QUESTION 1

Let:

$$\begin{aligned} A &= f^{-1}(2) \text{ if } f(x) = \frac{3x+3}{5} \\ B &= \text{the degree of } f(x) = 5x^5 - 6x^3 + 3x^2 - 2x^7 + 2 \\ C &= \text{the sum of the roots of } f(x) = x^6 + 5x^4 + 6x^3 + 8x^2 + 3x + 1 \\ D &= \text{the number of times } f(x) = x^3 - 4x + 2 \text{ intersects the } x\text{-axis} \end{aligned}$$

Find $A + B + C + D$.

QUESTION 2

Given the following matrix $\mathbf{J}$

$$\begin{bmatrix} 1 & 3 & 4 \\ 7 & 9 & 8 \\ 2 & 6 & 5 \end{bmatrix}$$

Let:

$$\begin{aligned} A &= |\mathbf{J}| \\ B &= (\mathbf{J}^T)_{3,3} \\ C &= (\mathbf{J}^A)_{2,2} \\ D &= (\mathbf{J}^{-1})_{1,1} \end{aligned}$$

Find $ABCD$.

QUESTION 3

Let:

$$
\begin{aligned}
A &= \frac{\log_5 \sqrt{3125\sqrt{625\sqrt[5]{5}}}}{\log_4 \sqrt[8]{4096} + \log_e e^{-20}} \\
B &= \text{the value of } (x+y)^3 - x^3 - y^3 \text{ given,} \\
&\quad \log y + \log(x+y) = 15 \\
&\quad \log 1 + \log((3x)^{-1}) = -30 \\
C &= (\log_3 5)(\log_7 243)(\log_{125} \sqrt{2401})(\log_{16} 121)(\log_{11} 2) \\
D &= \text{the minimum value of the function, } f(x) = 3(\log x)^2 - 6(\log x) + \log 1000
\end{aligned}
$$

Find $385A + \log B + 3C + D$.

QUESTION 4

Let:

$$\begin{aligned}
A &= \sqrt{210+\sqrt{210+\sqrt{\ldots}}}-\sqrt{210-\sqrt{210-\sqrt{\ldots}}} \\
B &= 3+\cfrac{4}{3+\cfrac{4}{3+\cfrac{4}{\ldots}}} \\
C &= \sum_{n=3}^{\infty}\frac{1}{n^2-n-2} \\
D &= \frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\ldots
\end{aligned}$$

Find $A+B+C+D$.

QUESTION 5

Let:

$$\begin{aligned}
A &= \text{the sum of the real roots of } f(x) = x^4 - x^3 - 11x^2 - 9x - 180 \\
B &= \text{the sum of the real roots of } f(x) = x^4 - 10x^3 + 25x^2 - 40x + 84 \\
C &= \text{the number of asymptotes of the graph of } f(x) = \frac{x^4 + x^3 - 19x^2 + 11x + 30}{x^4 + 6x^3 - 7x^2 - 36x + 36} \\
D &= \text{the ordinate of the } y\text{-intercept of the oblique asymptote of the graph of } f(x) = \frac{10x^3 + 3x^2 - x + 6}{5x^2 + 2x + 4}
\end{aligned}$$

Find $(D^A + B^C) + (A^B - D + C)$.

QUESTION 6

Let:

$$\begin{aligned} A &= \text{the value of } k \text{ that makes } x-2 \text{ a divisor of the the function } h(x) = 3x^3 + x^2 - kx + 12 \\ B &= \text{the number of trailing zeros in 2016!} \\ C &= \text{evaluate the following infinite geometric series: } 1 + \frac{5}{6} + \frac{25}{36} + \ldots \\ D &= \text{the harmonic mean of } \frac{1}{24} \text{ and } \frac{1}{48} \end{aligned}$$

Find $A + B + C + \dfrac{1}{D}$.

