*For all questions, answer choice (E) NOTA means that none of the given answers is correct. $i$ is defined as $\sqrt{-1}$. Let $cis\theta = \cos\theta + i\sin\theta$. Assume all inverse trigonometric functions have their traditional restricted ranges. Good Luck!*

1. Convert $\frac{7\pi}{16}$ from radians to degrees.

   (A) $\frac{315^\circ}{2}$ (B) $\frac{315^\circ}{4}$ (C) $108.75^\circ$ (D) $\frac{39^\circ}{375}$ (E) NOTA

2. Find $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

   (A) $\frac{3\pi}{4}$ (B) $\frac{3\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{-\pi}{4}$ (E) NOTA

3. Keyura is learning about complex problems and has just learned de Moivre's formula! What is $\left(\frac{\text{cis}\,(36^\circ)}{\text{cis}\,(18^\circ)}\right)^{10}$?

   (A) $-1$ (B) 1 (C) $\frac{(1-i)}{2}$ (D) $\frac{(1+i)}{2}$ (E) NOTA

4. Find the dot product of vectors $u$ and $v$, where $u =< 3, 4 >$ and $v =< 5, -2 >$.

   (A) 7 (B) 23 (C) 14 (D) 10 (E) NOTA

5. Given matrix A:

$$\begin{vmatrix} 1 & 5 \\ -6 & 3 \end{vmatrix}$$

   Find $det(A)$.

   (A) 66 (B) 33 (C) 0 (D) 25 (E) NOTA

6. Find the cosine of the angle between the vectors $< -6, 8 >$ and $< 4, 0 >$.

   (A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{-4}{5}$ (D) $\frac{3}{4}$ (E) NOTA

7. Olivia was watching the tides when she noticed that the pattern of the tides could be represented by the function $f(x) = \sin\left(\pi x - \frac{\pi}{6}\right) + 2$. If the period of the function represents the amount of time in minutes for the tide to come in, how often will the tide come in during a span of 8 minutes? (Hint: Find the frequency).

   (A) 2 (B) 4 (C) 1 (D) 8 (E) NOTA

8. Which of the following are even functions?

   I. $\cos(x)$
   II. $\tan(x)$
   III. $\tan(\cos(x))$
   IV. $\sin(x^2)$

   (A) I, II, III (B) I, II, III, IV (C) I, III, IV (D) I, IV (E) NOTA

9. Express $-e^{\frac{7i\pi}{2}}$ in rectangular form.

   (A) $i$ (B) $-i$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

**For questions 10 and 11, use the graph of $2x^2 - 4xy + 2y^2 + x + y - 8 = 0$**

10. Identify the shape of the graph stated above.
(A) Hyperbola (B) Circle (C) Parabola (D) Non-circular Ellipse (E) NOTA

11. What is the angle of rotation of the graph? (Answers are in degrees.)
(A) 90 (B) 30 (C) 45 (D) 60 (E) NOTA

12. If $\dfrac{x+18}{x^2+x-12} = \dfrac{A}{x-3} + \dfrac{B}{x+4}$ and A, B are real numbers, find $A^2 - B^2$.
(A) 5 (B) 13 (C) 1 (D) 8 (E) NOTA

13. Hazel was studying her linear algebra when she came across an interesting problem about vectors: The dot product of two vectors is 10. Their cross product is $\langle 9, 12, 20 \rangle$. Find the tangent of the acute angle between them.
(A) $\dfrac{5}{2}$ (B) 2 (C) $\dfrac{2}{5}$ (D) $\dfrac{1}{2}$ (E) NOTA

14. If $\sin(x) + \cos(x) = \dfrac{4}{5}$, then what is $\sin(2x)$?
(A) $\dfrac{-18}{25}$ (B) $\dfrac{-9}{25}$ (C) $\dfrac{18}{25}$ (D) 1 (E) NOTA

15. Given the graph $f(x) = \cos^3(x)\sin(x) - \sin^3(x)\cos(x)$, what is the period of $f(x)$?
(A) $\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$ (C) $\pi$ (D) $2\pi$ (E) NOTA

