

For all questions, answer choice (E) NOTA means that none of the given answers is correct. For all problems, $i = \sqrt{-1}$
Good Luck!

1. For real numbers x and y , $2^x = 27$ and $9^y = 64$. Find xy .

(A) $4\sqrt{3}$ (B) 18 (C) $6\sqrt{2}$ (D) 9 (E) NOTA

2. Let $x = i - 3$. Find the value of:

$$\begin{vmatrix} x^2 & x & 1 \\ x+4 & 0 & x \\ 2 & x & 1 \end{vmatrix}$$

(A) $-12 + 84i$ (B) $-60 + 84i$ (C) $60 - 82i$ (D) $36 - 82i$ (E) NOTA

3. Given the function $f(x) = 9x^2 - 18x + 9 + 16y^2 - 24y + 9 = 144$, how many of the following statements are correct?

- i. The area enclosed by the function is 12π .
- ii. The function is an ellipse.
- iii. The function's center is $(1, \frac{3}{2})$.
- iv. The distance between the foci is $2\sqrt{7}$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

4. What is the remainder when $-x^4 - 5x^2 + 2x - 4$ is divided by $x - 2$?

(A) 0 (B) 2 (C) -2 (D) 36 (E) NOTA

5. Vamsi is going to a carnival. In order to win one of the games, he must answer this question correctly: "What is the sum of $\frac{1}{3} + \frac{1}{2} + \frac{4}{9} + \frac{1}{2} + \frac{7}{27} + \frac{3}{8} + \frac{10}{81} + \frac{1}{4} + \dots$?" What must Vamsi answer in order to win?

(A) $\frac{5}{4}$ (B) $\frac{2}{3}$ (C) $\frac{13}{4}$ (D) $\frac{5}{6}$ (E) NOTA

6. What is the distance between the points of intersection of $f(x)$ and $g(x)$, if $f(x) = 2x^2 - 9x + 1$ and $g(x) = 7x - 23$?

(A) $2\sqrt{214}$ (B) $20\sqrt{29}$ (C) $2\sqrt{41}$ (D) $20\sqrt{2}$ (E) NOTA

7. Find the sum of all possible values of x , if

$$\begin{vmatrix} x & 0 & 2 \\ 3 & 2 & 4 \\ 2 & x & 1 \end{vmatrix} = -4$$

(A) -1 (B) -4 (C) 1 (D) 2 (E) NOTA

8. Rayyan, who loves the New England Patriots, aspires to be the next Tom Brady. While throwing a football, the football traveled in a perfect parabolic path. The equation of the path was $y^2 + 2y + 8 - 3x = 0$. What is the focal radius of the parabola?

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) $-\frac{8}{3}$ (D) $\frac{3}{4}$ (E) NOTA

9. What is the coefficient of the fifth term in the expansion of $(x^2 - \frac{1}{x})^{10}$?

- (A) 210 (B) -252 (C) -210 (D) 252 (E) NOTA

10. Given that $x + \frac{1}{x} = 3$, what is the value of $x^3 + \frac{1}{x^3}$?

- (A) 9 (B) 27 (C) 18 (D) Not Enough Info (E) NOTA

11. Joshua, to spite Nihar, decided to draw the graph of $|x| + |y| < 8$. What is the area enclosed by the graph of $|x| + |y| < 8$?

- (A) 64 (B) 16 (C) 32 (D) 128 (E) NOTA

12. Rohan, after getting in trouble with yet another teacher, is given this system of equations to solve:

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 2 \\ 2x + 4y + 5z &= 3 \end{aligned}$$

What value of y should Rohan get if he solved the system correctly?

- (A) 0 (B) -1 (C) -2 (D) 2 (E) NOTA

13. Let $a = \log 2$, $b = \log 3$, and $c = \log 7$. What is the value of $\log_{126}(588)$ in terms of a , b , and c ?

- (A) $\frac{2a + 2b + 2c}{a + 2b + c}$ (B) $\frac{a + 2b + 2c}{a + 2b + c}$ (C) $\frac{a + b + c}{a + 2b + c}$ (D) $\frac{2a + b + 2c}{a + 2b + c}$ (E) NOTA

14. Given that $f(x) = \frac{x^2 - 4x - 5}{x + 23}$, and the asymptotes are $y = x - a$ and $x = b$, find the value of $f(a + b)$?

- (A) $\frac{2295}{73}$ (B) $-\frac{5}{27}$ (C) $-\frac{5}{19}$ (D) $-\frac{2695}{27}$ (E) NOTA

15. Given that $f(x) = x^3 + 2x^2 + x$ has roots a , b , and c , what is the value of $\frac{1}{a-1} + \frac{1}{b-1} + \frac{1}{c-1}$?

