QUESTION 1

Let

$$\begin{aligned} A &= \|\langle 3,4\rangle\| \\ B &= \|\langle 9,40\rangle\| \\ C &= \|\langle 3,4,12\rangle\| \\ D &= \|\langle 111,148\rangle\| \end{aligned}$$

Find $A+B+C+D$.

QUESTION 2

Let

$$\begin{aligned} A &= \sin\frac{\pi}{12}\cos\frac{\pi}{12} \\ B &= \cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} \\ C &= \tan\frac{5\pi}{12} \\ D &= 4\cos^3\frac{\pi}{9} - 3\cos\frac{\pi}{9} \text{ (Hint: expand } \cos(2\theta+\theta)) \end{aligned}$$

Find $4A + 4B + 4C + 4D$.

QUESTION 3

Let

$$\begin{aligned} A &= \sqrt{-2} \cdot \sqrt{-8} \\ B &= \text{the real component of } \sum_{a=1}^{100} \frac{a}{i^a} \\ C &= \cfrac{4}{5 + \cfrac{4}{5 + \cfrac{4}{5 + \dots}}} \\ D &= \text{the real component of } \sqrt{e^{7i\pi}} \end{aligned}$$

Find $A + B + 2C + D$.

QUESTION 4

Let

$$\begin{aligned} A &= \text{the sum of the 5th roots of unity} \\ B &= \text{the product of the 4th roots of unity} \\ C &= \text{the product of the 7th roots of 128} \\ D &= \text{the sum of the 6th roots of 64} \end{aligned}$$

Find $A - B + C + D$.

QUESTION 5

Let

A	$=$	the area of triangle ABC when $AB = 7$, $BC = 7$, and $\angle BAC = 30^\circ$
B	$=$	the largest possible diagonal with integer length in a quadrilateral with sides 3, 3, 4, and 4
C	$=$	the length of AB in triangle ABC when $BC = 10$, $\angle BAC = 30^\circ$, and $\angle ACB = 60^\circ$
D	$=$	the distance from Awnish to John in yards, if Siddarth is standing 10 yards from John, and Siddarth can see Awnish 5 yards away if he turns his head by 120°

Find $\sqrt{3}\,(4A + C) + B + D^2$.

QUESTION 6

Consider the function $f(x) = -3\cos(4x - 6) - 3$.

$$\begin{aligned} A &= \text{the period of } f(x) \\ B &= \text{the amplitude of } f(x) \\ C &= \text{the value of } x \text{ such that } f(x) = 0 \text{ on the interval } [0, 2) \\ D &= \text{the sum of the phase shift and the vertical shift of } f(x) \end{aligned}$$

Find $\dfrac{AB}{C+D}$.

QUESTION 7

Consider the parametric equations $x(t) = 9\sec t$ and $y(t) = 4\tan t$. Let

$$\begin{aligned} A &= \text{the length of a latus rectum} \\ B &= \text{the eccentricity} \\ C &= \text{sum of the } x \text{ and } y \text{ coordinates of the center} \end{aligned}$$

Find ABC

QUESTION 8

Let vector I be $\langle 5, 24\rangle$ and vector J be $\langle 7, 12\rangle$. Let

$$\begin{aligned} A &= I \cdot J \\ B &= \text{the sum of the components of } I \times J \\ C &= \|I\| \\ D &= \text{the cosine of the angle between } I \text{ and } J \end{aligned}$$

Find $A + B + C^2 + \dfrac{D\sqrt{772}}{646}$.

QUESTION 9

Let

$$\begin{aligned} A &= f\left(\frac{1}{2013}\right) + f\left(\frac{2}{2013}\right) + \cdots + f\left(\frac{2013}{2013}\right), \text{ given that } f(x) = \frac{4^x}{4^x+2} \\ B &= \text{the value of } \log_3\left(\frac{3^x}{3^2\left(3^x+3^x+3^x\right)}\right) \text{ if } x = 2013 \end{aligned}$$

Find $3A - B$.

QUESTION 10

Let

A = the number of positive 4-digit integers such that the digits are strictly increasing from right to left

B = $f(8)$, given that $f(x)$ is a polynomial that has the values $f(3) = 28$, $f(4) = 44$, $f(5) = 72$, $f(6) = 113$, and $f(7) = 168$

Find $A + B$.

QUESTION 11

Let

$$\begin{aligned} A &= x+y, \text{ if } x \text{ and } y \text{ are real numbers such that } \lfloor x+y \rfloor + y = 22.7 \text{ and } x+y-\lfloor x \rfloor = 4.9 \\ B &= \text{the hypotenuse of a Pythagorean triple with 13 as the shortest side (Hint: notice the following: } (3,4,5), \\ & \quad (5,12,13),\ (7,24,25),\ (9,40,41), \ldots) \end{aligned}$$

Find $A+B$.

QUESTION 12

For the following question, let the answer of each part be 1 if the graph of a cardioid, 2 if it is a lemniscate, 3 if it is a rose, and 4 if it is none of the above.

$$\begin{aligned} A &: \quad r = e + e\cos\theta \\ B &: \quad r = 2a\cos\theta \\ C &: \quad r^2 = 9\cos 2\theta \\ D &: \quad r = \cos\theta \\ E &: \quad 2r = 1 + \sin\theta \end{aligned}$$

Find $A + B + C + D + E$.

QUESTION 13

Let

$$\begin{aligned} A &= \sin^4 \frac{\pi}{12} - \cos^4 \frac{\pi}{12} \\ B &= \sin^4 \frac{7\pi}{12} + \cos^4 \frac{7\pi}{12} \\ C &= \text{the value } a \text{ such that } \frac{\sin^3 x + \cos^3 x}{5\sin x + 5\cos x} = \frac{1}{5} - a\sin 2x \\ D &= \sin^6 \frac{\pi}{12} + \cos^6 \frac{\pi}{12} \end{aligned}$$

Find $\dfrac{10A}{\sqrt{3}} + 80B + 100C + 16D$.

QUESTION 14

Let

$$\begin{aligned} A &= r_1^3 + r_2^3 + r_3^3, \text{ if } r_1,\ r_2, \text{ and } r_3 \text{ are the roots of } x^3 - x + 2 \\ B &= \text{the minimum possible value of } \frac{50b}{a} + \frac{140c}{b} + \frac{49a}{c} \text{ where } a,\ b,\ c \text{ are positive integers} \\ C &= \text{the number of positive integers relatively prime to 100 that are less than 100} \end{aligned}$$

Find $A + B + C$.