QUESTION 1

Let:

A = the length of the shortest altitude of a triangle with side lengths 5, 12, and 13

B = the apothem of a regular hexagon with a perimeter of 24

C = the area of a regular hexagon with a perimeter equal to the perimeter of a regular octagon with side length of 6

D = the number of faces in a polyhedron with 20 vertices and 30 edges

Find $13A + B + C + D$.

QUESTION 2

Let:

$$\begin{aligned} A &= \text{the sum of all the interior angles in a regular octagon} \\ B &= \text{the sum of all the exterior angles in a regular heptagon} \\ C &= \text{the lateral surface area of a right cone with a base area of } 25\pi \text{ and height of } 3 \\ D &= \text{the number of cubes with side length 2 that can fit into larger cube with side length 8} \end{aligned}$$

Find $A + B + C + D$.

QUESTION 3

Let:

A = the number of possible positive integer values for the third side of a triangle with two sides of a length of 5 units each

B = the area of an isosceles triangle with side lengths of 5, 5, and 6

C = the area of parallelogram ABCD, given that $AB = 9$, $AD = 6$ and angle $ADC = 60$ degrees

D = the area of a regular octagon with side length of 2 units

Find $A + B + C + D$.

QUESTION 4

Three circles are placed inside one another so that they all have the same center, and then these circles are all placed within a square of side length 10 . The circles have radii of 3, 4 and 5. Let:

$$
\begin{aligned}
A &= \text{the area of the annulus formed by the largest circle and the second largest circle} \\
B &= \text{the probability of throwing a dart into the smallest circle if the dart always lands inside the largest circle} \\
C &= \text{the probability of throwing a dart inside the largest circle if the dart will always hit the square} \\
D &= \text{the area inside the square, but outside the largest circle}
\end{aligned}
$$

Find $A + 100BC + D$.

QUESTION 5

Let:

$$\begin{aligned} A &= \text{the area of a triangle with side lengths 4, 13, and 15.} \\ B &= \text{2 if the following statement is true and -2 if the following statement is false:} \\ &\ \text{When given a true conditional, then the contrapositive is always true.} \\ C &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \\ D &= \text{Find the area of a polygon with coordinates of } (1,1)\ ,\ (-1,3)\ ,\ (4,4)\ ,\ (2,5)\ ,\text{ and } (1,5) \end{aligned}$$

Find $A + B + C + D$.

QUESTION 6

The following statements have point values indicated by the numbers within the parentheses by each statement. Starting with 0, add the points of every true statement, and subtract the points of every false statement.

(8) The area of any quadrilateral can be expressed as $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is the semiperimeter and a, b, c, and d are the side lengths of the quadrilateral.

(-4) The sum of the exterior angles of a triangle is equal to the sum of the exterior angles of a hexagon.

(-17) Any angle inscribed in a semicircle is an obtuse angle.

(3) The minimum number of points required to define a plane is 1.

(-1) The maximum inclosed area that can be formed by a rope of length 16 units is $\frac{32}{\pi}$.

(-6) The incenter is located at the intersection of all the angle bisectors of a triangle.

What is the final number of points?

QUESTION 7

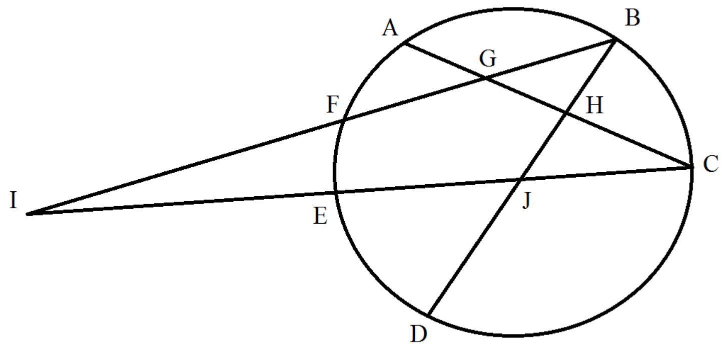
Let:

$$\begin{aligned} A &= \text{the surface area of a right pyramid with a height of 8, and a base with lengths 12 and 18} \\ B &= \text{the height of a regular tetrahedron with a side length of 8} \\ C &= \text{the lateral surface area of a right, circular frustum with a height of 6, and radii of 6 and } \frac{3}{2} \\ D &= \text{the surface area of a hemisphere with a radius of 3} \end{aligned}$$

Find $A + 3B + 4C + D$.