16. How many solutions does the equation $x + \sin(x) = 1$ have?
(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

17. The polynomial $6x^3 - 25x^2 - Kx + 8$ has a remainder of $-9$ when divided by $x - 1$ and a remainder of $-25$ when divided by $x - 2$. What is the value of $K$?
(A) $-1$ (B) 1 (C) 2 (D) 3 (E) NOTA

18. Let $x^2 = x + 1$. If $a$ and $b$ are its roots, evaluate: $(a + b)(a^2 + b^2)$.
(A) $-1$ (B) $-2$ (C) 2 (D) 4 (E) NOTA

19. Find the sum of the solutions in the interval $[0, 2\pi)$ of the equation: $\cos(3x + \pi) + \cos(3x - \pi) = 0$.
(A) $2\pi$ (B) $\dfrac{15\pi}{2}$ (C) $6\pi$ (D) $7\pi$ (E) NOTA

20. Express $(\sin^2\theta)^{log_{\sin\theta}\cos\theta}$ in terms of $\cos\theta$.
(A) $2\cos\theta$ (B) $1 - \cos^2\theta$ (C) $\cos\theta$ (D) $\cos^2\theta$ (E) NOTA

**For problems 21 and 22, use the ellipse defined by $x = 3\sin t + 2$ and $y = 6\cos t - 5$.**

21. Find the area of the ellipse.

(A) $18\pi$ (B) $324\pi$ (C) 18 (D) $10\pi$ (E) NOTA

22. Find the center of the ellipse.

(A) $(5, 2)$ (B) $(-5, 2)$ (C) $(2, 5)$ (D) $(2, -5)$ (E) NOTA

23. $x + \dfrac{1}{x} = 2\cos\left(\dfrac{\pi}{2014}\right)$. Find $x^{1007} + \dfrac{1}{x^{1007}}$. (Hint: Keyura knows how to solve this now!)

(A) 1 (B) $-1$ (C) 2 (D) $-2$ (E) NOTA

24. $A$, $B$, $C$ are the angles of a triangle where $\sin(A + B) = \dfrac{3}{4}$. Find $|\cos(C)|$.

(A) $\dfrac{3}{8}$ (B) $\dfrac{9}{16}$ (C) $\dfrac{\sqrt{7}}{4}$ (D) $\dfrac{1}{2}$ (E) NOTA

25. Find the smallest angle formed by the lines $y = -3x + 2$ and $y = 2x + 5$.

(A) $30^\circ$ (B) $45^\circ$ (C) $60^\circ$ (D) $75^\circ$ (E) NOTA

26. Which conic is described by the equation $r = \dfrac{7}{8 + 6\cos\theta}$?

(A) parabola (B) circle (C) ellipse (D) hyperbola (E) NOTA

27. Jenny is eating a sandwich on a park bench. On her left, Siddarth is taking a nap underneath a tree 10 meters away. She turns her head $105^\circ$ to the right and sees Puneet doing his homework in his car 5 meters away. What is the distance between the tree and Puneet's car in meters? (Hint: $\sin 15^\circ = \dfrac{\sqrt{6} - \sqrt{2}}{4}$)

(A) $\sqrt{125 - 25(\sqrt{6} - \sqrt{2})}$ (B) $\sqrt{125 + 25(\sqrt{6} + \sqrt{2})}$
(C) $\sqrt{125 + 25(\sqrt{6} - \sqrt{2})}$ (D) $\sqrt{125 + 50(\sqrt{6} - \sqrt{2})}$ (E) NOTA

28. What is the product of the factors of 2014?

(A) 8169178744 (B) $2014^4$ (C) 2014 (D) $2014^2$ (E) NOTA

29. Find $(\tan 20^\circ + \tan 30^\circ)(\tan 40^\circ + \tan 30^\circ)$.

(A) $\dfrac{1}{2}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{4}{3}$ (E) NOTA

**For problem 30 it is given that $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$**

30. Evaluate $\dfrac{1}{3!} + \dfrac{2}{4!} + \dfrac{3}{5!} + \dfrac{4}{6!} + \ldots$

(A) $2e - 5$ (B) $\dfrac{e}{4}$ (C) $e - 2$ (D) $3 - e$ (E) NOTA