

For all questions, answer choice (E) NOTA means that none of the given answers is correct. The $\sqrt{-1} = i$. Good Luck!

- What is the area enclosed by the conic represented by the equation $4x^2 + 5xy + y^2 = 1$ in the argand plane?
 (A) $\frac{2\pi}{3}$ (B) The conic is degenerate. (C) 6π (D) $\frac{2\pi}{\sqrt{3}}$ (E) NOTA
- What is $\cos(36^\circ)$?
 (A) $\frac{1 - \sqrt{5}}{4}$ (B) $\frac{1 + \sqrt{5}}{4}$ (C) $\frac{1 - \sqrt{5}}{2}$ (D) $\frac{1 + \sqrt{5}}{2}$ (E) NOTA
- Find the acute angle formed by the intersection of the two planes $x + y + z = 5$ and $2x - y - z = 2017$. (Hint: use vectors.)
 (A) $180^\circ - \arccos\left(\frac{\sqrt{2}}{3}\right)$ (B) $\arccos\left(\frac{\sqrt{5}}{5}\right)$ (C) $\arccos\left(\frac{\sqrt{2}}{3}\right)$ (D) $180^\circ - \arccos\left(\frac{\sqrt{5}}{5}\right)$
 (E) NOTA
- Which of the following summations converges?
 I. $\sum_{x=1}^{\infty} \frac{1}{\sqrt{x^3 + 2x}}$
 II. $\sum_{x=3}^{\infty} \frac{1}{\sqrt[3]{x^3 + 2x^2}}$
 III. $\sum_{x=1}^{\infty} \frac{1}{\sqrt[4]{x^7 + 5x}}$
 (A) I only (B) I and II only (C) I, II, and III (D) I and III only (E) NOTA
- Evaluate the following summation: $\sum_{x=1}^{\infty} \frac{x}{3^x}$.
 (A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{3}{4}$ (D) $\frac{5}{9}$ (E) NOTA
- Find the cosine of the acute angle formed by the intersection of the lines $y = 4x + 10$ and $y = 6x - 35$.
 (A) $\frac{25}{\sqrt{629}}$ (B) $\frac{2}{625}$ (C) $\frac{4}{\sqrt{235}}$ (D) $\frac{36}{\sqrt{293}}$ (E) NOTA
- Classify the following polar curve: $r = 4 - 3 \sin(\theta)$.
 (A) Cardioid (B) Convex Limacon (C) Dimpled Limacon (D) Lemniscate (E) NOTA
- Find the area of the triangle defined by the points $(1, 3, 6)$, $(4, 5, 7)$, and $(9, 4, 10)$.
 (A) $\frac{5\sqrt{34}}{2}$ (B) $\frac{83\sqrt{3}}{4}$ (C) 83 (D) $\frac{3\sqrt{26}}{2}$ (E) NOTA
- What is the eccentricity of the polar curve $r = \frac{8}{5 - 10 \cos(\theta)}$?
 (A) $\frac{8}{5}$ (B) 2 (C) $\frac{4}{5}$ (D) $\frac{1}{2}$ (E) NOTA
- Find the area of the annulus defined by two concentric circles given that a chord of length 46 of the larger circle is tangent to the smaller circle.
 (A) 200π (B) 625π (C) 529π (D) 324π (E) NOTA
- Find the volume of the parallelepiped determined by the vectors $\langle 1, -9, -1 \rangle$, $\langle 4, 7, -9 \rangle$, and $\langle 2, 3, 2 \rangle$.
 (A) 287 (B) 50 (C) $\frac{235}{6}$ (D) $\frac{190}{6}$ (E) NOTA

