## QUESTION 1

Consider the following function: $f(x) = 2x^7 - 4x^6 + x^5 + 14x^4 - 32x^3 - 8x^2 + 3x - 1$. Let:

$$\begin{aligned}
A &= \text{the sum of the roots of } f(x) \\
B &= \text{the product of the roots of } f(x) \\
C &= \text{the sum of the reciprocals of the roots of } f(x) \\
D &= \text{the sum of the roots of } f(x) \text{ taken four at a time}
\end{aligned}$$

Find $A - 2B + C + D$.

## QUESTION 2

Josh and Nihar are running in woods and find their trail blocked by a bunch of logs. Sadly they are all too weak to push them out of the way, so the only way to go by is to solve them. Help them solve the following logarithmic problems! Let:

$$\begin{aligned} A &= \log_{\frac{1}{125}} 729 \cdot \log_8 125 \\ B &= \log_{\frac{3}{4}} 27 \cdot \log_9 64 \cdot \log_8 \frac{81}{256} \\ C &= \frac{\ln 25}{\ln 3} \cdot \frac{\log 27}{\log 5} \\ D &= \log 15 + \log 2 \log 10 - \log 3 \end{aligned}$$

Compute $AB - CD$.

## QUESTION 3

Consider the functions $f(x) = x^2 - 5x + 6$ and $g(x) = -3x + 14$. Let:

$$\begin{aligned} A &= \text{the ordinate of the intersection } f(x) \text{ and } g(x) \text{ in first quadrant} \\ B &= \text{the ordinate of the intersection } f(x) \text{ and } g(x) \text{ in second quadrant} \\ C &= (f \circ g)(3) \\ D &= \text{the slope the line passing through the intersections of } f(x) \text{ and } g(x) \end{aligned}$$

Find $\dfrac{ABC}{D}$.

## QUESTION 4

Rayyan and Anirudh are absolutely obsessed with The Matrix and decided that they are experts at everything that has to do with matrices. Seeing their confidence, Nihar decided he is going to give them a few questions to see if they truly are the Matrix Masters they claim to be. Let:

$$A = \begin{vmatrix} 4 & -1 & 1 \\ 4 & 5 & 3 \\ -2 & 0 & 0 \end{vmatrix}$$

$$B = \text{the value of } x \text{ given the following system of equations: } \begin{cases} x + z = -1 \\ -2x + y + 8z = 1 \\ -15x + 6y + 11z = 4 \end{cases} \text{(Hint: use Cramer's Rule.)}$$

$$C = \text{the determinant of the coefficient matrix for the following system of equations: } \begin{cases} 15x + 12y - 7z = 2018 \\ -x - z = 3013 \\ 8x + 3y - 13z = 32301 \end{cases}$$

It turns out, they are not the Matrix Masters they thought they were. Maybe you are. Find $C - 5A - 34B$.

## QUESTION 5

Let:

$$\begin{aligned} A &= \text{the sum of the coefficients in the expansion of } (2x-7y)^4 \\ B &= \text{the constant term in the expansion of } (x^3-(1/x))^4 \\ C &= \text{the number of terms in the expansion of } (3x+7y-4z)^8 \\ D &= \text{the coefficient of the } x^2y^3z \text{ term in the expansion of } (2x+y+3z)^5 \end{aligned}$$

Find $A^{(0.25)} + B + C - D$

## QUESTION 6

Let:

$$\begin{aligned} f(x) &= x^2 - 8x + 12 \\ g(x) &= x + 4 \\ h(x) &= -x^2 + 9x - 18 \end{aligned}$$

The following statements have point values indicated by the numbers within the parentheses by each statement. Starting with a value of 1, multiply by the points of every true statement, and multiply by the reciprocal of the points of every false statement.

(2) One of the intersections of $f(x)$ and $g(x)$ is at $(1, 5)$.

(3) When all three functions are graphed there are a total of 4 intersections.

(12) The graph of $h(x)$ intersects the $x$ axis at $(4, 0)$.

(4) The sum of the distinct values of $x$ for which $x$ is a zero of at least two of the three functions $f(x)$, $g(x)$, and $h(x)$ is 6.

(9) The $y$ intercept of $g(x)$ is at $y = 4$.

(6) The axis of symmetry of $f(x)$ and $h(x)$ intersect at a point on $y = 3$.

What is the final value?

## QUESTION 7

Varun, Vamsi, and Rohan are all competing to see who can bike the furthest without stopping. All three think they biked the furthest.

Varun says the number of miles he biked is equal to the distance between the zeroes of the equation: $y = x^2 + 8x - 37$.

Vamsi says the number of miles he biked is equal to the sum of the values of $x$ that satisfy the equation: $\sqrt{x-1}+4 = x-3$.

Rohan says the number of miles he biked is equal to one more than three times the sum of the $x$ values that satisfy the equation: $8^{2x} - 20(8^x) + 67 = 3$.

