8} \\ B &= \frac{x^{-5}y^2}{x^2y^3} \cdot \frac{x^6y^{-3}}{x^2y^{-5}} \\ C &= \frac{6x^4 3y^7 4z^{-4}}{2x^4y^2 5z^3} \cdot \frac{10x^{-5}y^2z^5}{2x^3 8yz^{-2}} \\ D &= \left(\frac{(x)^{(7)^2}}{(y)^{(3)^4}} \cdot \frac{(y)^{(4)^3}}{(x)^{(6)^2}}\right)^{-1} \end{aligned}$$

Find $ABCD$.

QUESTION 11

Your mother wants you and your 5 siblings to paint your fence before she arrives home from her 8 hour work day. Before she left for work she showed you and your siblings how to paint a fence. You and your siblings copy her technique exactly, so that each of you can paint at the same rate as her. Your mother can paint a fence in 20 hours. Assuming you and your siblings start painting as soon as she leaves for work, let A be the number of minutes in advance of your mother's arrival home that you finish your job.

If a varies jointly with c and the square of b, and $a = 36$ when $b = 4$ and $c = 6$, let B be the value of a when $b = 2$ and $c = 12$.

Steve and his coworkers Tyrone and Betsy all work at a paper company. Tyrone makes 225 dollars in 15 hours at the office. Betsy makes 165 dollars in 11 hours at the office. Assuming they are all equally hard working people, equally skilled, and paid the same per hour, let C be the number of cents Steve makes for 8 hours of work?

If y varies jointly with x and z, and $y = 1$ when $x = 6$ and $z = 5$, let D be the value of z when $y = 6$ and $x = 15$.

Find $A + B + (C - D)$

QUESTION 12

Let:

A = the ordinate of the y-intercept of the line that passes through the point $(6, 2)$ and is perpendicular to the line $y = 3x + 6$

B = the slope of the line that is the perpendicular bisector of $(-2, 0)$ and $(0, 4)$

C = the slope of the line that passes through the points of $(8, 14)$ and $(-8, -10)$

D = the abscissa of the intersection of the lines $x = 6$ and $y = 9$

Find $ABCD$.

QUESTION 13

Let:

$$\begin{aligned} A &= \text{the number of real solutions to } 2x^2 - 3x + 4 \\ B &= \text{the number of real solutions to } 15x^2 + 9x - 6 \\ C &= \text{the number of real solutions to } 4x^2 - 12x + 9 \\ D &= \text{the number of real solutions to } 18x^2 - 7x - 1 \end{aligned}$$

Find $\dfrac{B+D}{A+C}$.

QUESTION 14

Let:

$$\begin{aligned} A &= \text{the } 10^{\text{th}} \text{ prime number} \\ B &= \text{the } 10^{\text{th}} \text{ triangular number} \\ C &= \text{the } 3^{\text{rd}} \text{ smallest number that is both a cube and a square of a positive real number} \\ D &= \text{the harmonic mean of 3, 5, and 6} \end{aligned}$$

Find $\dfrac{D}{A-(C-B)}$.