12. How many petals does the rose curve $r = \sin(14\theta)$ have?
 (A) 28 (B) 14 (C) 13 (D) 261 (E) NOTA
13. What is the 6th pentagonal number?
 (A) 105 (B) 51 (C) 8 (D) 16 (E) NOTA
14. Find the period of $f(x) = \cos(\cos x)$.
 (A) 6π (B) 2π (C) $\frac{\pi}{2}$ (D) π (E) NOTA
15. What is the amplitude of $f(x) = 4\sin(x) + \sqrt{26}\cos(x)$?
 (A) $\sqrt{42}$ (B) $\frac{\sqrt{26}}{2}$ (C) 2 (D) $\frac{4 + \sqrt{26}}{2}$ (E) NOTA
16. How many solutions are there to $\sin(8x) + \sin(12x) = 0$ over the interval $(0, 2\pi]$?
 (A) 25 (B) 48 (C) 24 (D) 50 (E) NOTA
17. Find the determinant of the following 4×4 matrix:
- $$\begin{bmatrix} 2 & 5 & 12 & 10 \\ 7 & -5 & 9 & 11 \\ 3 & \frac{15}{2} & 17 & 15 \\ -1 & 4 & 8 & -10 \end{bmatrix}$$
- (A) 543 (B) 0 (C) -537 (D) 592 (E) NOTA
18. How many times do the graphs of $y = \frac{x}{20}$ and $y = \sin(x)$ intersect?
 (A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA
19. A cylinder of radius 8 and height 27 is laying on its side and is filled with water. The water rises to a height of 4. What is the volume of the water? (Keep in mind, the cylinder is on its side, not standing on its base.)
 (A) $288\pi - 432\sqrt{3}$ (B) $576\pi - 432\sqrt{3}$ (C) 216π (D) $288\pi - 216\sqrt{3}$ (E) NOTA
20. What is the equation of the plane defined by the points $(1, 3, 5)$, $(-4, 7, 6)$, and $(5, 8, -2)$?
 (A) $5x - 4y - z = -12$ (B) $9x + y - 8z = -28$ (C) $33x + 31y + 41z = 331$ (D) $53x + 40y + 32z = 333$ (E) NOTA
21. A regular hexagon is graphed on the Cartesian plane, centered on the origin. One vertex can be represented by the polar coordinate $(-6, 240^\circ)$. What is the area enclosed by this hexagon?
 (A) $27\sqrt{3}$ (B) $9\sqrt{3}$ (C) $15\sqrt{3}$ (D) Cannot be determined. (E) NOTA
22. The function $\sinh(x) = \frac{e^x - e^{-x}}{2}$ is called the Hyperbolic Sine Function. What does the inverse of the Hyperbolic Sine Function evaluate to at $x = 4$?
 (A) $\ln(4 + \sqrt{17})$ (B) $\ln(17)$ (C) $\pm \ln(\sqrt{17} - 4)$ (D) $\ln(2\sqrt{17})$ (E) NOTA
23. Given that $\log(2) + \log(\cos \theta) + \log(\sin \theta) = \sqrt{3}$, solve for $\sin(2\theta)$.
 (A) $\frac{10^{\sqrt{3}}}{2}$ (B) $\frac{10^{2\sqrt{3}}}{2}$ (C) $5^{\sqrt{3}}$ (D) $10^{\sqrt{3}}$ (E) NOTA

24. What are the eigenvalues of the following matrix?

$$\begin{bmatrix} 3 & 9 \\ -5 & 17 \end{bmatrix}$$

- (A) 3, 17 (B) 8, 12 (C) 9, 11 (D) 7, 13 (E) NOTA

25. Evaluate:

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + \frac{3}{4}x} - \sqrt{x^2} \right)$$

- (A) ∞ (B) $\frac{3}{4}$ (C) $\frac{3}{8}$ (D) $\frac{3}{2}$ (E) NOTA

26. Classify the following conic: $3x^2 + 12xy - 6y^2 + 23x + 420y - 5 = 0$.

- (A) Hyperbola (B) Parabola (C) Ellipse (D) Circle (E) NOTA

27. What type of graph do the following parametric equations define?

$$x = 3 \sin(\theta) - 5$$

$$y = 7 \cos(\theta) + 8$$

- (A) Circle (B) Parabola (C) Hyperbola (D) Ellipse (E) NOTA

28. Given that $\sin \theta = \frac{2}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$, find $\tan \frac{\theta}{2}$.

- (A) $\frac{5 + \sqrt{21}}{2}$ (B) $\frac{5}{2}$ (C) $\frac{\sqrt{21}}{2}$ (D) $\frac{5 - \sqrt{21}}{2}$ (E) NOTA

29. Assuming the function $\text{cis}(x) = \cos x + i \sin x$ has its domain restricted to $[0, 2\pi]$, simplify $(-i)^{\frac{1}{2}}$.

- (A) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ (B) $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ (C) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ (D) $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ (E) NOTA

30. Congratulations! You have made it to the end of the test. What is $\cos^{-1}(\cos(\frac{7\pi}{6}))$?

- (A) $\frac{7\pi}{6}$ (B) $-\frac{\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) Undefined. (E) NOTA