- (A) $\frac{1}{2}$ (B) -1 (C) -2 (D) 0 (E) NOTA

16. A polynomial $f(x)$, has three roots $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$. Given that the value of $f(2)$ is 55, what is the value of $f(4)$?

- (A) 110 (B) 60 (C) 220 (D) 120 (E) NOTA

17. Solve for all values of x where $\sqrt{8x+1} - \frac{12}{\sqrt{8x+1}} = -1$.

- (A) 1 (B) $\frac{15}{8}$ (C) $\{1, \frac{15}{8}\}$ (D) No solutions (E) NOTA

18. Tanvi, Tanusri, Sanjita and Deekshita, who are all at the point (17,38), decide to get dinner at Moe's. Moe's lies on the line $3x + 4y = 8$. What is the shortest distance between the girls and Moe's?

- (A) 38 (B) $\frac{211}{5}$ (C) 39 (D) $\frac{203}{5}$ (E) NOTA

19. Use the following table for #19:

x	$f(x)$	$g(x)$
-4	4	3
-3	2	4
-1	5	1
3	-2	4

Given that $f(x)$ is an odd function and $g(x)$ is an even function. What is $f(g(-1)) + g(f(-4))$?

- (A) -8 (B) -2 (C) 8 (D) 7 (E) NOTA

20. Let $\frac{A}{x-1} + \frac{B}{x-3} = \frac{15x+9}{(x-1)(x-3)}$. Find $A+B$.

- (A) 15 (B) -15 (C) 39 (D) -39 (E) NOTA

21. Simplify the following expression: $\frac{\sqrt{8}}{\sqrt{3} + \sqrt{5} - \sqrt{8}}$

- (A) $\frac{5\sqrt{10} + 3\sqrt{6} + 4\sqrt{15}}{15}$ (B) $\frac{4\sqrt{10} + 5\sqrt{6} + 3\sqrt{15}}{15}$ (C) $\frac{3\sqrt{10} + 4\sqrt{6} + 5\sqrt{15}}{15}$ (D) $\frac{3\sqrt{10} + 5\sqrt{6} + 4\sqrt{15}}{15}$ (E) NOTA

22. Find the sum of all the elements of the cofactor matrix of the following:

$$\begin{bmatrix} 3 & 9 & 4 \\ 2 & 1 & 6 \\ 5 & -1 & -7 \end{bmatrix}$$

- (A) -155 (B) 127 (C) -127 (D) 155 (E) NOTA

23. Let $f(x) = -2x^2 + 13x$. What is the determinant of the resulting matrix of $f(A)$, when $A = \begin{bmatrix} 3 & 5 \\ 1 & 8 \end{bmatrix}$?

- (A) 31 (B) 779 (C) -405 (D) 374 (E) NOTA

24. Find the solution of the following equation: $\log_9(\log_6(\sqrt{x-4} + \sqrt{x+8})) = 0$

- (A) 16 (B) 32 (C) 54 (D) 8 (E) NOTA

25. What is the directrix of $x^2 + 3x + 10 - y = 0$?

- (A) $x = -8$ (B) $y = -8$ (C) $y = 7.5$ (D) $x = 7.5$ (E) NOTA

26. What transformations describe the graph of $f(x) = x^3 - 3x$ to $f(x) = -(x^3 - 3x) + 1$?

- (A) Reflection across the x -axis and a translation of one unit down
 (B) Reflection across the y -axis and a translation of one unit up
 (C) Reflection across the y -axis and a translation of one unit down
 (D) Reflection across the x -axis and a translation of one unit up
 (E) NOTA

27. Which of the following are odd functions?

i. $f(x) = x^3$

ii. $f(x) = \frac{1}{x^3 + 1}$

iii. $f(x) = x^{10} + 10$

- (A) I only (B) II only (C) III only (D) I, II, III (E) NOTA

28. Identify this conic: $3x^2 + 4y^2 - \frac{3}{2}x + 8y = -12$.

- (A) Circle (B) Hyperbola (C) Parabola (D) Ellipse (E) NOTA

29. What is the smallest number of times that a fair coin must be flipped such that the probability of getting at least two tails is at least 0.50?

- (A) 6 (B) 5 (C) 4 (D) 3 (E) NOTA

30. What is the eccentricity of $25x^2 - 150x - 36y^2 + 72y + 189 = 0$?

- (A) 0 (B) $\frac{1}{9}$ (C) 1 (D) $\frac{10}{9}$ (E) NOTA