QUESTION 7

Let:

$$\begin{aligned}
A &= \text{the sum of the abscissa and twice the ordinate of the vertex of: } y = 2x^2 + 16x - 9 \\
B &= \text{the product of the slopes of the asymptotes of: } x^2 - 10x - y^2 + 4y = 123 \\
C &= \text{the area of the ellipse defined by the equation: } 25x^2 + 4y^2 + 100x - 40y + 100 = 0 \\
D &= \text{the sum of the distances between a point on the equation: } 4x^2 + 16y^2 - 8x - 64y + 4 = 0 \text{ and the two foci}
\end{aligned}$$

Find: $A + 18B + C + D$.

QUESTION 8

Let:

$$
\begin{aligned}
A &= \text{the sum of the solutions to this system of equations:} \\
&\qquad 5x + 3y - 7z = 0 \\
&\qquad \frac{7}{2}x + \frac{5}{2}y - \frac{3}{2}z - 4 = 4 \\
&\qquad 6x - 10y + 4z = -16 \\
B &= \text{the coefficient of the third term of the expansion of } (x-14)^{11} \\
C &= |5 + 12i| \\
D &= \text{one plus the constant value of the expansion of } (7x+6)^5
\end{aligned}
$$

Find $A - \dfrac{B}{140} + C + D$.

QUESTION 9

Given the function: $f(x) = 2x^4 - 16x^3 - 5x^2 + x + 2$

Let:

$$\begin{aligned} A &= \text{the product of the roots.} \\ B &= \text{the sum of the squares of the roots.} \\ C &= \text{the sum of the roots taken three at a time.} \\ D &= \text{the sum of the reciprocals of the roots of } g(x) \text{ where } g(x) = (2x-3)f(x) \end{aligned}$$

Find: $ABCD$.

QUESTION 10

Let:

$$\begin{aligned}
A &= \text{the coefficient of the } x^4y \text{ term in the simplified expansion of } (2x+3y)^5 \\
B &= \text{the sum of the coefficients in the simplified expansion of } (3x-y)^{12} \\
C &= \text{the number of terms in the simplified expansion of } (5w+3x-2y+7z)^4 \\
D &= \text{the coefficient of the } x^{-\frac{1}{4}}y \text{ term in the simplified expansion of } (x+y)^{\frac{3}{4}}
\end{aligned}$$

Find $A+B+C+4D$.

QUESTION 11

Let:

$$
\begin{aligned}
A &= \text{the characteristic of } \log 987 \\
B &= \text{the eccentricity of a circle} \\
C &= \text{the number of letters in the name (singular) of a conic that is the locus of points where the} \\
&\quad\ \text{absolute value of the difference of the distances to two fixed points is constant} \\
D &= \text{the minimum number of real roots of a cubic with real coefficients}
\end{aligned}
$$

Find $1000A + 57B + C + 7D$.

QUESTION 12

Let:

$$
\begin{aligned}
A &= \text{the distance between the points } -2+3i \text{ and } -6-3i \text{ on the Argand plane.} \\
B &= \sum_{n=3}^{2016} i^{-n} \\
C &= \text{the sum of the complex roots of } ix^3+2x^2+ix+2 \\
D &= \text{the sum of the magnitudes of the roots of } x^4+41x^2+400
\end{aligned}
$$

Find $A+B+C+D$.

QUESTION 13

Given the function $f(x) = \dfrac{2x^4 + 4x^3 - 24x^2 + 12x - 90}{x^3 - 7x^2 + 7x + 15}$

Let:

(A, B) be the point of discontinuity when x approaches A.

$x = C$, $x = D$ be the equations of the vertical asymptotes of $f(x)$

$y = Ex + F$ be the oblique asymptote of $f(x)$

Find $A + B + C + D + E + F$.

QUESTION 14

The following sentences are presented as a series of true/false questions about the types of numbers. Starting with 99, subtract 12 for every true statement, and add 13 for false one.

I. 0 belongs to the sets of real, rational, imaginary, and complex numbers
II. $\mathbb{C}$ is the standard notation for complex numbers, and $\mathbb{Z}$ is the notation for imaginary numbers
III. e, but not π, is an example of a transcendental number
IV. 1 is a composite number
V. 11810 is the same as 3146 and 7616
VI. $\mathbb{R}/\mathbb{Q}$ = the set of irrational numbers
VII. The Golden Ratio is $\dfrac{1+\sqrt{5}}{2}$
VIII. 28 is a perfect number

What is the final number?