QUESTION 1

Suppose $f(x) = xe^{-x^2+3}$.

Let:

$$\begin{aligned} A &= \text{the sum of the critical values of } f(x) \\ (B, C) \cup (D, \infty) &= \text{the interval where } f(x) \text{ is concave up} \\ (E, F) &= \text{the coordinate of the absolute maximum of } f(x) \end{aligned}$$

Find $AC + BD + E^2 + \ln(\sqrt{2}F)$.

QUESTION 2

Given the functions $f(x) = 3x^3 + 4x^2 - 10$ and $g(x) = 7x^2 - 6x - 5$, evaluate the following:

$$
\begin{aligned}
A &= f(g(1)) \\
B &= \frac{d}{dx}(f(g(x)) \text{ at } x = 2 \\
C &= \frac{d}{dx}(g(f(x)) \text{ at } x = 2 \\
D &= \frac{d}{dx}\frac{f(x)}{g(x)} \text{ at } x = 2
\end{aligned}
$$

Find $A + B + C + 121D$.

QUESTION 3

Let:

$$\begin{aligned} A &= \int_0^{\frac{2\pi}{3}} \sin(x)\,dx \\ B &= \int_\pi^{\frac{3\pi}{2}} \sin^3(x)\cos^6(x)\,dx \\ C &= \int_{\frac{\pi}{3}}^{\pi} \sin(5x)\cos(4x)\,dx \\ D &= \int_0^{\frac{\pi}{2}} e^{\sin(x)}\cos(x)\,dx \end{aligned}$$

Find $ABC \cdot \ln(D+1)$.

QUESTION 4

Given that $f(x) = 1 + x^2$, let:

$$\begin{aligned} A &= \text{the left hand Riemann sum using four equal sub intervals over the domain } [-1, 1] \\ B &= \text{the right hand Riemann sum using four equal sub intervals over the domain } [-1, 1] \\ C &= \text{the midpoint Riemann sum using four equal sub intervals over the domain } [-1, 1] \\ D &= \text{the trapezoidal Riemann sum using four equal sub intervals over the domain } [-1, 1] \end{aligned}$$

Find $8(A + B + C + D)$.

QUESTION 5

Assume that the range of $\arccos(\theta)$ is limited to $[0, \pi]$, and that the range of $\arctan(\theta)$ is limited to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
Let:

$$\begin{aligned}
A &= \lim_{x\to\infty} \arccos\left(\frac{1}{x}\right) + \lim_{x\to\infty} \arctan(x) \\
B &= \lim_{x\to\infty} \frac{4x^3 + 5x^2 - 5x^5 + 12x + 2}{10x^5 + 3x^3 - 4x + 3} \\
C &= \lim_{x\to\infty} \frac{\sqrt{x^2 + 12x + 5} - (x + 2)}{2} \\
D &= \lim_{x\to 3} \frac{x - 3}{\sqrt{x} - \sqrt{3}}
\end{aligned}$$

Find $ABCD^2$.

QUESTION 6

Let:

$$\begin{aligned} A &= \int_1^2 x\sqrt{x-1}\,dx \\ B &= \int_1^{\sqrt{7}+1} \frac{1}{x^2-2x+8}\,dx \\ C &= \int_0^1 e^{\sqrt{x}}\,dx \\ D &= \int_{-2}^2 \frac{1}{\sqrt{2x+5}}\,dx \end{aligned}$$

Find $\sqrt{7}ABCD$.

QUESTION 7

A ladder of length 10 m lies against a wall, so that it slides horizontally away from the wall at a rate of 5 m/s.

Let:

A = the speed that the top of the ladder slides down the wall, when the bottom of the ladder is 3 m from the wall (Keep in mind that speed must be nonnegative.)

Water is poured into an inverted cone (that is the base is at the top) with a height of 8 m and radius of 10 m at a rate of 10 m^3/s. However, the water also drips out of the tip of the cone at a rate of 3 m^3/s.

Let:

B = the instantaneous rate of change of the height of the water in the cone, when the height is 3 m

Find $\dfrac{1}{AB}$.