Order Varun, Vamsi, and Rohan in descending order of how many miles each biked.

## QUESTION 8

Given $f(x) = 2x^2 + 3x - 21$ and $g(x) = x - 10$, let:

The equation for the oblique asymptote of $\dfrac{f(x)}{g(x)}$ be $y = Ax - B$.

The equation for the vertical asymptote of $\dfrac{f(x)}{g(x)}$ be $x = C/3$.

The sum of the roots of $f(x)$ be $D$.

Calculate $A + B + C + 2D$.

## QUESTION 9

Note: an "elliptical cone" is defined as a regular cone with an elliptical base rather than a circular one. Let:

$$\begin{aligned}
A\pi + B\sqrt{3} &= \text{the area of the overlap of the graphs of the equations: } (x-3)^2 + (y+2)^2 = 36 \text{ and } (x-3)^2 + (y-4)^2 = 36 \\
C\pi &= \text{the volume of an elliptical cone with a base defined by } 9x^2 - 36x + 39 + 4y^2 - 24y = 3 \text{ and a height of } 5 \\
D &= \text{he sum of the focal radius, eccentricity, and the coordinates of the vertex of } 8x - 6y + 47 = y^2
\end{aligned}$$

Find $\dfrac{A+C}{B+D}$.

## QUESTION 10

Tanvi, Sanjita, and Tanusri decided to binge watch all the TV series available. As they were watching they ran into an error which read:

"IN ORDER TO CONTINUE WATCHING YOU MUST ANSWER THE FOLLOWING ALGEBRAIC SERIES QUESTIONS."

Let:

$$\begin{aligned} A &= 1 + \frac{2}{5} + \frac{1}{2} + \frac{4}{25} + \frac{1}{4} + \frac{8}{125} + \ldots \\ B &= \sum_{n=1}^{\infty} \frac{n}{3^n} \\ C &= 1 + 1 + 2 + 5 + 4 + 9 + \ldots + 64 + 25 \\ D &= \sum_{n=1}^{2019} i^n \end{aligned}$$

You can help the girls continue to watch TV. Find $60A - 4B - C + D$.

## QUESTION 11

Let:

$$\begin{aligned}
A &= \text{the magnitude of } 2(3+2i)^2 - 3i + 10 \\
B &= \text{the value of } b \text{ when } \frac{6-4i}{2i+3} \text{ is written in form } a+bi \\
C &= \text{the number of complex numbers in the set: } \{2, 3+i, \pi, e, 4-3i\} \\
D &= a+b \text{ where } a+bi \text{ is the solution to the equation } -34 = x^2 - 10x \text{ with } b<0
\end{aligned}$$

Find $A - BCD$.

## QUESTION 12

Isaiah and Siddharth are on the Argand Plane, hoping to play Fortnite together, and are located at $2 + 10i$ and $7 + 2i$ respectively. Siddharth decides to go to Isaiah but must first go to the gargantuan new Gamestop, which is the entirety of the real axis, to get an extra gaming controller. Let:

$A$ = the shortest distance Siddharth can travel to pick up a controller and go to Isaiah

$B$ = the shortest distance each would have to travel if they decided they would travel an equal distance to meet each other (one of them must pick up the extra controller from Gamestop)

$C$ = the shortest distance Siddharth would have to travel if Gamestop was the entirety of the imaginary axis instead of the real axis

Find $A + 2B - C$.

## QUESTION 13

Let:

$$\begin{aligned}
A &= \text{the sum of the solutions to the equation } |3x-12| = |2x+8|.\\
B &= \text{the minimum value of } f(x) \text{ where } f(x) - 83 = |x-11|.\\
C &= \text{the greater value for } x \text{ which satisfies } \frac{1}{3}|x-3| + 2 = |x-4| - 5.\\
D &= \text{the number of integer values of } x \text{ which satisfy the inequality: } |\frac{1}{3}x+2| > \frac{1}{4}x^2
\end{aligned}$$

Find $AC - BD$.

## QUESTION 14

Let:

$$A = \text{the number of times the graphs of } \frac{1}{2}x^2 + \frac{1}{3}y^2 - 2x + 2y + 11 = 0 \text{ and } y = 2x^2 - 3x + 1 \text{ intersect}$$

$$B = \text{the value of } x \text{ on } \frac{1}{3}x^2 - \frac{1}{2}y^2 + 2x - 2y + 1 = 0 \text{ which has only one possible } y \text{ coordinate}$$

$$C = \text{the determinant of the following matrix: } \begin{bmatrix} 21 & 13 & 74 & 59 \\ 13 & 11 & 42 & 23 \\ 19 & 31 & 7 & 49 \\ 11 & 7 & 8 & 91 \end{bmatrix}$$

Calculate $AC + B$.