## QUESTION 1

Let:

$$\begin{aligned}
A &= \lim_{x\to 0} \frac{\sin x}{x} \\
B &= \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{h} \text{ where } f(x) = \sqrt{\frac{\sin x + 1}{2}} \text{ at } x = 0 \\
C &= \lim_{x\to -\infty} xe^{x} \\
D &= \lim_{x\to 2} \frac{\sqrt{x-1}-1}{\tan(x^2-3x+2)}
\end{aligned}$$

Find $A^2 + B^2 + C^2 + D^2$

## QUESTION 2

Let $f(x) = x^3 - 7x^2 + 4x + 12$

$$\begin{aligned}
A &= \text{the left hand Riemann sum using four equal sub intervals over the domain } [2,6] \\
B &= \text{the right hand Riemman sum using four equal sub intervals over the domain } [2,6] \\
C &= \text{the trapezoidal Riemman sum using four equal sub intervals over the domain } [2,6] \\
D &= \text{the midpoint Riemman sum using two equal sub intervals over the domain } [2,6]
\end{aligned}$$

Find $A + B + C + D$

## QUESTION 3

- A ladder is propped up against the side of a building. The ladder is 13 feet tall, and the wall and floor are perfectly flat. Let $A =$ the velocity at which the midpoint of the ladder is falling if the bottom is moving away from the wall at 1 ft/s and is 5 feet from the base of the wall.
- A rectangular prism has a constant volume. The length is increasing at a rate of 4 units/sec, and the width is decreasing at a rate of 3 units/sec. Given that at this moment, the length of the rectangular prism is 8, the width is 12, and the height is 9. Let $B =$ the rate that the surface area is changing in units$^2$/sec.

Find $2AB$.

## QUESTION 4

Let:

$$\begin{aligned}
A &= \frac{dy}{dx} \text{ at } x = 1 \text{ of } 3y + 2xy = -5 \\
B &= \frac{dx}{dy} \text{ at } x = 3 \text{ of } y = e^{x-3}\sin(2x-6) + 6\ln(x) \\
C &= \frac{d^2y}{dx^2} \text{ at } (x,y) = (0,2) \text{ given the relationship } 2e^x\sin(x) + 3x^2y = 4y^2 - 16 \\
D &= \frac{dy}{dx} \text{ at } x = 3 \text{ given the relationship } y = \frac{(x-4)^3(x+1)^2}{(x-1)}
\end{aligned}$$

Find $5A + B + 128C + D$

## QUESTION 5

Start with 0. For each of the following statements, if the statement is true, add 5 to the total. If the statement is false, subtract 7 from the total. Submit the final total.

*I* The slope of the tangent to $y = e^x$ at $y = 2$ is $e^2$.

*II* If $f$ is continuous on the closed interval $[a, b]$ and $k$ is a number between $f(a)$ and $f(b)$ then there is at least one number $c$ in $[a, b]$ such that $f(c) = k$.

*III* $\frac{d}{dx}[fgh] = f'gh + g'fh + h'fg$ given $f$, $g$, and $h$ are functions of $x$ and $f'$ denotes $\frac{\mathrm{d}f}{\mathrm{d}x}$.

*IV* The indefinite integral of $\sec^3(x)dx$ is $\frac{\sec(x)\tan(x) + \ln|\sec(x) + \tan(x)|}{2}$

*V* The integral of the derivative of a function is that function.

*VI* If enough repetitions are performed, Euler's Method won't always approximate a root to a given degree.

*VII* If $f(x)$ is an even function, the $\lim_{x\to 1^+} f(x) \neq \lim_{x\to -1^+} f(x)$

*VIII* $\lim_{n\to\infty} \frac{1}{n} \cdot \sum_{i=0}^{n} \left(\frac{i}{n}\right) = \frac{1}{2}$. Hint: Definition of an integral.

## QUESTION 6

Given $h(x) = f(x)g(x)$, let:

| | $x=1$ | $x=2$ | $x=3$ | $x=4$ |
|---|---|---|---|---|
| $f(x)$ | 2 | 4 | 3 | 5 |
| $f'(x)$ | 1 | 2 | 2 | 3 |
| $g(x)$ | 3 | 5 | 2 | 1 |
| $g'(x)$ | 1 | 1 | 1 | 4 |

$$\begin{aligned} A &= (f' \circ g)(x) \text{ at } x = 3 \\ B &= \frac{d}{dx}(f \circ g)(x) \text{ at } x = 4 \\ C &= (h' \circ f)(x) \text{ at } x = 2 \\ D &= \frac{d}{dx}(h \circ f)(x) \text{ at } x = 2 \end{aligned}$$

Find $A + B + \dfrac{D}{C}$

## QUESTION 7

Carson needs to get some paper to print the Calculus Team Round that he wrote. Thankfully, there is a store nearby.