QUESTION 8

In the following diagram $\widehat{AB} = 61^\circ$, $\widehat{AF} = 18^\circ$, $\widehat{FE} = 20^\circ$, $\widehat{ED} = 112^\circ$, $\widehat{DC} = 107^\circ$, $IF = 6$, $FG = 3$, $GB = 7$, $IE = 4$, $GH = 4$ and BD bisects EC. The diagram is not drawn to scale.



Let:

$$
\begin{aligned}
A &= \text{the measure of } \angle BIC \\
B &= \text{the measure of } \angle AHD \\
C &= \text{the area of triangle } BGH \\
D &= \text{the length of } EJ
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 9

Let:

A = the area of a square inscribed in a semicircle with a radius of 8

B = There exists a point X in the interior of rectangle $OPQR$. If $OX = 5$, $PX = 2\sqrt{11}$, and $QX = 8$, what is RX?

C = the length of u in the Figure 1 (not drawn to scale), if $t = 7$, $s = 6$, $AB = 16\sqrt{3}$, and the triangle is equilateral

D = the shortest distance an ant must travel to get from point A to point G in Figure 2, given that Figure 2 is a cube with a side length of 4, and that the ant must travel on the surface of the cube

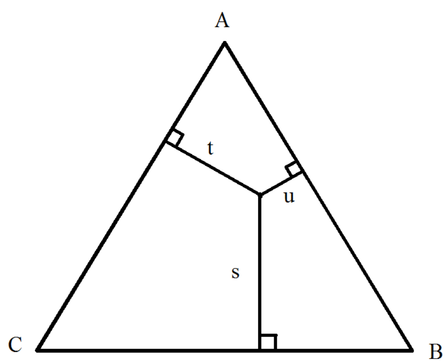


Figure 1

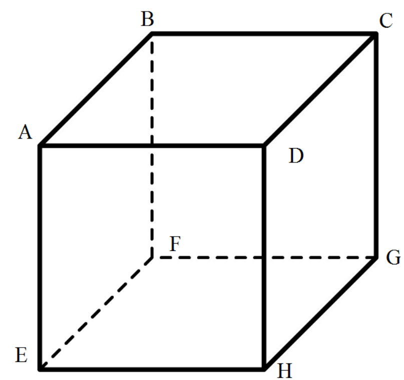


Figure 2

Find $5A + B + C + D$.

QUESTION 10

Triangle ABC has an angle bisector, AD, extending from A to side BC. $AB = 6$, $BD = 4$, $DC = 2$, and $\angle ABC = 29^\circ$.

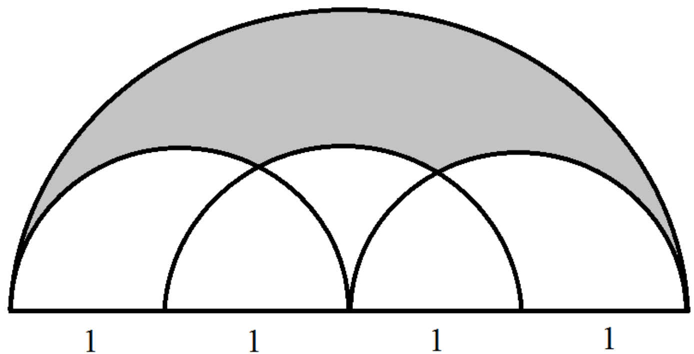
Let:

$$
\begin{aligned}
A &= \text{the length of } AC \\
B &= \text{the measure of } \angle DAC \\
C &= \text{the length of } AD
\end{aligned}
$$

Find $A + 4B + C$.

QUESTION 11

Three semicircles with radii 1 are layed on the diameter of a larger semicircle with a radius of 2, as shown in the following diagram. What is the area of the shaded region? The diagram is not drawn to scale.



QUESTION 12

A triangle's vertices are located at the points $(12, 3)$, $(16, 3)$, and $(7, 15)$.

Let:

$$\begin{aligned} A &= \text{the sum of the coordinates of the centroid} \\ B &= \text{the area of the triangle} \\ C &= \text{the length of the inradius} \\ D &= \text{the length of the circumradius} \end{aligned}$$

Find $3A + B + C + D$.

QUESTION 13

Let:

$$\begin{aligned} A &= \sin(30^\circ) \\ B &= \cos(60^\circ) \\ C &= \tan(30^\circ) \\ D &= \tan(60^\circ) \end{aligned}$$

Find $A + B + C + D$.

QUESTION 14

Let:

$$\begin{aligned} A &= \text{the number of books in Euclid's } \textit{Elements} \\ B &= \text{the number of faces on an icosahedron} \\ C &= \text{the ratio of a circle's circumference to its diameter} \\ D &= \text{the maximum number of distinct sections that a circle can be divided into using 6 lines} \end{aligned}$$

Find $A + B + C + D$.