QUESTION 8

Let:

$$A = f\left(\frac{\pi}{3}\right) \text{ given that } f'(x) = \sec(x)(\sec(x) + \tan(x)) \text{ and } f\left(\frac{\pi}{4}\right) = -1$$

$$B = \lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \ldots + \sqrt{\frac{n}{n}}\right)$$

$$C = F'(0) \text{ where } F(x) = \int_0^x (4 - t^2)e^{t^3}\, dt$$

$$D = g'(1) \text{ where } g(x) = \frac{x^2 + 2}{x^3 + 4}$$

Find $AB + \frac{\text{C}}{\text{D}}$.

QUESTION 9

Let:

A = the value of x that satisfies Rolle's Theorem for $f(x) = x^2 + 4x + 5$ on the interval $[-4, -1]$
B = the value of x that satisfies the Mean Value Theorem for derivatives for $g(x) = x^2 - 2x + 12$ on the interval $[-2, 6]$
C = the average value of $h(x) = x^3 + 8x$ on the interval $[2, 4]$.

Using Newton's method, find the third approximation, $x_3 = D$, of the root for $y = x^3 + 2x - 4$, given that $x_1 = 1$. Round your answer to the nearest tenth.

Find $A + B + C + 10D$.

QUESTION 10

A particle follows the path modeled by the polar function r, where $r = 3 - 4\cos(\theta) + 2\sin(\theta)$.

Let:

$$\begin{aligned} A &= \frac{dx}{d\theta} \text{ at } \theta = \frac{\pi}{6} \\ B &= \frac{dy}{d\theta} \text{ at } \theta = \frac{\pi}{2} \\ C &= \text{the slope of the tangent line to the curve at } \theta = \frac{\pi}{4} \end{aligned}$$

Find ABC.

QUESTION 11

Let:

A = the volume of the solid formed by rotating the region bounded by $y = 2x^3$, $x = 0$, $x = 2$, and $y = 0$ about $y = 0$

B = the volume of the solid formed by rotating the region bounded by $y = -x^2 + 3x - 2$ and $y = 0$ about $x = 0$

C = the volume of a solid given that its base is the region bounded by $4x^2 + 25y^2 = 100$ with cross sections perpendicular to the $x-$axis that are squares with sides on the base

D = the volume of a solid given that its base is the region bounded by $4x^2 + 25y^2 = 100$ with cross sections perpendicular to the $x-$axis that are isosceles right triangles with hypotenuses on the base

Find $\dfrac{7A}{\pi} + \dfrac{2B}{\pi} + 3C + 3D$.

QUESTION 12

Each of the following statements have point values assigned to them, indicated by the number within the parentheses. Starting with 0 points, add the point values of all the true statements, and subtract the point values of the false statements.

(3) If $f(x)$ is continuous on a certain interval, then $f(x)$ is always differentiable on that interval.

(2) The Chain Rule states that $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$.

(4) $\int \frac{dx}{x \ln x} = \ln|\ln x|$

(-6) Given that f is a differentiable function over all x, has a critical value at $x = 2$, and is concave down on the interval $(-1, 3)$, then f has a local minimum at $x = 2$.

(1) $\lim_{x\to\infty} [f(x) + g(x)] = \lim_{x\to\infty} f(x) + \lim_{x\to\infty} g(x)$

What is the final number of points?

QUESTION 13

Let $f(x) = 2x^3 - 18x^2 + 50x - 6$. Let A and B be equal to the values of x for which the tangent line to $f(x)$ is parallel to $y = 2x + 1$.

Suppose that $f(x+h) - f(x) = hx^2 + 3hx + 5h^2x + h^2 - 3h^2$. $f'(x)$ can be written as $Cx^D + Ex$.

Find $A + B + C + D + E$.

QUESTION 14

The position of a Nexus is given by the parametric equations $x(t) = t^4 + 3t - 5$ and $y(t) = 3t^2 - 4t$.
Let:

A = the time, t, when the Nexus' horizontal acceleration is changing at twice that of its vertical acceleration

B = the slope of the equation of a rocket's path at $t = 5$, if the rocket's path is normal to the Nexus' path at that instant

C = the distance the Nexus travels vertically from $t = 5$ to $t = 7$

D = the average horizontal speed of the Nexus from $t = 2$ to $t = 4$

Find $A + 26B + C + D$.