Unfortunately for Carson, there is traffic around the store. The line below represents where the traffic starts. If Carson can go at 2 mph without traffic, and 1.5 mph with traffic, how far from point A, in miles, should Carson cross the line to get to the store in the least amount of time?

## QUESTION 8

$A$ = the area of the figure bounded by $y = -x^2 + 2x$ and $y = 0$

$B$ = $\displaystyle\int_0^6 |x-3| + 7dx$

$C$ = the volume of the solid formed by rotating the region bounded by $y = -x^2 + 2x$ and $y = 0$ about the $y-$axis.

$D$ = the volume of the solid formed by placing equilateral triangle cross-sections perpendicular to the $y-$axis on a base that is bounded by $y = \sqrt{4 - x^2}$ and the $x-$axis

Find $\dfrac{D}{A} + \dfrac{9C}{\pi} + B$

## QUESTION 9

Let:

$$
\begin{aligned}
A &= \lim_{t\to 0} \frac{\int_0^t e^x(\sin x + \cos x)dx}{\int_1^{t+1} xe^x dx} \\
B &= \int_1^{\frac{\pi}{4}} x \cdot \ln x \cdot \sec^2 x + \int_1^{\frac{\pi}{4}} \ln x \cdot \tan x dx + \int_1^{\frac{\pi}{4}} \tan x dx
\end{aligned}
$$

**Hint:** Combine the terms and try to integrate the resulting integrand.

Find $A + B$

## QUESTION 10

Let:

$$\begin{aligned}
A &= \int_1^2 \ln x dx \\
B &= \int_2^3 x^x(\ln x + 1)dx \\
C &= \int_0^1 \frac{dx}{x^2 - 5x + 6} \\
D &= \int_0^1 x^2 e^x dx
\end{aligned}$$

Find $A - C + B + D$

## QUESTION 11

If an integral diverges, use 7 in place of the integral. If there is more than one solution, **use the higher one**. Let:

$$\begin{aligned} A &= \int_A^A x\,dx \\ B &= \int_A^B x\,dx \\ C &= \int_B^C \frac{dx}{x-7} \\ D &= \int_C^D (2x+1)\,dx \end{aligned}$$

Find $\int_A^B D^2 e^{Cx}\,dx$.

## QUESTION 12

A particle is traveling along the $x$-axis. Its position is modeled by the function $x(t) = 7t^6 + 3t^5 - 12t^4 + 2t^3 + 5t^2 + \sin x + 7$. Also, for reference, the order of derivatives of position are, respectively: velocity, acceleration, jerk, snap, crackle, and pop.

$A$ = The position of the particle at $t = 0$.

$B$ = The jerk of the particle at $t = 1$.

$C$ = The crackle of the particle at $t = 1$.

$D$ = The force of the particle at t = 0, if it has a mass of 0.5 kilograms, and the $x$-axis is in meters. (Hint: Newtons Second Law!)

Evaluate $A + B + C + 40D$

## QUESTION 13

Let:

$$\begin{aligned}
A &= \text{The average value of } 2x+3 \text{ from } 2 \leq x \leq 5 \\
B &= \text{The average value of } \cos(x) \text{ from } 0 \leq x \leq \frac{\pi}{2} \\
C &= \text{The average value of } \sqrt{1-x^2} \text{ from } -1 \leq x \leq 1 \\
D &= \text{The average value of } \frac{1}{x\ln(x)} \text{ from } e \leq x \leq e^e
\end{aligned}$$

Find $A + B\pi + 4C + \dfrac{1}{D}$

## QUESTION 14

Given $f(x) = x^3$ and $g(x) = x^3 + 2x + 1$, let:

$$\begin{aligned} A &= \text{the Riemman approximation of } f(x) \text{ using Simpson's rule from } 0 \leq x \leq 5 \text{ with } n = 5 \text{ sub-intervals.} \\ B &= \text{the Riemman approximation of } f(x) \text{ using Simpson's rule from } 0 \leq x \leq 5 \text{ with } n = 468 \text{ sub-intervals.} \\ C &= \text{the Riemman approximation of } g(x) \text{ using Simpson's rule from } 0 \leq x \leq 7 \text{ with } n = 3 \text{ sub-intervals.} \\ D &= \text{the Riemman approximation of } g(x) \text{ using Simpson's rule from } 0 \leq x \leq 7 \text{ with } n = 67 \text{ sub-intervals.} \end{aligned}$$

Find $A - B + C - D$.